ON THE SEISMIC ANALYSIS OF UNSYMMETRICAL STORIED BUILDINGS

by

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SYNOPSIS

The seismic analysis of unsymmetrical storied buildings must take into account that at those constructions transverse and torsional vibrations will be always coupled. For the analysis of those coupled vibrations, Holzer's method is generalized. It is shown that in case of close periods of torsional and transverse vibrations (neglecting their coupling), a resonance like increase of the vibrational amplitudes can appear, particularly dangerous for buildings with small torsional stiffness.

THE GENERALISATION OF HOLZER'S METHOD

The seismic analysis of unsymmetrical storied buildings requires the investigation of their vibrations. In the case of unsymmetrical buildings, at which the mass centers of sundry stories (M in fig.1) do not coincide with their rigidity centers (S in fig.1), the analysis must take into account that transverse vibrations will be always coupled with torsional vibrations and vice versa.

For the analysis of those vibrations Holzer's method is generalized in [1,2,3] , indicating the following group of three recurrence relations between the horizontal orthogonal deflections y^u and y^v of two adjacent stories j and j+1 and their rotation θ round the rigidity center :

$$y^u_j=y^u_{j+1}-\frac{\omega^2}{R_{j,j+1}}\left[R^{vv}_{j,j+1}\sum_{k=j+1}^{n}M_k(y^u_k-v^m_k\theta_k)-R^{uv}_{j,j+1}\sum_{k=j+1}^{n}M_k(y^v_k+u^m_k\theta_k)\right]$$

$$y^v_j=y^v_{j+1}-\frac{\omega^2}{R_{j,j+1}}\left[R^{uu}_{j,j+1}\sum_{k=j+1}^{n}M_k(y^v_k+u^m_k\theta_k)-R^{uv}_{j,j+1}\sum_{k=j+1}^{n}M_k(y^u_k-v^m_k\theta_k)\right] \qquad (1)$$

$$\theta_j=\theta_{j+1}-\frac{\omega^2}{R^t_{j,j+1}}\sum_{k=j+1}^{n}\left[I_{Mk}\theta_k-M_k(v^m_k y^u_k-u^m_k y^v_k)\right]$$

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In relations (1) there is denoted as ω the natural circular frequency of the vibrations, as M_k the lumped mass at the floor level k, as I_{Mk} its moment of inertia about the rigidity center of the story, as $R^{uu}_{j,j+1}$, $R^{vv}_{j,j+1}$, $R^{uv}_{j,j+1}$ the stiffnesses of the story j,j+1 in the directions u,v, as $R^{t}_{j,j+1}$ the torsional stiffness of the story and as n the number of stories. Farther $R_{j,j+1} = R^{uu}_{j,j+1}\ R^{vv}_{j,j+1} - (R^{uv}_{j,j+1})^2$. The meaning of u^m, v^m is shown in fig.1.

Starting with an appreciated value ω and a couple of values y^v_n/y^u_n, θ_n/y^u_n arbitrarily chosen, the deflected shape of the structure can be determined by trial, using relations (1). If in this way $y^u_o = y^v_o = \theta_o = 0$ is obtained, the introduced values ω, y^v_n/y^u_n, θ_n/y^u_n are true.

For computer utilization, relations (1) can be written by means of transfer matrices as

$$V_j=A_{j,j+1}V_{j+1} = A_{j,j+1}A_{j+1,j+2}\cdots\cdots A_{n-1,n}V_n \tag{2}$$

with

$$V_j=\begin{bmatrix} y^u_j \\ Q^u_{j-1,j} \\ y^v_j \\ Q^v_{j-1,j} \\ \theta_j \\ M^t_{j-1,j} \end{bmatrix},\quad V_{j+1} = \begin{bmatrix} y^u_{j+1} \\ Q^u_{j,j+1} \\ y^v_{j+1} \\ Q^v_{j,j+1} \\ \theta_{j+1} \\ M^t_{j,j+1} \end{bmatrix},\ldots\quad V_n = \begin{bmatrix} y^u_n \\ 0 \\ y^v_n \\ 0 \\ \theta_n \\ 0 \end{bmatrix}$$

$$A_{j,j+1}=\begin{bmatrix} 1 & -b_j & 0 & c_j & 0 & 0 \\ m_j & 1-m_jb_j & 0 & m_jc_j & m_jv^m_j & m_jv^m_jf_j \\ 0 & c_j & 1 & -a_j & 0 & 0 \\ 0 & m_jc_j & m_j & 1-m_ja_j & m_ju^m_j & -m_ju^m_jf_j \\ 0 & 0 & 0 & 0 & 1 & -f_j \\ -m_jv^m_j & m_jd_j & m_ju^m_j & m_je_j & i_j & 1-i_jf_j \end{bmatrix} \tag{3}$$

In relations (3) $Q^u_{j-1,j}$, $Q^v_{j-1,j}$ and $M^t_{j-1,j}$ mean shear forces and torsional moment at the story j-1,j, farther there were introduced the notations

$a_j = R^{uu}_{j,j+1}/R_{j,j+1}$; $b_j = R^{vv}_{j,j+1}/R_{j,j+1}$; $c_j = R^{uv}_{j,j+1}/R_{j,j+1}$
$d_j = v^m_j b_j + u^m_j c_j$; $e_j = v^m_j c_j + u^m_j a_j$; $f_j = 1/R^t_{j,j+1}$; $m_j = \omega^2 M_j$;
$i_j = \omega^2 I_{Mj}$. An approximate determination of the values y^v_n/y^u_n, θ_n/y^u_n is given in [1,2,3].

THE DYNAMICAL EFFECT OF VIBRATION COUPLING

For obtaining some qualitative results, the case of a one-mass-system with $v^m = 0$, $u^m = u$, $y^v = y$, $R^{vv} = R$ is examined. Denoting $t^2 = \omega_v^2/\omega_t^2 = RI_M/R^t M$, ($\omega_v$, ω_t are the natural circular frequencies of transverse and torsional vibrations by neglecting their coupling) it follows

$$c_{1,2} = \theta/y = \frac{1}{2u}\left[t^2 - 1 \pm \sqrt{(t^2-1)^2 + 4u^2t^2M/I_M}\right] \qquad (4)$$

$$\omega^2_{1,2} = \omega^2_v/(1 + uc_{1,2}). \qquad (5)$$

Putting u=0, transverse and torsional vibrations are decoupled and it follows from (4), - for $t^2 > 1$, $uc_1 = t^2 - 1$, $uc_2 = 0$, therefore $\omega_1^2 = \omega_t^2$, $\omega_2^2 = \omega_v^2$; - for $t^2 < 1$, $uc_1 = 0$, $uc_2 = t^2 - 1$, therefore $\omega_1^2 = \omega_v^2$, $\omega_2^2 = \omega_t^2$. For $t^2 > 1$, the values c_1, ω_1 correspond consequently to vibrations, due to the dynamical action of a torsional moment and the values c_2, ω_2 correspond to vibrations, due to the dynamical action of a shear force. Conversely for $t^2 < 1$, the values c_1, ω_1 correspond to vibrations due to the dynamical action of a shear force and the values c_2, ω_2 to vibrations, due to the dynamical action of a torsional moment (for the general case $u^m \neq 0$, $v^m \neq 0$ a similar reasoning can be made, identifying the solutions of the secular equation).

In the following, torsional vibrations, due to the dynamical action of a shear force, will be examined.

By neglect of the dynamical effect of torsion, the rotation of the building round the rigidity center would be $\theta' = yRu/R^t$, therefore relation (4) would be replaced by the relation $c' = \theta'/y = Ru/R^t$. Correspondingly the ratio

$$k = \frac{c}{c'} = \frac{1}{2u^2t^2}\,\frac{I_M}{M}\left[t^2 - 1 \pm \sqrt{(t^2-1)^2 + 4u^2t^2M/I_M}\right] \qquad (6)$$

is a dynamical factor, showing the dynamical effect of the coupling of transverse and torsional vibrations. Fig.2 represents k-factor diagrams for a square building, depending on t, fig.3 represents curves of ku-values, proportional to the torsional moments M^t. As is shown in fig.2, in case of close periods of transverse and torsional vibrations (neglecting their coupling), a resonance like increase of the vibrational amplitudes can appear. For u=0, relation (6) becomes $\lim_{u \to 0} k = 1/(1-t^2)$.

RESULTS AND CONCLUSIONS

It is shown that in case of certain values of the ratio between the torsional and the transverse stiffness of an unsymmetrical structure, a dangerous increase of vibrational amplitudes can appear. This increase appears particularly at buildings with a small torsional stiffness in comparison with their transverse stiffness. Using the relations given in the paper, the stiffness ratios can be chosen in such a manner, that dangerous resonance like effects are avoided.

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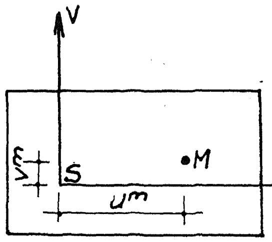

Fig. 1

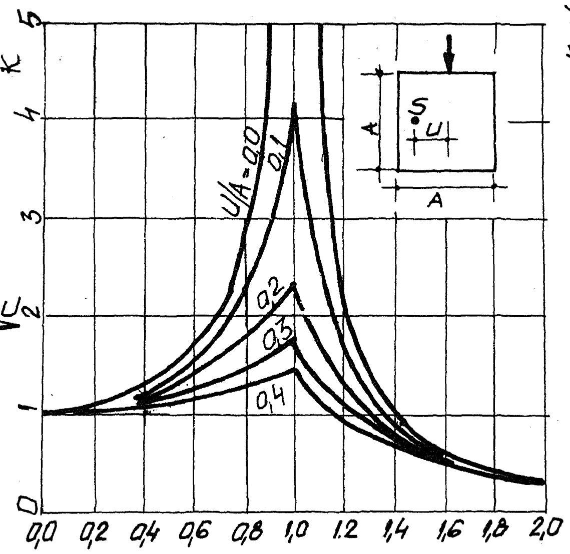

Fig. 2

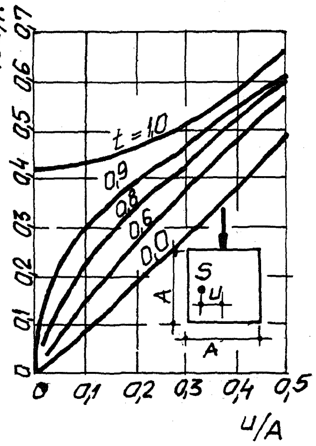

Fig. 3