HYSTERESIS LOOPS FOR REINFORCED CONCRETE BEAM-COLUMN CONNECTIONS

by

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SYNOPSIS

A mathematical model for the hysteresis loops of reinforced concrete beam-column connections is derived from test data. These hysteresis loops are presented as parabolic arcs in terms of displacement level, column axial loads, and the number of inelastic cycles of displacement.

INTRODUCTION

During an earthquake the ground moves in two horizontal directions as well as vertically in an intricate combination of frequencies and displacements. This ground movement causes complex dynamic response of multistory structures. Numerical integration of the equations of motion of the structure which utilize the nonlinear structural member hysteresis loops can be used to predict the response of multistory structures to earthquakes. The accuracy of this integration procedure in predicting response is limited by the accuracy of the individual structural member load-displacement curves and the accelerogram record of the anticipated earthquake. This paper presents equations for reinforced concrete beam-column connection hysteresis loops which were derived from test results of twenty-two connections subjected to simulated earthquake conditions (4).

EXPERIMENTAL PROGRAM

The three parameters considered to be most important in describing this nonlinear behavior are: 1) column axial load, 2) displacement level, and 3) the number of cycles of inelastic load. The connections were designed so that steel yielding occurred first in the ten inch (25.4 cm) deep beam at the column face and so that the columns remained elastic throughout the test.

The twenty-two beam-column connections tested were constructed in a T shape with the column loading applied at the ends of the T. Specimens were designed according to current United States earthquake resistant building code recommendations and built using intermediate grade reinforcing steel and 4000 psi (281 kg. per sq. cm.) concrete. Earthquake lateral forces cause many columns to have nearly equal but opposite bending moments at their ends producing a point of contraflexure near the column mid-point. The eight inch (70.3 cm.) square specimen columns were constructed to span from mid-height to mid-height and were loaded so that zero bending moment existed at each end. Column axial loading was held constant while cyclic loading was applied at the beam tip. A

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continuous record of the force-deformation hysteresis loops provided data which were used to analyze the rate of stiffness degradation and loss of energy-absorbing capacity of the connections.

Column Axial Load

Lateral forces produce overturning moments which increase the column stresses on one side of a structure while reducing them on the other side. This reduction in column stresses may occur during an instant of high vertical acceleration, which is ignored by most engineers, producing high column tensile loads. Goel (1), using a Ramberg-Osgood structural member model, found that overturning moment alone was great enough to cause column tension stresses equal to 2.5 times the gravity load stresses. Workman (5), using an elastoplastic model for a ten story frame, found total column tensile stresses as great as one third the yield stress of steel. Further evidence is provided by the Petunia II apartment building which was severely damaged during the 1967 Venezuela earthquake. This twenty-one story structure had reinforced concrete columns which cracked in tension (2). There is little doubt that large tension loads can occur in columns during a large earthquake. This tension load is significant for reinforced concrete because concrete cracks at relatively low tensile stress requiring the entire tension load to be carried by the reinforcing steel. The resulting steel strains tend to open cracks in the connection reducing the stiffness and energy absorption of the connection. This effect of column tension, which has not been reported in previous reinforced concrete research, is the first parameter investigated here.

Displacement Level

Displacement level, which is defined as the hinge rotation divided by that hinge rotation which would cause the top beam steel to yield, is the second parameter investigated. As loading is applied to the hinge, cracks form in concrete before the reinforcing bars yield. As loading increases the cracks grow and new ones are formed, reducing the stiffness of the connection. When the load is reversed, these cracks close and new ones form. The opening and closing of cracks dissipates energy whether the steel yields or not. As the cracks open and close, particles are ground or crushed, further reducing the stiffness of the connection. The amount of stiffness reduction and energy dissipation is dependent upon the displacement level.

Cycles of Inelastic Load

As the structure vibrates the concrete is ground and crushed changing the load-displacement curves. Several researchers who have tested model frames of reinforced concrete loaded beyond the yield point of steel (3) report rapid changes in stiffness during the first several inelastic cycles. The number of cycles of inelastic deformation is the third parameter considered in this research.

HYSTERESIS LOOPS

The equations derived from the analysis of these experimental hysteresis loops are converted to dimensionless variables for simplicity of presentation and utilization. The load coordinate for the hysteresis loops is presented as M/M_y where M_y is the computed moment which tensions the top steel to the design yield stress. Column axial load, σ, is the dimensionless ratio σ_T/f'_c where f'_c is the design compressive strength of concrete and σ_T is the axial stress on the transformed column section. Displacement level, γ, is defined as ϕ/ϕ_y where ϕ_y is the beam rotation ten inches (25.4 cm.) from the column face caused by the moment M_y. The derived hysteresis loop equations are presented as four parabolas (Fig. 1) of the form:

$$\frac{M}{M_y} = a_i + b_i\gamma + c_i\gamma^2$$

where,

$$a_1 = .461 - .319 \log_{10}\rho + .0060\,\gamma_m - .671\,\sigma \quad (\dot{\gamma}>0) \qquad (2)$$

$$a_2 = -.388 + .343 \log_{10}\rho - .015\,\gamma_m + .548\,\sigma \quad (\dot{\gamma}<0) \qquad (3)$$

$$b_1 = .685 - .125\,\gamma_m + .00591\,\gamma_m^2 \qquad (4)$$

$$c_1 = -.00755 + .00625 \log_{10}\rho - .00049\,\gamma_m + .03311\,\sigma \quad (\gamma>0 \text{ and } \dot{\gamma}>0) \qquad (5)$$

$$c_2 = .029 - .00764 \log_{10}\rho - .00209\,\gamma_m - .01276\,\sigma \quad (\gamma>0 \text{ and } \dot{\gamma}<0) \qquad (6)$$

$$c_3 = .03628 - .01625 \log_{10}\rho - .00242\,\gamma_m - .00283\,\sigma \quad (\gamma<0 \text{ and } \dot{\gamma}<0) \qquad (7)$$

$$c_4 = .00090 - .00284 \log_{10}\rho - .00011\,\gamma_m + .01634\,\sigma \quad (\gamma<0 \text{ and } \dot{\gamma}>0) \qquad (8)$$

provided that

$$\gamma_m \leq 12.7 \text{ and } 0.25 \leq \rho \leq 10$$

where,

σ is positive for tension loads, γ_m is the maximum γ which occurred in any previous cycle, and ρ is an integer representing the total number of cycles. A regression analysis of the first quarter-cycle with $\rho = 1$ and $\gamma_m = 0$ indicates that column loading has a minor effect on the shape of the curve. The resulting equation for the first quarter cycle is

$$\frac{M}{M_y} = .172 + 1.03\,\gamma - .167\,\gamma^2 + .00846\,\gamma^3 \qquad (9)$$

The derivative of M/M_y with respect to γ gives the stiffness of the connection which equals b_1 at zero displacement. The value of b_1 is determined from Equation 9 for the first quarter-cycle of loading and from Equation 4 for yielded connections. Table 1 summarizes the stiffnesses at zero displacement for connections that have been loaded to various displacement levels. It is apparent that large displacement levels cause the zero displacement stiffness of these connections to deteriorate rapidly. This could cause lateral responses to vary considerably from levels predicted by previous researchers.

CONCLUSIONS

Analysis of the hysteresis loops for twenty-two reinforced concrete beam-column connections loaded beyond yield indicates that:

1. Lower column axial compression forces produce smaller hysteresis loops for beam hinges near the beam-column interface.

2. Continued cyclic loading of the connection causes reduction in the size of the hysteresis loop.

3. An increase in the level of yielding causes a rapid decrease in the stiffness of the connection of zero displacement.

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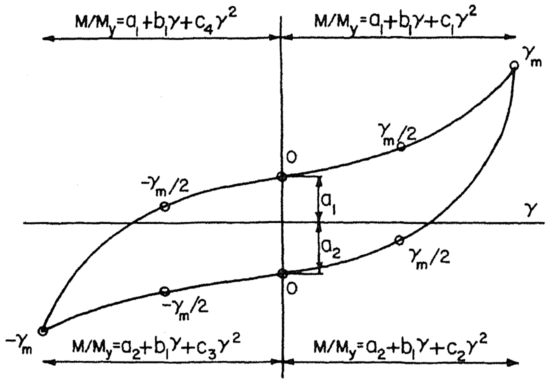

FIG. 1. - PARABOLAS IN A HYSTERESIS LOOP

TABLE 1: STIFFNESS AT ZERO DISPLACEMENT

Displacement Level	Stiffness	% Uncracked Stiffness
0	3.370	100.0
0 (Equation 9)	1.030	30.5
1	.566	16.8
2	.458	13.6
3	.363	10.7
5	.208	6.1
10	.026	0.8