Experimental Study on Low Cycle Fatigue of a Structural Member Subjected to Earthquake Loads

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Synopsis

Elastoplastic responses of a steel frame idealized to be a one degree of freedom system subjected to the stationary random disturbances were calculated by an analog computer, and then the uniaxial elastoplastic displacement associated with the above-obtained responses was applied to thin walled cylinders of structural steel by means of an electrohydraulic fatigue testing machine. $\Delta\varepsilon N^{0.346}=0.0624$ was obtained in the constant amplitude tests. The average value of $\Sigma(n_i/N_i)$ was 1.51 for the elastic responses and 1.89 for the elastoplastic responses. Failure due to the elastoplastic response was found to be between static and cyclic.

Introduction

Failure of structures subjected to earthquake is considered to be due to random low-cycle fatigue during elastoplastic vibrations. Then, in this study the elastoplastic displacement associated with the earthquake responses of a steel frame was applied to thin walled cylinders of structural steel by means of an electrohydraulic fatigue testing machine. The total number of cycles to fracture, cummulative damage theories, etc. were discussed by using the amplitude-mean value occurrence distributions and the constant amplitude fatigue test results.

Earthquake Response

The equation of motion of a steel structure idealized to be a single degree of freedom system as shown in Fig.1 can be written in the form

$$m\ddot{x} + Q(x) = -m\ddot{x}_0 \qquad \cdots\cdots(1)$$

in which m=the mass of the structure, Q(x)=the stiffness force, $\ddot{x}_0$=the exciting acceleration of the ground, x=the displacement relative to the ground. The relationship between the stiffness force and the displacement of the structure is assumed to be fully elastoplastic, and the yield stiffness force Q_y occurs when the extreme fiber strain at the critical section attains the yield strain ε_y as shown in Fig.2. If the top and the bottom of the column remain in the vertical direction during vibration, the relation between the yield stiffness force Q_y and the bending moment M_y develops at eight sections labeled A, such that $QH=4M_y$, and $Q_y=24EI\delta_y/H^3$ and also $M_y=2EI\varepsilon_y/d$, therefore $\varepsilon_y=3d\delta_y/H^2$ $\cdots\cdots(2)$
in which k=the spring constant, E=Young's modulus, I=the moment of inertia of the cross section of the column, d=depth of the column, H=height of structure, and δ_y=yield relative displacement. In the response analysis, the following data were used: $p^2=k/m=100$ rad/sec: the elastic circular frequency of the structure or $T=2\pi/p=0.628$sec: the natural period of the structure, $\varepsilon_y=0.02$ from the static tension test of the specimen, and δ_y = 2cm. Hence, the yield base shear coefficient $C_y=Q_y/mg=k\delta_y/mg=0.2$. As the external disturbance of $\ddot{x}_0$ the band limited white noise generated by the white noise generator and cut by a filter was adopted. Its frequency range was chosen to be 0.15∿9.6Hz in relation to 1.6Hz of the natural frequency of the structure and its maximum amplitude was chosen to be 240 gal and

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300 gal so as to break the specimen by low cycle fatigue. Three shapes of external disturbances were used in the elastic analysis. One of the three shapes was used in the elastoplastic analysis. The power spectra of these three disturbances of actual length used in each analysis are shown in Fig.3. The following responses were calculated by the analog computer:

(a) Elastic Response:(i)No.1 Disturbance of 215 gal, (ii)No.2 Disturbance of 215 gal, (iii)No.3 Disturbance of 215 gal, (iv)No.3 Disturbance of 190 gal, (v)No.3 Disturbance of 120 gal

(b) Elastoplastic Response:(i)No.3 Disturbance of 240 gal and (ii)No.3 Disturbance of 300 gal

Responses for the cases (a)(iii) and (b)(ii) are shown in Figs.4 (a) and (b), respectingly. The characteristic feature of responses is that in the elastic analysis both force and displacement amplitudes are of almost complete reversal and the response increases as time elapses, while in the elastoplastic analysis, the force amplitude is of almost complete reversal but the mean strain increases almost monotonically and the strain amplitude is rather small. Herein, necessity of low cycle fatigue test exists.

Design and Method of Experiment

In the fatigue test, the push-pull displacement related with the above mentioned horizontal floor displacement responses by relation (2) were applied to the thin walled cylinder of structural steel SS41 which has the dimension of 2.54cm in inner diameter, 0.127cm in thickness, and 2.54cm in gage length as shown in Fig.5 by means of the electrohydraulic fatigue testing machine, under the consideration that the strain in this specimen represents the extreme fiber strain of the column of the structure expressed in Eq.2. The following tests were conducted:

(1) Thirteen constant strain amplitude tests covering the range of 10 to 2000 cycles to fracture,

(2) Five preliminary random fatigue tests in which the strain varies like as the elastic response stated in the previous section, although the strain on the specimen can not be directly related to the elastic response (Test Nos.1∿5), and

(3) Three random fatigue tests in which the strain varies like as the elastoplastic response stated in the previous section. In the response of (b)(i), the strain concentration factor of 2 was considered, while in the response of (b)(ii), the strain concentration factors of 2 and 2.5 were considered. (Test Nos.6∿8).

During each test, time histories of (i) the strain on the specimen, (ii) the displacement in the gage length measured by the extensometer, (iii) the stress by the load cell on which four foil strain gages are put, and (iv) the motion of the head of the hydraulic cylinder were recorded by means of an ink writing oscillograph. At the same time, the stress-strain hysteresis loops were recorded by the X-Y recorder as shown in Fig.6. Arrangement of the measuring instruments is shown in Fig.7. The cyclic rate in the test was reduced to a quater of the response. Failure was determined when considerable stress reduction occurs due to increasing strain on the specimen.

Experimental Results and Discussions

(1) Results of the Constant Strain Amplitude Fatigue tests: The total strain range vs. the number of cycles to fracture has been plotted on the

log-log paper as shown in Fig.8. From this diagram, the relationship

$$\Delta\varepsilon N^{0.346}=0.0624 \qquad \cdots\cdots(3)$$

has been obtained by the method of least square. In addition, the fracture ductility $\varepsilon_{ft}=0.18$ was obtained in the simple tension test of the specimen.

(2) Analysis of the Number of Occurrence of the Strain Amplitude and the Mean Strain: There are several methods of analysis of random strain time histories. In this study, the range count method was adopted in the tests Nos.1∿5 using the elastic response, and the range-mean count method was adopted in the tests Nos.6∿8 using the elastoplastic response, by watching the pattern of the responses in Fig.4. The number of occurrence vs. the strain amplitude from minimum to maximum in the tests Nos.1∿5 has been plotted in Figs.9(a)∿(e), respectively. The ductility factor that is the strain divided by the yield strain has been also written on the abscissa. The strain amplitude from minimum to maximum vs. the number of occurrence in the tests Nos.6∿8, respectively, was plotted by taking the mean strain as parameter, but omitted in this paper.

(3) Examination of the Cummulative Damage Theory: The damage factor $\Sigma(n_i/N_i)$ for the tests Nos.1∿5, is shown in Table 1, in which N_i and n_i are the number of cycles to fracture and the number of cycles at the strain range i, respectively. The average value of $\Sigma(n_i/N_i)$ is 1.51,which is bigger than 1. This may be because the strain increases almost monotonically as shown in Fig.4(a). For the test Nos.6∿8, the cummulative damage was examined based on two theories. The values of $\sum_i\sum_m(n_{im}/N_{im})$ based on the Miner's and the Sachs' theories and $\Sigma(\Delta\varepsilon_t/\Delta\varepsilon_{t1})^a$ based on the Yao's theory are shown in Table 2, in which m denotes the mean strain level, $\Delta\varepsilon_t$ and $\Delta\varepsilon_{t1}$ denote the tensile strain amplitude and that for N=1, respectively, and $a=1-(a_0-1)r$ where $r=\Delta\varepsilon_t/\Delta\varepsilon_c$ and a_0 is $1/\alpha$, a for r=-1 and $\Delta\varepsilon_c$ is the compressive strain amplitude. The average values of $\sum_i\sum_m(n_{im}/N_{im})$ and $\Sigma(\Delta\varepsilon_t/\Delta\varepsilon_{t1})$ are 1.89 and 3.66, respectively.

(4) Fatigue Life: The experimental values of the total number of cycles to fracture N_t are shown in the left column of Table 3 for the tests Nos.1∿8. The analytical values in the right columns of Table 3 were calculated by means of

$$N_t=1/4\sum_i\sum_m f_{im}\{\Delta\varepsilon_{im}/2(\varepsilon_f-\varepsilon_f\varepsilon_m/\varepsilon_{ft})\}^{1/\alpha} \qquad \cdots\cdots(4)$$

derived based on the Miner's cummulative damage theory and the Sachs-Weiss' theory of mean strain, by using the results of the constant amplitude test and the simple tension test and the occurrence distribution of the strain amplitude and the mean strain. In Eq.(4), $f_{im}=n_{im}/N_{im}$, $\alpha=0.346$ and $\varepsilon_f=4^\alpha\times 0.0624/2=0.05$ from the constant amplitude test and $\varepsilon_{ft}=0.18$ from the simple tension test. There are some differences between the experimental values and the analytical values in Table 3.

(5) Mode of Fracture: Both the maximum and the final strains were found to be the same in the experiment. The experimental final strains together with their percentages to $\varepsilon_{ft}=0.18$ are shown in Table 4. Failure in the random fatigue tests using the elastoplastic response shows the failure between static and cyclic ones. The mode of failure such as two cracks at the middle and the end of the gage length were found in the test using the elastoplastic response,while in the constant amplitude test a crack occurs at the end of the gage length and in the static tension test a crack occurs at another place than the end.

Conclusions: As shown in Synopsis.

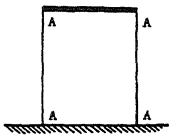


Fig.I A Single Degree of Freedom System

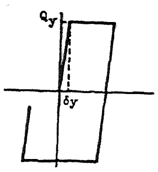


Fig.2 Elastoplastic Force-Deflection Hysteresis Loop

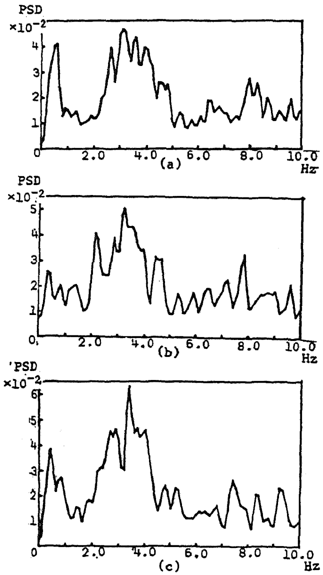


Fig.3 Power Spectra of the External Disturbances

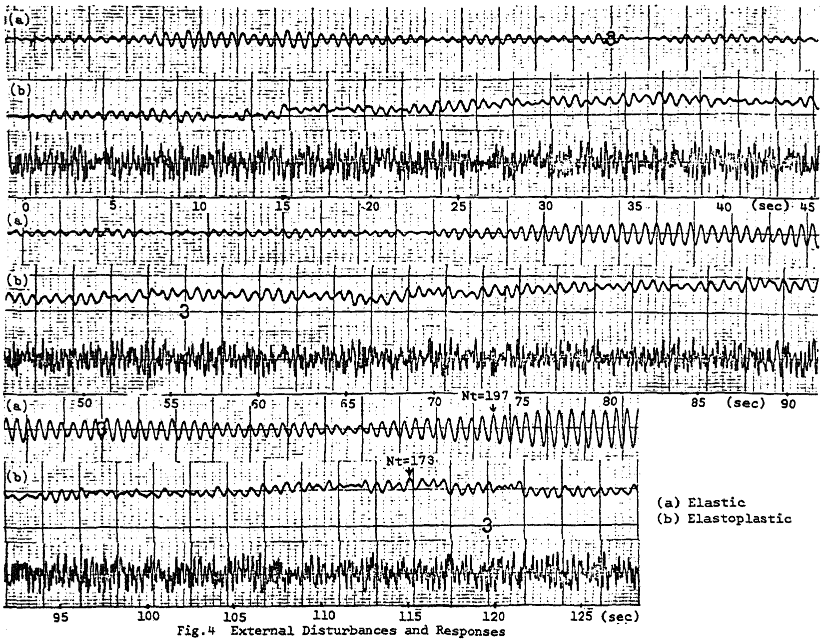


Fig.4 External Disturbances and Responses

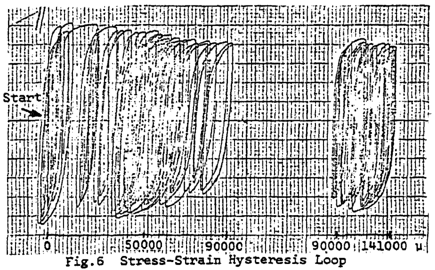


Fig.6 Stress-Strain Hysteresis Loop in the Random Fatigue Test No.8

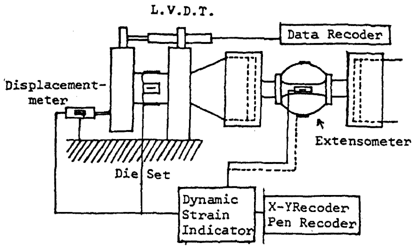


Fig.7 Arrangement of the Measuring Instruments

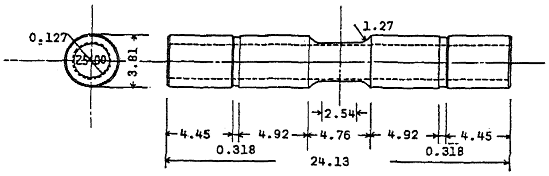


Fig.5 Specimen for the Fatigue Tests

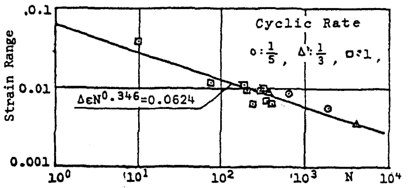
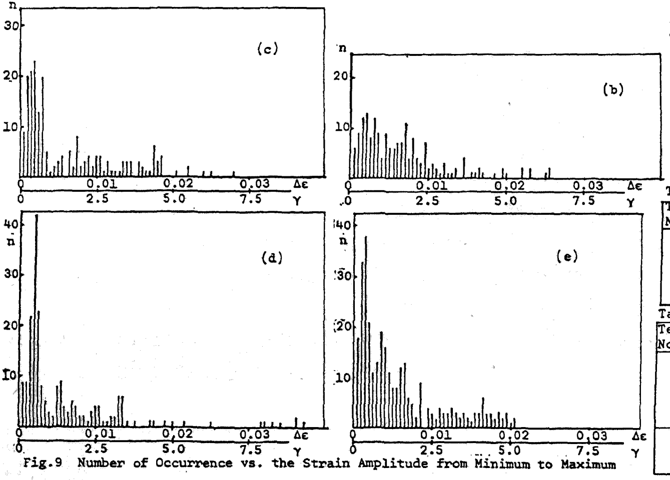


Fig.8 Total Strain Range vs. the Number of Cycles to Fracture



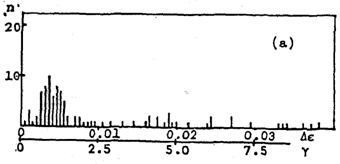


Fig.9 Number of Occurrence vs. the Strain Amplitude from Minimum to Maximum

Table 1. $\Sigma(n_i/N_i)$ value

Test No.	min.to max.	max.to min.
1	2.094	2.238
2	0.994	1.045
3	1.168	1.213
4	1.892	1.947
5	1.241	1.257

Table 2. Damage Factor Value

Test No.	$\Sigma\Sigma(n_{im}/N_{im})$	$\Sigma(\Delta\varepsilon_t/\Delta\varepsilon_{tl})^a$
6	1.610	4.303
7	1.317	2.993
8	2.736	3.690

Table 3. Total Number of Cycles

Test No.	Experimental	Analytical	
		min.to max	max.to min.
1	96	45.41	42.97
2	182	180.73	173.32
3	197	168.48	162.23
4	205	112.86	115.17
5	312	246.79	248.45
6	216	134.12	
7	232	176.22	
8	173	63.23	

Table 4. Final Strain

Test No.	Final	
	Strain	Per Cent
6	0.084	46.9
7	0.107	60.0
8	0.141	78.8