EARTHQUAKE RESPONSE ANALYSIS AND DATA PROCESSING BY THE FOURIER TRANSFORM

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SYNOPSIS

When the input mechanism of earthquake energy to a structure is considered, it is to be assumed that the earthquake motions are transmitted through some transfer media, the transfer functions of which are generally expressed as functions of frequency. In this system, it is not easy to analyze the response of a structure to random motions by a direct time-integration method such as the linear acceleration method or the Runge-Kutta method.

The method presented here is to expand the original random motions into Fourier series, and responses to all individual harmonic components are calculated. The total structural response is then given as the sum of component responses. It was confirmed that the earthquake response of a system having any complex transfer functions could be computed with good accuracy and economical speed using the Fast Fourier Transform (FFT).

This Fourier technique can be effectively applied to the data processing of earthquake records. Many earthquakes have been recorded in displacement from older times in Japan. These precious records have been little used for the engineering purposes since they are the records of displacement. It is well-known that accelerations obtained from displacement records by means of numerical differentiation are erroneous that they are unsuitable for any practical use. However, in the present case, results are much better. An acceleration time-history is obtained as a sum of all component second derivatives of the original series which was fit to the original displacement record. For example, an acceleration curve thus obtained is shown in Fig. 2, together with the original displacement record which was obtained during the main shock of Kanto-earthquake at Hongo, Tokyo in 1923.

1. RESPONSE ANALYSIS OF SINGLE DEGREE OF FREEDOM SYSTEM BY THE FOURIER TRANSFORM METHOD

It is well-known that the response analysis of any complex structure can be reduced to a problem of single degree of freedom system on the basis of the modal analysis. The acceleration function of earthquake motion $\ddot{Z}(t)$ can be expanded into the Fourier series

$$\ddot{Z}(t)=\frac{a_o}{2}+\sum_{n=1}^{\infty}\left\{ a_n \cos\frac{2n\pi}{T}t + b_n \sin\frac{2n\pi}{T}t \right\} \qquad (1)$$

where

$$a_n=\frac{2}{T}\int_o^T \ddot{Z}(t)\cos\frac{2n\pi}{T}tdt \quad , \quad b_n=\frac{2}{T}\int_o^T \ddot{Z}(t)\sin\frac{2n\pi}{T}tdt \qquad (2)$$

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The equation of motion for each frequency component is written as

$$\ddot{x}_n + 2h\omega_o\dot{x}_n + \omega_o^2 x_n = -(a_n \cos\omega_n t + b_n \sin\omega_n t) \tag{3}$$

where x_n is the displacement of mass for the frequency component ω_n, h is the damping ratio, ω_o is the undamped natural circular frequency of the system, and $\omega_n = 2\pi n/T$. The general solution of Eq (3) is

$$x_n = x'_n + x''_n \tag{4}$$

where x'_n is the complementary function, x''_n is the particular solution and they are expressed respectively as follows.

$$x'_n = e^{-h\omega_o t}(A_n \cos\sqrt{1-h^2}\ \omega_o t + B_n \sin\sqrt{1-h^2}\ \omega_o t) \tag{5}$$

$$x''_n = -\frac{a_n}{\omega_o^2}\frac{\cos(\omega_n t - \phi_n)}{\sqrt{(1-r_n^2)^2 + 4h^2 r_n^2}} - \frac{b_n}{\omega_o^2}\frac{\sin(\omega_n t - \phi_n)}{\sqrt{(1-r_n^2)^2 + 4h^2 r_n^2}} \tag{6}$$

where

$$r_n = \frac{\omega_n}{\omega_o}, \quad \tan\phi_n = \frac{2hr_n}{1-r_n^2} \tag{7}$$

The arbitrary constants A_n and B_n of the transient term can be determined by setting $x_n = \dot{x}_n = 0$ at $t = 0$ as the initial conditions. The transient term $g_1(t)$ and the steady state term $g_2(t)$ of the response can be rearranged respectively in a form of the Fourier series as follows.

$$g_1(t) = \frac{a'_o}{2} + \sum_{n=1}^{\infty}\{a'_n \cos\omega_n t + b'_n \sin\omega_n t\} \tag{8}$$

$$g_2(t) = \frac{a''_o}{2} + \sum_{n=1}^{\infty}\{a''_n \cos\omega_n t + b''_n \sin\omega_n t\} \tag{9}$$

in which the constants a'_n and b'_n are derived by

$$a'_n = \frac{2}{T}\int_o^T(\sum_{n=0}^{\infty} x'_n)\cos\omega_n t\,dt, \quad b'_n = \frac{2}{T}\int_o^T(\sum_{n=0}^{\infty} x'_n)\sin\omega_n t\,dt \tag{10}$$

and a''_n and b''_n are determined from substituting Eq (7) into Eq (6). Thus the response of the mass x(t) is expressed in the Fourier series form as

$$x(t) = \frac{a'_o + a''_o}{2} + \sum_{n=1}^{\infty}\{(a'_n + a''_n)\cos\omega_n t + (b'_n + b''_n)\sin\omega_n t\} \tag{11}$$

If a frequency dependent transfer function $\psi(\omega)$ is taken into consideration before the given earthquake signal reaching to the base of the system, the response of the mass may be calculated by

$$x(t) = \frac{a'_o + a''_o}{2} + \sum_{n=1}^{\infty}\psi(\omega_n)\{(a'_n + a''_n)\cos\omega_n t + (b'_n + b''_n)\sin\omega_n t\} \tag{12}$$

2. DATA PROCESSING BY THE FOURIER TRANSFORM

A random motion f(t) can be expressed in the form

$$f(t) = \frac{A_o}{2} + \sum_{n=1}^{\infty}\{A_n \cos\omega_n t + B_n \sin\omega_n t\} \tag{13}$$

where

$$A_n = \frac{2}{T}\int_o^T f(t)\cos\omega_n t dt \quad , \quad B_n = \frac{2}{T}\int_o^T f(t)\sin\omega_n t dt \tag{14}$$

(1) Differentiation: Time derivative of Eq (13) makes

$$f'(t) = \frac{df(t)}{dt} = \sum_{n=1}^{\infty} \{ A'_n \cos\omega_n t + B'_n \sin\omega_n t \} \tag{15}$$

where

$$A'_n = \omega_n B_n \quad , \quad B'_n = -\omega_n A_n \tag{16}$$

(2) Integration: Time integration of Eq (13) yields

$$F(t) = \int f(t)dt = \frac{A''_o}{2} + \sum_{n=1}^{\infty} \{ A''_n \cos\omega_n t + B''_n \sin\omega_n t \} \tag{17}$$

Setting F(t) = 0 at t = 0 as an initial condition, the constants are determined as

$$A'_n = -\frac{B_n}{\omega_n} \quad , \quad B''_n = \frac{A_n}{\omega_n} - \frac{A_o T}{2n\pi} \quad , \quad A'_o = \frac{A_o}{2} T + 2\sum_{n=1}^{\infty}\frac{B_n}{\omega_n} \tag{18}$$

(3) Filter processing: The frequency contents higher than ω_{nc} are filtered out by

$$f(t) = \frac{A_o}{2} + \sum_{n=1}^{nc} \{ A_n \cos\omega_n t + B_n \sin\omega_n t \} \tag{19}$$

The frequency contents lower than ω_{nc} are filtered out by

$$f(t) = \sum_{n=nc}^{\infty} \{ A_n \cos\omega_n t + B_n \sin\omega_n t \} \tag{20}$$

3. NUMERICAL EXAMPLES

(1) Comparison of the responses calculated by the direct time-integration and the Fourier transform:

In order to investigate the applicability of the present method, sample calculations were made for various conditions in comparison with those obtained from the linear acceleration method. The comparisons are shown in Table 1, and the responses corresponding to the case III are shown in Fig. 1. As a result, it was found that the present method could be satisfactorily applied to the response analyses of structures.

(2) Acceleration record of the Kanto-earthquake in 1923:

As a sample application of the data processing, the acceleration curve of the Kanto-earthquake in 1923 was computed from the E-W component of the displacement record obtained at Hongo in Tokyo during the main shock of the earthquake. The acceleration record thus computed is shown in Fig. 2 in parallel with the displacement record. The maximum acceleration is assumed to be 238 gals. For the reference, the acceleration response spectrum of the record is shown in Fig. 3 compared with that of the major after shock of the earthquake.

REFERENCES

1. W. T. Cochran, J. W. Cooley et al "What is the Fast Fourier Transform?", Tran. IEEE. AU-15 No. 2, pp. 45-79 (1967)
2. T. Itoh and H. Yamahara "Earthquake Response Analysis of Structure by the Fourier Transform", Tran. A. I. J. No. 190, pp. 35-41 (1971)

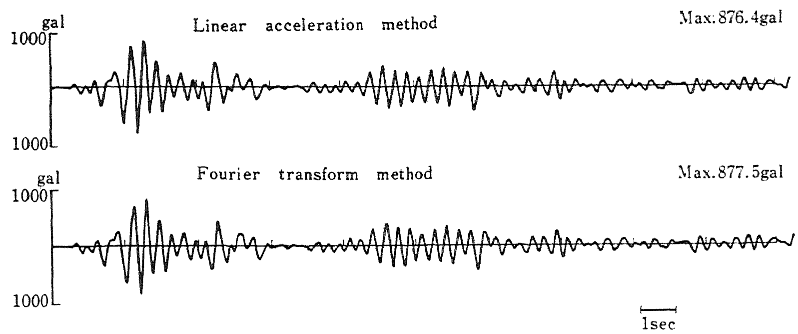

Fig. 1 Comparison between the responses (Case III in Table 1) obtained from the linear acceleration and the Fourier transform methods

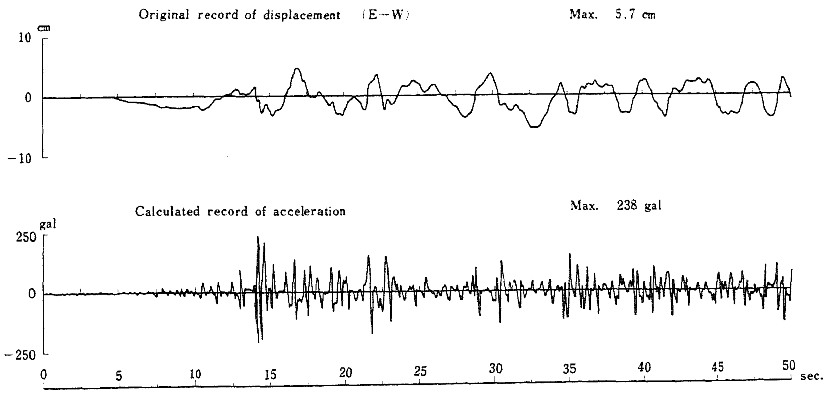

Fig. 2 Accelerations presumed from the displacement record obtained during the main shock of the Kanto-earthquake Sep. 1st, 1923

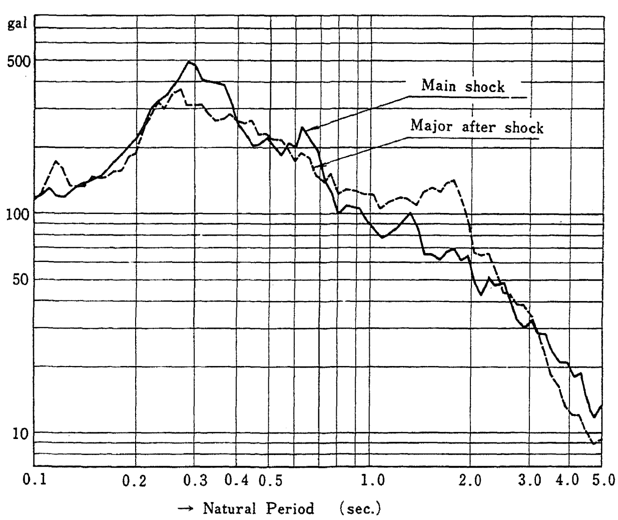

Fig. 3 Acceleration response spectra of the Kanto-earthquake 1923 (Maximum acceleration of the earthquakes are normalized to be 100 gals, h = 0.05)

CASE	Natural freq. Hz	Damping ratio	Method of analysis*	Number of sample	Max. accn gal
I	1	0.05	B		481
			A	1024	478
				2048	482
II	3	0.01	B		1225
			A	1024	1231
				2048	1230
III	3	0.05	B		876
			A	1024	877
				2048	878
IV	3	0.10	B		646
			A	1024	647
				2048	646
V	5	0.05	B		1002
			A	1024	1003
				2048	1003

*A: The Fourier transform method
B: The linear acceleration method

Table 1
Comparison between the maximum responses of a single degree of freedom system obtained from the linear acceleration and the Fourier transform methods