IDENTIFICATION OF DAMPING COEFFICIENTS FROM SYSTEM RESPONSE

Paolo Caravani[I] and William T. Thomson[II]

SYNOPSIS

Damping is generally an unknown quantity in a structure. This paper develops an optimization procedure to identify the damping coefficients from the measured response of the structure. Successive iteration from the equation developed is found to converge rapidly to the correct damping coefficients associated with the measured response.

INTRODUCTION

The parameters which characterize the linear model of an engineering structure are the mass, stiffness, and the damping coefficients. In repeated structures these reduce to the natural frequencies and the damping coefficients. A considerable interest has been shown recently for techniques likely to identify system parameters starting from the measured response of the structure. Several investigators[1,2] have successfully attacked the problem of identifying natural frequencies from earthquake records. The work presented here illustrates a numerical technique which identifies in an optimal sense the damping coefficients of a structure whose earthquake response is known over some frequency range. Although the method is limited to linear systems, it can be applied to various types of damping. The generality of the method exceeds the limits of the present brief discussion which is specifically referred to a multi-story high-rise building.

SYSTEM MODEL

Figure (1) shows the model of a multi-story building. In terms of the coordinates adopted, the equations of motion

---

I Research Assistant, Mechanical Engineering, Univ. of Calif. S.B.
II Professor, Mechanical Engineering, Univ. of Calif. Santa Barbara.

expressed in matrix form are;

$$M\ddot{X} + C\dot{X} + KX = f \tag{1}$$

Assuming the ground acceleration to be a stationary random process, the frequency domain representation of the above equation is

$$H(j\omega)\,X(\omega) = F(\omega) \tag{2}$$

where $H(j\omega)$ is the dynamic impedance matrix which is a function of m_ℓ, k_ℓ, and c_ℓ. The quantities $X(\omega)$ and $F(\omega)$ are the response and forcing vectors respectively.

Introducing a vector c containing the n damping coefficients,

$$c = (c_1\ c_2\ c_3\ \ldots\ c_n)^T \tag{3}$$

the response

$$X(\omega) = H^{-1}(j\omega)\,F(\omega) \tag{4}$$

can be regarded as a function of only c and ω.

IDENTIFICATION ALGORITHM

Let $\bar{X} = \bar{X}(\omega)$ be the measured response when $c = \bar{c}$, the actual damping of the structure, and $X_i = X_i(\omega)$ the response related to the i^{th} estimate of the vector $\bar{c}$. We propose to identify the vector $\bar{c}$ by recursively minimizing the function

$$L_{i+1} = \int_{\Omega_1}^{\Omega_2} (X_i-\bar{X})^T A\,(X_i-\bar{X})\,d\omega + c_i^T B\,c_i \tag{5}$$

where Ω_1, Ω_2 is the frequency range of interest. A is a diagonal weighting matrix which accounts for the different reliabilities in the measurements at the various stations.

B is a diagonal matrix which restrains the c's to physically plausible values.

Since we cannot express X_i in terms of c_i directly from Eq. (2) or (4), we write it in terms of the first order Taylor expansion;

$$X_i = X_{i-1} + J_{i-1}(c_i - c_{i-1}) \tag{6}$$

where J_{i-1} is the Jacobian matrix $\left[\partial X_k / \partial c_i\right]$. If Eq.(6) is substituted into Eq.(5) and its first derivatives with respect to c_i are set equal to zero, the following algorithm results.

$$c_i = \left[\int_{\Omega_1}^{\Omega_2} J_{i-1}^T A\, J_{i-1} d\omega + B\right]^{-1} \left[\int_{\Omega_1}^{\Omega_2} J_{i-1}^T A\, J_{i-1} d\omega\, c_{i-1} - \int_{\Omega_1}^{\Omega_2} J_{i-1}^T A\, (X_{i-1} - \bar{X}) d\omega\right] \tag{7}$$

At every iteration i, the Jacobian J_{i-1} is evaluated column by column with standard control techniques[3] from Eq.(2).

The convergence of the model response is demonstrated in Fig.(2) for a two story building. From an initial guess of 3.75% critical damping, the final value was identified as 5% critical for both damping coefficients in three iterations.

---

(1) Collins, J.D. and W.T.Thomson "The Eigenvalue Problem for Structural Systems with Statistical Properties". AIAA Jour. Vol.7, No.4, Apr. 1969, pp642-648.

(2) Ibanez, P. "Identification of Dynamic Structural Models from Experimantal Data". Ph.D. Dissertation, UCLA, Mar.1972.

(3) Caravani,P. and W.T.Thomson "Frequency Response of a Dynamic System with Statistical Damping" - scheduled for AIAA Jour. Feb. 1973

This study is supported by NSF Grant GK 21403.

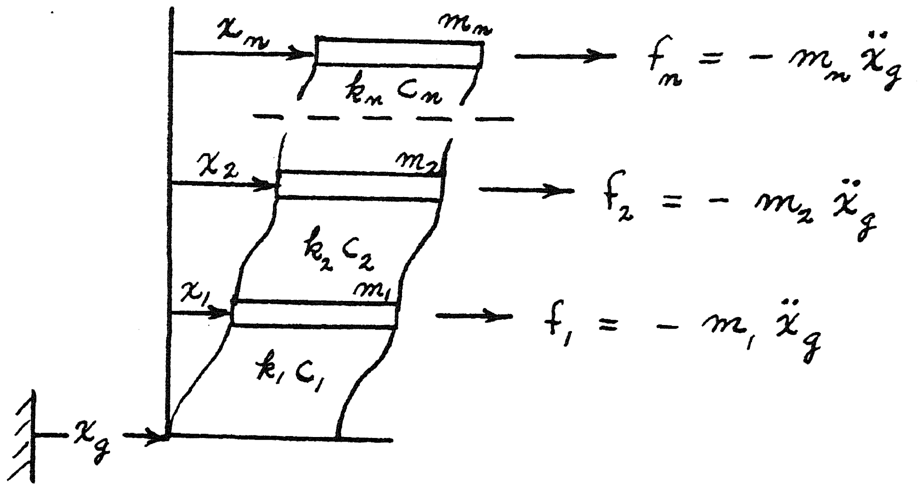

FIG. 1 - MODEL OF MULTI-STORY BUILDING

FIG. 2. DYNAMIC AMPLICATION COEFFICIENT (dots indicate measured response)