DETERMINATION OF ISOACCELERATION LINES BY SLIDING AND OVERTURNING OF OBJECTS

by

Jai Krishna*, Anand S.Arya* & Krishen Kumar*

SYNOPSIS

An attempt has been made to solve the problem of determining peak ground acceleration near the epicentre and its attenuation with distance, in the absence of strong motion instruments, by studying the sliding and overturning behaviour of thousands of small household and other objects during an earthquake. It is shown that the lower bound values of maximum ground acceleration are definitely determined in this way and if a large number of observations are available at a place, which is usually the case, reasonable estimates to actual values are possible. The information obtained on 900 objects after the Broach earthquake of 1970 has been utilised to draw an "isoacceleration" diagram of the area. The iso-acceleration lines are compared with isoseismals as well as the tectonic features. The acceleration attenuation curves are compared with those determined by accelerographs in Parkfield and San Fernando earthquakes which show similar trends. Thus much needed information for design purposes regarding accelerations can be obtained at small cost.

INTRODUCTION

How to obtain data on peak ground acceleration near the epicentre and its attenuation with distance, in the absence of strong motion instruments arranged specifically for the purpose, is a common problem. Even in well instrumented areas,instruments spaced close enough are not available which would give the attenuation of the ground motion from the epicentre in all directions. Yet for design purposes, such information is of basic importance. As seen from results published by Hershberger (1), the Modified Mercalli Intensity of any value corresponds to too wide a range of accelerations and does not lead to acceptable design criteria. Since very close network of accelerographs is not possible except in limited areas due to prohibitive cost, it appears that the problem will stay for all time in most countries and an alternative solution is necessary. With this objective in mind, attention has been paid to the earthquake behaviour of thousands of small household objects and others which either slide or overturn or donot move at all. Such information becomes available in abundance after a moderate or severe earthquake and, if scientifically interpreted, can give extremely useful results at small cost. The authors have made such studies after the Koyna earthquake of Dec. 11, 1967 (2) and Broach earthquake of March 23, 1970 (3) in India. This paper presents the results of such observations, their interpretation and scientific evaluation. Particular reference is made here to the Broach earthquake. Data obtained from Koyna earthquake was presented earlier (4).

* School of Research & Training in Earthquake Engineering, University of Roorkee, Roorkee, U.P. INDIA

BASIC CONCEPTS

For determining the peak ground acceleration at a place several observations of movements and non - movements are required. In the case of movement of an object, the force applied by the earthquake must be more than the minimum force required to cause motion. On the contrary the force required to move an object which did not move during the earthquake will give an upper limit on the earthquake acceleration. Thus by determining the forces in the two limits, lower and upper bounds on the peak ground acceleration are obtained. If a number of observations are possible at a place (village or town) the range between the upper and lower bounds gets narrowed and the peak ground acceleration is estimated with sufficient accuracy.

There are several factors which affect the mechanism of overturning or sliding of objects. The earthquake motion may be considered to have two mutually perpendicular components and a vertical component. The overturning of an object will depend upon that horizontal component against which the stability is the least including the effect of vertical component. It is likely that the component in the vertical direction decreases with distance from the epicenter at a different rate compared to the decrease in horizontal component and that the frequency pattern of the accelerations may also change. If the effect of this variation is considered to be small, a constant ratio of the vertical to the horizontal component may be assumed. Where an accelerogram of the earthquake is available in the epicentral area, the critical combination of vertical and horizontal components may be worked out for worst effect on overturning and sliding motions and used for evaluating the observed data.

Other factors which affect the results depend upon the location of the observed object. Objects on the ground floor resting on rigid supports will experience the ground motion but those on higher elevations on first or higher floors, or on relatively less rigid supports, will experience an amplified motion due to the motion of the supporting member itself relative to the ground. Moreover some objects may be more favourably placed than the others for their movement to occur. Then small amount of sliding may escape observation whereas for overturning of an object the chances of wrong information are not high. Therefore care has to be exercised in selecting an object for determination of peak ground acceleration from its reported behaviour during the earthquake.

CONDITIONS FOR SLIDING AND OVERTURNING

The parameters entering the calculations for sliding and overturning of rigid objects are the coefficient of friction between the contact surfaces and the shape of the object, and the earthquake motion.

a) Sliding of Objects: A rigid object resting on even horizontal surface, is shown in Fig. 1 with the forces marked on it. For sliding motion, the horizontal inertia force must exceed the frictional force, that is,

$$mk_h\, g > m(1-k_v)g$$

$$\text{or} \quad \frac{k_h}{1-k_v} > \mu \qquad \text{.... (1)}$$

where μ is the coefficient of friction, k_h and k_v are acceleration coefficients of horizontal and vertical components respectively, m is the mass of the object and g is acceleration due to gravity.

b) Overturning of Objects: For overturning motion, the moment of the horizontal inertia force about base must exceed the stabilizing moment due to modified weight under the vertical acceleration of the ground. From the forces marked in Fig. 2, this condition can be expressed as:

$$mk_h gh > m(1-k_v)ga$$

$$\text{or} \quad \frac{k_h}{1-k_v} > \left(\frac{a}{h}\right) \qquad \text{.... (2)}$$

From Eq. (1) and (2) it may be noted that the smaller of the two quantities μ and (a/h) will be the governing condition. That is if $\mu < (a/h)$ the object would slide and if $\mu > (a/h)$ the object would overturn provided of course that $k_h / (1-k_v)$ exceeds either μ or (a/h) as the case may be.

c) Effect of Vertical Acceleration: As indicated by Eq. (1) and (2), the horizontal acceleration required to cause overturning or sliding will be reduced due to simultaneous action of downward vertical motion of the ground. At a place where an accelerogram is available, the critical combination of the two accelerations can be ascertained for defining the sliding and overturning of rigid objects. For example, for Koyna and El Centro earthquakes, the following results were obtained by scanning their vertical and horizontal accelerograms:

	Koyna (Long.)	El Centro (NS)
Factor $[k_h/(1-k_v)]_{max}$	0.684	0.329
Corresponding horizontal acceleration coefficient, k_h	0.542	0.319
Corresponding vertical acceleration coefficient, k_v	0.207	0.029
Acceleration ratio k_v/k_h	0.382	0.091
Factor $(k_h)_{max}$ as recorded	0.630	0.319

Although the factor $[k_h/(1-k_v)]_{max}$ will always be larger than $(k_h)_{max}$, but the difference between them is expected to be small (as seen above for both earthquakes) unless a large positive peak of the vertical component happens to coincide with the largest peak of the horizontal acceleration component. Therefore for practical purposes, effect of k_v may be neglected in determining $(k_h)_{max}$.

DYNAMIC ANALYSIS OF FRICTION SUPPORTED MASSES

The overturning or extent of sliding displacement of a rigid object under earthquake motion would not only depend on the maximum acceleration but also the duration of the acceleration peaks. For establishing the scope and limitations of Eq. (1) and (2) in predicting the peak ground acceleration, a dynamic analysis is necessary using base motion as input data. The equations of motion, considering one horizontal and one vertical component of ground acceleration acting simultaneously, may be written as follows with the notations shown in Fig. 3 and 4.

For sliding

$$\dot{v} + \phi(v) = -\ddot{y} \qquad \text{.... (3)}$$

$$\left.\begin{array}{lll} \text{where} & \phi(v) = \mu(g + \ddot{u})\{\text{sign}(v)\} & \text{when } v \neq 0 \\ \text{or} & \phi(v) = -\ddot{y} & \text{when } v = 0 \end{array}\right\} \qquad \text{.... (4)}$$

$$\text{and} \quad v = x - y; \quad \dot{v} = \dot{x} - \dot{y} \qquad \text{.... (5)}$$

For overturning

$$I\ddot{\theta} + m(g + \ddot{u})(a\cos\theta - h\sin\theta) = m\ddot{y}(a\sin\theta + h\cos\theta) \text{.. (6)}$$

In Eq. (6) whenever θ changes sign there is an impact with reduction in velocity so that

$$\dot{\theta}_{(\text{after impact})} = -\vartheta\, \dot{\theta}_{(\text{before impact})} \qquad \text{.... (7)}$$

where the coefficient of elastic restilution, ϑ, has to be determined experimentally and is a function of the velocity $\dot{\theta}$ itself besides the nature of the surface of the object and the surface on which it rests.

Results from Eq. (3) and (6) for several pulses as well as Koyna accelerogram are described below:

a) Sliding of Objects: For single rectangular and triangular pulses, the sliding motion gives closed-form solutions as shown in Fig. 5. Taking a single triangular pulse from an accelerogram for the sake of understanding, the sliding movement is computed for two values of time base, 0.05 and 0.10 sec, two coefficients of friction, 0.2 and 0.3 and several values of acceleration peak as shown in Table 1. It is noted that the extent of sliding is a function of the square of the duration t_o provided of course that peak acceleration, Ag, exceeds μg. It is also seen that the sliding movement is very small when A exceeds μ slightly. For instance when A = 0.3, μ = 0.2, and t_o = 0.05 sec, the movement will only be 0.11mm. This movement is too small to be noticeable and the object may be taken as if it did not slide. Thus the fact of sliding gives a lower bound but upper bound may actually be higher than observed.

Sliding movements of objects with various friction coefficients for Koyna earthquake (Long. Comp.) are shown in Table 2. Here also it is seen that as μ approaches $A_{max} = 0.63$, the extent of movement goes on decreasing and the movement may be difficult to observe in the field for $\mu > 0.5$. Therefore the determination of minimum value of upper bound will require very careful observation.

b) <u>Overturning of Objects</u>: The overturning movement of a rectangular prism 40mm x 40mm x 200mm high having $a/h = 0.2$ was computed from Eq. (3) by applying a single triangular pulse having peak acceleration coefficient A and time base t_o. The object would start tilting about one edge of base as soon as the acceleration coefficient exceeded 0.2, but it would fall back to its vertical equilibrium position unless the area under the pulse was large enough to tilt the object to an unstable equilibrium position. For $t_o = 0.05$ sec, the overturning of the prism occurred when A exceeded 1.15 . For $t_o = 0.1$ sec, A would be 0.67 . The overturning oscillations of the same prism were computed by using Eq. (3) and (6) for Koyna earthquake (Long. and vert. Comps.). The results are plotted in Fig. 6 for $\vartheta = 1.0$, 0.9 and 0.7. A value of $\vartheta = 1.0$ means no energy loss in the impact when velocity changes sign. It is seen that the prism overturns in the earthquake with $A_{max} = 0.63$ in the case when $\vartheta \geq 0.9$ but does not overturn when $\vartheta = 0.7$. Thus the fact of overturning gives a definite lower bound but the absence of it may give an upper bound which is too low and the actual upper bound may be much higher.

From the dynamic analysis of sliding and overturning of objects, it turns out that the peak ground accelerations determined by the static approach, Eq. (1) and (2), will definitely represent the minimum that occurred during the earthquake. For achieving closeness of the computed accelerations to the actual value, as many observations must be taken at a given site as possible.

EXPERIMENTAL OBSERVATIONS

Solution of Eq. (1) and (2) for estimating $(k_h)_{max}$ requires a knowledge of the properties μ and (a/h). These properties are experimentally determined for the rigid objects. An index map of Broach region is shown in Fig. 7 and the shapes of some typical objects studied are shown in Fig. 8.

For collecting data during Broach earthquake, more than 1200 objects were examined in an area of about 20 km radius around the epicentre at Broach and after scanning through their appropriateness, about 900 objects were selected for experimental observations.

A few of these observations giving the narrowest limits on the horizontal ground acceleration are presented in Table 3. The placing of the observed object is also given; k_s and k_o are coefficients of horizontal ground acceleration corresponding to sliding and overturning respectively. The effect of vertical ground acceleration is neglected.

ISOACCELERATION MAP

The maximum ground accelerations at various places obtained from analysis of rigid objects, such as listed in Table 3, have been utilised to prepare an isoacceleration map of the area as explained below:

Since only the limits on the maximum acceleration at any place are obtained, the probable value of maximum ground acceleration cannot be directly predicted. Therefore a reasonable estimate is obtained on the assumption that the decrease of seismic intensity would be more or less gradual rather than abrupt. The curves of gradually reducing ground accelerations with distance from the epicenter are drawn passing within the acceleration bounds along various directions which are first marked by straight lines on plan radiating out from a point near the epicenter (Broach) so that each passes near to as many places of observation as possible (Fig.9). The observation points are transferred to the nearer radial line by projecting them, as a first approximation, on circular arcs with Broach as center. The limits for the peak ground acceleration at each place are marked on the cross-sections by arrow-heads and a smooth curve is fitted to remove small discrepancies. Two such cross-sections are shown in Fig. 10 and 11 for lines 7 and 8 respectively. Other sections are not included here. From these curves it is easy to measure the distances in plan corresponding to any desired value of acceleration, and plotting these points on the corresponding radial lines in plan . Since the correct projecting arc in plan should be parallel to the acceleration contours, the cross-sections based on earlier projections with circular arcs need adjustment. The final accelerations in sections and plan are arrived at by trials. Finally the contours of equal acceleration or the isoacceleration lines are constructed giving the map shown in Fig. 12. In the map, isoseismals base on intensity observations (5) are also shown for comparison.

CORRELATION OF RESULTS

<u>Isoacceleration map and Seismotectonics</u>: The general trend of the isoacceleration lines indicates their elongation along the course of river Narmada in the ENE-WSW direction which represents slower attenuation of ground acceleration in this direction, a phenomenon indicative of the direction of fault from which the earthquake originated. South of Broach the existence of an ENE-WSW fault is confirmed by exploration surveys carried out by Oil and Natural Gas Commission of India (Fig. 7 and 12). Tectonic movements along the Narmada fault are therefore the most probable cause of the Broach earthquake.

The ground acceleration attenuation curves along and transverse to the course of river Narmada are shown in Fig. 13. For comparison, the attenuations available for three other earthquakes - Koyna (4), Parkfield (6) and San Fernando (7) are also given. Two empirical relationships (1,8) are also plotted for the data related to the Broach earthquake. It is seen that the attenuation in both the directions is contained within the range of the Koyna and Parkfield earthquakes. Now Koyna earthquake occurred where the basaltic trap deposits are very dense with wave velocity of 6 km/sec and the absorption of energy will naturally be slow. On the other hand Parkfield earthquake had a shallow focus and occurred in an area consisting of alluvium of unknown depth having wave velocities 2.8 to 4.0 km/sec in the upper four km depth. Perhaps the attenuations in these earthquakes could be taken as the limits of attenuation curves.

Comparison with Isoseismal map: The isoseismals have an E-W trend roughly fitting with the orientation of the Narmada fault, and like the isoacceleration lines, corroborate with the tectonic features of the region. However, the intersection of an isoseismal with several isoacceleration lines confirms considerable variation of peak ground acceleration in a given isoseismal.

CONCLUSIONS

1) The large amount of data available on the sliding and overturning response of house-hold and other objects in an earthquake can be usefully utilized in estimating peak ground accelerations.

2) Reasonably accurate determination of the acceleration attenuation curves in all directions is made possible by this method.

3) The accelerations determined in this manner actually represent the lower bound on maximum ground accelerations and should be looked upon as the probable minimum values.

4) Pattern of the isoacceleration lines is likely to reveal the tectonic features of the site.

5) Attenuation of peak ground accelerations away from the causative fault is obtained at small cost which is not possible to obtain through instrumentation alone.

REFERENCES

1. Hershberger J., "A comparison of Earthquake Accelerations with Intensity Ratings", Bull. Seism. Soc. of Am., Vol 46, No. 4, Oct. 1956, pp. 317 - 320.

2. Krishna, J., Arya A.S. and Kumar K., "Distribution of Maximum Intensity of Force in the Koyna Earthquake of Dec. 11, 1967", Earthquake Engineering Studies, School of Res. & Trg. in Earthquake Engg., University of Roorkee, Aug. 1969.

TABLE 1				
SLIDING MOVEMENT UNDER SINGLE TRIANGULAR PULSE				
A	Displacement (mm)			
	t_o = .05 Sec		t_o = .10 Sec	
	μ = 0.2	μ = 0.3	μ = 0.2	μ = 0.3
.3	0.11	0	0.44	0
.4	0.60	0.02	2.40	0.08
.5	1.32	0.38	5.28	1.52
.6	2.44	0.90	9.76	3.60
.8	5.27	2.54	21.08	10.16

A = Peak acceleration coefficient

μ = Coefficient of friction

t = Duration of the pulse

TABLE 2

SLIDING DISPLACEMENTS UNDER KOYNA EARTHQUAKE (LONG. COMP.)

μ	Maxm. disp. (mm)		Residual disp. (mm)	
	Horz. ground motion	Horz.+ Vert. ground motion	Horz. ground motion	Horz.+ Vert. ground motion
0.1	23.66	38.14	-.885	25.04
0.2	7.07	24.84	5.66	22.28
0.3	1.25	9.75	1.07	9.57
0.4	0.34	2.13	0.34	2.13
0.5	0.064	0.36	0.064	0.36
0.6	0.0002	0.01	0.0002	0.0005

TABLE 3

LIMITING ACCELERATION COEFFICIENTS FROM OBSERVED SLIDING AND OVERTURNING AT SOME OF THE PLACES

S.No.	Place	Distance from Broach (km)	Object	Action	k_s or k_o	Place of object
1	Dehgam	6	Glass bottle	Overturned	0.18+	Concrete shelf at 1.95 m
			Earthen water pots	Not overturned	0.21-	Concrete shelf at 0.9 m
2	Bhorbhata	4	Earthen water pots	Overturned	0.23+	Concrete shelf at 0.75 m
			Earthen water pots	Not overturned	0.31-	Floor (Plinth level)
3	Hinglot	10	Earthen water pots	Overturned	0.27+	Wall 0.7 m high
			Glass bottle	Not overturned	0.29-	Wood shelf 1.7 m high
4.	Dhanturia	13	Earthen water pots	Overturned	0.20+	Wall 1.0 m high
5	Kaswa	18	Metal plate	Overturned	0.18+	Wood shelf at 2.07 m
			Earthen water pots	Not overturned	0.23-	Wall 0.8 m high

Note: '+' sign means lower bound and '-' sign means upper bound.

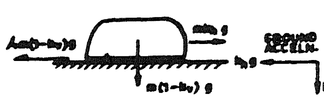


Fig.1_Forces at threshold of sliding

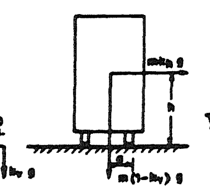


Fig.2_Forces at threshold of overturning

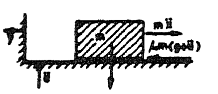


Fig.3_Forces on object in sliding motion

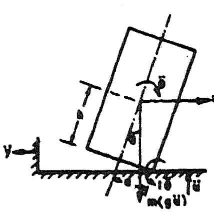


Fig.4_Forces on object in overturning motion

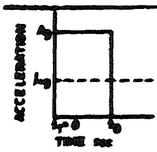
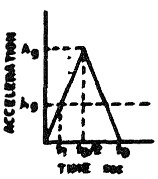
Pulse	Relative velocity, v	Relative Displacement, u
	(i) $0 < t \leq t_o$ $v_1 = (A - \mu) g t$	$u_1 = \frac{1}{2}(A - \mu) g t^2$
	(ii) $t > t_o$ $v_2 = g(A t_o - \mu t)$	$u_{max} = \frac{g}{2\mu} A^2 t_o^2 (1 - \mu/A)$
	(i) $t_1 < t \leq t_o/2$ $v_1 = \mu g(t - \frac{\mu t_o}{4A}) - Agt^2/t_o$	$u_1 = \frac{\mu^3 g t_o^2}{24A^2} - \mu g(\frac{\mu t_o t}{4A} - \frac{t^2}{2}) - \frac{Agt^3}{3t_o}$
	(ii) $t_o/2 \leq t \leq t_o$ $v_2 = v_1 + Ag(\frac{t_o}{2} - 2t + 2t^2/t_o)$	$u_2 = u_1 - Ag(\frac{t_o^2}{12} - \frac{t_o t}{2} + t^2 - \frac{2t^3}{3t_o})$
	(iii) If motion continues after t_o $v_3 = v_o - \mu g(t - t_o)$ where v_o = velocity at $t = t_o$	$u_{max} = u_o + \frac{v_o^2}{2\mu g}$ where u_o = u at $t = t_o$

Fig. 5 Maximum Displacements under Rectangular and Triangular Pulses

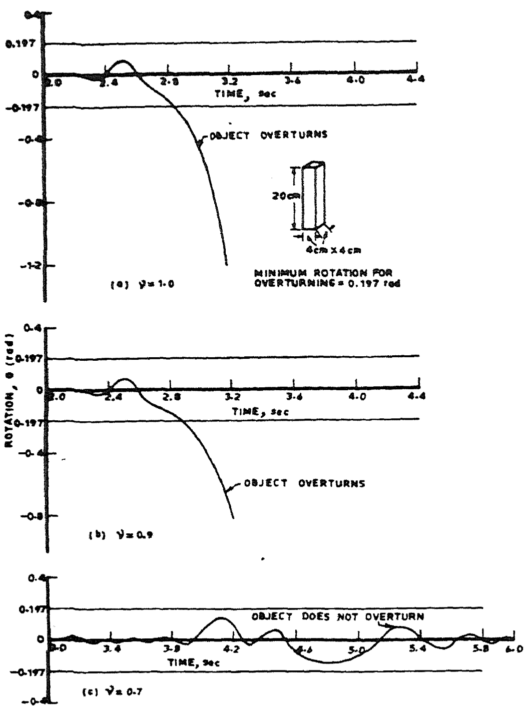


FIG. 6 _OVERTURNING RESPONSES FOR KOYNA EARTHQUAKE LONGITUDINAL AND VERTICAL MOTIONS COMBINED

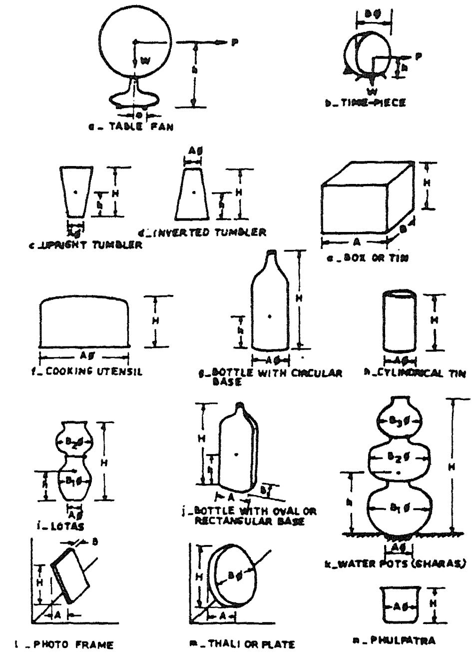


FIG. 8 _OBJECTS OBSERVED FOR OVERTURNING AND SLIDING MOTION

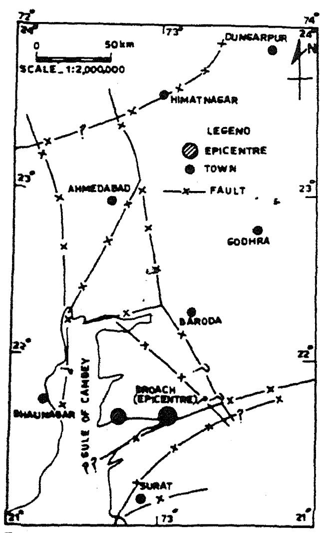


FIG. 7 _INDEX MAP SHOWING FAULTS & THE EPICENTRE OF THE MARCH 23,1970 EARTHQUAKE

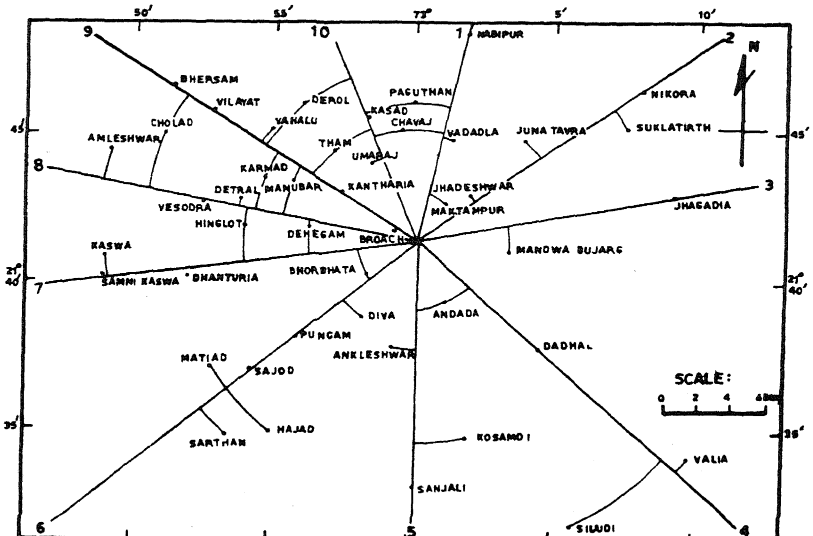


FIG. 9 _RADIAL DIRECTIONS 1 TO 10 CHOSEN FOR CONSTRUCTING CROSS-SECTIONS OF VARIATION OF GROUND ACCELERATION SHOWING PROJECTIONS OF OBSERVATION STATIONS ON THEM BY CIRCULAR ARCS AS FIRST APPROXIMATION

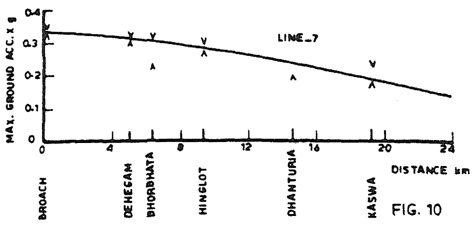


FIG. 10

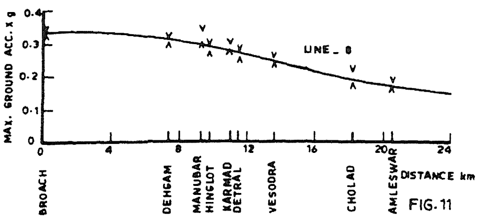


FIG. 11

ATTENUATION OF GROUND ACCELERATION ALONG RADIAL LINE 7 AND 8

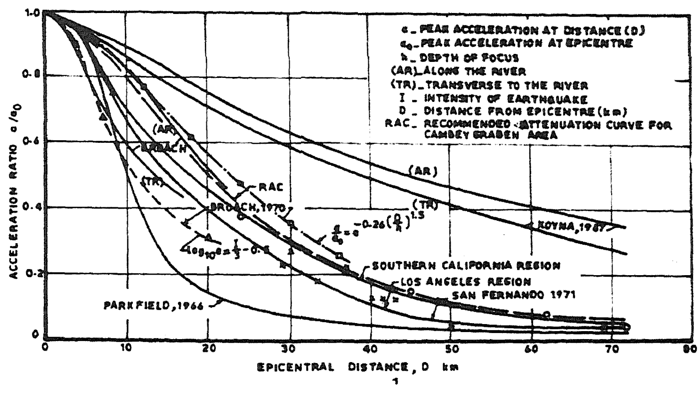


FIG. 13 _ATTENUATION OF GROUND ACCELERATION OBSERVED IN VARIOUS EARTHQUAKES AND COMPUTED BY EMPIRICAL FORMULAE

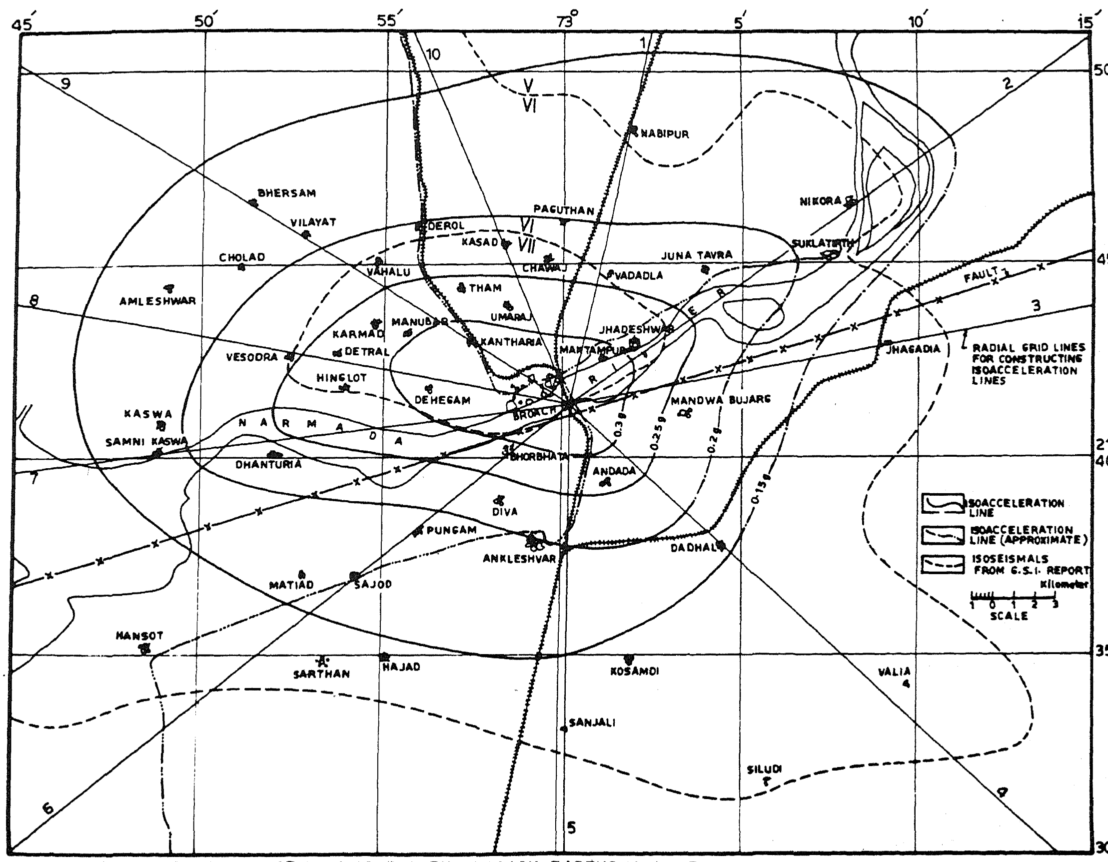


FIG. 12 _ISOACCELERATION MAP FOR THE BROACH EARTHQUAKE OF MARCH 23 1970