EARTHQUAKE SIMULATION BY A SHAKE TABLE

Enzo Lauletta (I) and Aldo Castoldi (II)

SYNOPSIS

The operation of a shake table with an excitation unit capable of generating random vibrations whose spectral density is of a predetermined shape is first described.

The general criteria and employment technique of the shake table for earthquake effect reproduction on elastic models by stationary random signals are then discussed.

INTRODUCTION

The reasons for existence and the rate of advancement of structural model engineering depend on the fidelity at which the model reproduces the structure, avoiding oversimplified schematizations.

This implies:

> construction of mechanically "faithful" models, i.e., reproducing all the structurally significant details and made of materials capable of simulating the behavior of the prototype also beyond the elastic range.

(I) Dr., Director, Istituto Sperimentale Modelli E Strutture (ISMES), Bergamo, Italy.

(II) Dr., Istituto Sperimentale Modelli E Strutture (ISMES), Bergamo, Italy.

The models thus result of great size, requiring large, intrinsically rigid and, therefore, massive and difficult to move shake tables, and materials endowed with specific properties;

"faithful" application of the external loads. While not difficult for static loads, this is hard to achieve for seismic loads both because of simulation troubles and arduous theoretical definition.
In fact, it is well known that recordings of severe earthquakes near the epicenter are exceedingly few in number.
Therefore, the simplest and most direct method of investigation would be subjecting the structures under study to earthquakes identical with those which actually occurred in situ. In itself, this would certainly be of great meaning and in many cases satisfactory; but it is rather scanty and liable to criticism as regards its statistical reliability with respect to future earthquakes.
Hence the wide-open novelty of having discovered random characteristics in earthquakes (at least in many zones). In fact, in a random process as many single samples as one pleases can easily be obtained.

This theoretical discovery and the progress made in the construction of electrodynamic exciters capable of producing large forces of any shape make possible a big stride forward in dynamic structural model engineering.

A description is presented below of the new seismic unit set up at ISMES, Bergamo, for simulating both real and random earthquakes and of its employment technique.

THE SEISMIC UNIT

Fig. 1 shows schematically the new unit.

Substantially, the unit is made up of a large slab moved by two electrodynamic exciters controlled by a signal generator, a motion control system and instrumentation for recording and analyzing the response of the model.

Some details:

a) The slab is of aluminium, 3 m x 2 m in size and weighs 600 Kg. It rests on a very rigid granite block through a film of oil which is continuously changed by a pump.
The film of oil is an efficient transverse restraint which practically eliminates the flexural vibrations of the slab without the inconvenience of inertial and frictional resistances harmful to the motion.

b) The electrodynamic exciters are connected in push-pull and are supplied by two power amplifiers.
The response of the amplifiers is flat for frequencies of 5 to 800 cps.
The maximum phase difference between the input voltage and the output current is 15°
The limiting performances of the set of exciters and amplifiers are shown in fig. 2.

c) To obtain a given spectrum acceleration on the slab it is generally necessary to produce a voltage of a different spectrum. This is because of the resonance of the structure under test.
The solution consists in using an automatic multi-band equalizer operating as follows (fig. 3).
The random signal produced by the noise generator A, properly trasferred into the high frequency range by the input signal converter B by means of the modulation technique, is sent to the random equalizer C.
The original signal spectrum is there divided into bands by a set of filters whose variable gain is changed by an external DC signal.
The output voltages of the filters, properly added in the mixer D, are

sent to the signal level control E and thence to the power amplifiers. The DC signal, required to control the filter gain, is obtained as follows.

The signal corresponding to the acceleration of the slab is sent to the analyzer F which divides its spectrum into bands identical with those of the equalizer.

A DC signal proportional to the spectrum of each band is then obtained by means of detector and averaging circuits.

The difference between the value of this signal and the predetermined one controls the filter gain in each channel.

The principal features of the system are the following:

number of filters: the original random signal spectrum is divided into 48 equal bands, each of 12.5 cps;

response time: the equalization time is 1 - 2 seconds and it is mainly supplied by the averaging circuit response time.
In fact, this circuit must have its response time proportional to the filter band width so as to decrease the variations of the DC signal about the true value of the spectral density.
The system is thus capable of operating in the best possible way with stationary signals for which the initial equalization phase may be neglected.

Evidently the equalizer is unnecessary and may be disconnected in the case of small models for which it may be assumed that the mechanical impedance variation at the different frequencies is small compared with the mechanical impedance of the motion generating set.

EMPLOYMENT TECHNIQUE

The equipment just described can first be used with an input of magnetic tape recordings of real earthquakes.

The fair fidelity of the whole permits a sufficiently correct reproduction, provided no intervention of the equalizer, which is not made for instantaneous compensations of this type, is required. This is the case of small models only, useful for basic research.

It is mainly due to the fact that the earthquake spectrum is restricted to a rather narrow frequency band. For example, with a time scale of 1 : 10, the band is $5 < \Delta f < 100$ cps.

This is corroborated by the comparison, shown in fig. 4, between the acceleration wave shape recorded on the table with no model on it and without using the equalizer and the wave shape of a signal at the power amplifier input.

As already mentioned, the principal use of the new seismic unit is for model excitation by random vibrations.

However, like all simulated random oscillations (and most probably differing from real earthquakes), the vibrations generated by the unit are of the "pure random" type and, therefore, lack periodic components, shocks or transients; they are, so to speak, uniform for the entire duration of the phenomenon.

This is the limit of validity of the unit, at least in its usual operation. As a consolation, indeed a meager one, it can be stated that this limitation is also common to all the analytical methods hitherto devised.

GENERAL TEST CRITERIA

Let y indicate a generic earthquake of duration T, whose development in a Fourier series notoriously is:

$$y(t) = K \cdot \sum_{1}^{\infty}{}_{n} (a_n \cos \omega_n t + b_n \sin \omega_n t) \qquad (\ast)$$

where $$K = \left(\frac{1}{T}\int_0^T y^2(t)\,dt\right)^{\frac{1}{2}}$$

The random process, of which the single earthquake is a sample, is known when the random parameters T , K , a_n , b_n (which can be taken as characteristics of the phenomenon) are known.

With this in mind, the correct procedure for a general simulation of the effects of earthquakes of the ($\ast$) type is as follows:

a) production of the desired random process;

b) selection of a certain number of samples of this process;

c) application of the samples to a model (or to a set of models if the single tests are believed to modify its characteristics);

d) statistical analysis of the results.

Obviously the number of tests should be sufficient to supply a statistical reliability, i.e., to identify at an acceptable accuracy all the probabilistic quantities of interest.

This procedure is therefore quite laborious, in particular for models beyond the elastic range, and should be used for very plain models only.

For more general purposes, simpler and yet not too inaccurate methods ought to be found.

A distinction will now be made between tests on elastic and elastic-plastic models respectively.

The most logical criterion for selecting the random process seems to be the following.

A random process may be assumed as equivalent to one formed of the entire set of earthquakes if the responses produced by it in the structures present a peak probability distribution function $P(x)$ and an expected number of peaks that are equal to those given by the responses of the same structures to real earthquakes.

Let us now examine the influence of each of the parameters K, T, a_n and b_n on the probability distributions $P(x)$.

a) Intensity K

When K, T, a_n, b_n are taken to be independent of one another, as is usually assumed while expecting the analysis of the real earthquakes to possibly indicate otherwise, the earthquake intensity proposed for elastic tests is an arbitrary parameter.

b) Duration T

When, in accordance with current theories, the earthquake is regarded as a stationary process, the influence of its duration T on $P(x)$ is negligible, and the tests may therefore be conducted on a single model subjected to an earthquake the duration of which is meaningful from a probabilistic point of view.

However, for the method to be correct, the fatigue phenomenon for alternate loadings must be negligible (which is usually reasonable in view of the number of loading cycles in play) and the transient part of the structure must also be negligible.

The latter hypothesis was verified as follows.

A one-degree-of-freedom oscillator (natural frequency 16 cps, damping $\zeta = 0.095\%$) was excited for one minute by a random signal the spectral density of which was constant in the 5-100 cps band and whose probability distribution of the instantaneous values (at least for $x \leq 3\sigma$) was of the gaussian type.

Diagram 2 of fig. 5 a shows the probability distribution of the response peaks.

The test was then repeated by exciting the oscillator with 25 samples of the same signal, each lasting 3 seconds.

The response of each of these samples was analyzed and the average peak probability distribution curve was plotted (diagram 1 of fig. 5 a).

The same experiment was repeated for two other oscillator damping values ($\gamma = 1.65$ and $\gamma = 5.0$) and the results are given in figs. 5 b and 5 c.

For practical purposes, the difference of the two curves is negligible for dampings which are not too small and, in any case, the results corresponding to the long excitation are always, though little, on the side of safety, as indeed ought to be.

c) Parameters a_n, b_n

These parameters have a direct influence on both the spectral density of the process and the probability distribution of the instantaneous values.

The spectral density must necessarily be selected so as to agree with that of the real earthquakes (e.g., Housner and Jennings select it in a way as to obtain velocity response spectra in agreement with the actual ones).

On the other hand, it does not seem of importance to also reproduce the earthquake probability distribution. In fact, it is experimentally proved that an oscillator response to a one degree of freedom with not too great a damping has a peak probability distribution practically coinciding with the Rayleigh distribution, however the original process probability distribution may be.

As a confirmation, fig. 6 shows that the measured probability density of the response of a 100 cps band width filter to a broad band signal having a non-gaussian probability density is practically gaussian.

It is also known that the peak value probability density of a gaussian narrow band random signal is that of a Rayleigh density.

TESTS ON ELASTIC-PLASTIC MODELS

The setting up a criterion of equivalence of the earthquake random process and another one is handicapped by the lack of information on the behavior of elastic-plastic structures with respect to real earthquakes.

Tests on small elementary structures subjected to actual earthquakes and to random motions are now in progress at ISMES.

ACKNOWLEDGMENT

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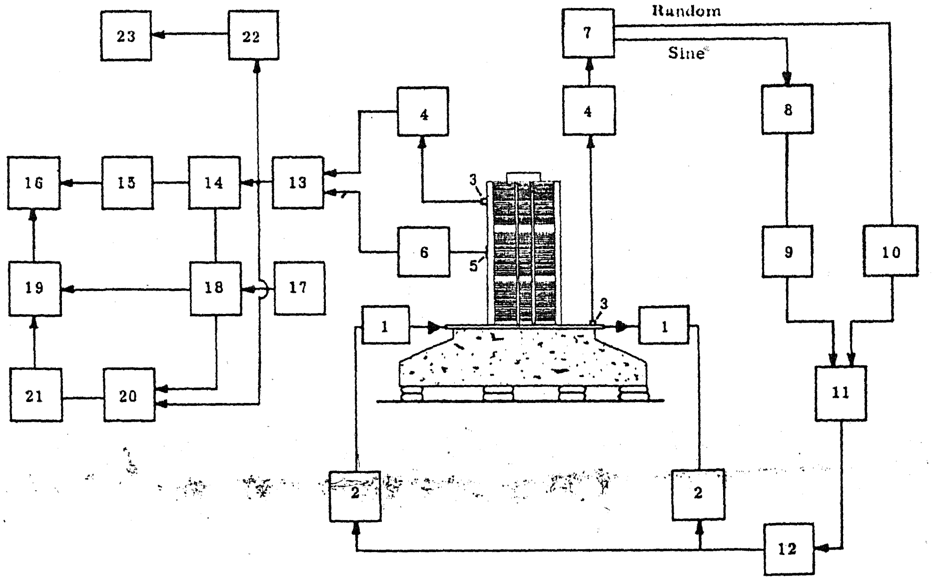


1 Exciter
2 Power Amplifier
3 Accelerometer
4 Charge Amplifier
5 Strain-gauge
6 Measuring Amplifier
7 Tracking Filter
8 Vibration Meter
9 Sine Generator
10 Random Equalizer and Analyzer
11 Mixer
12 Displacement Limiter
13 Tape Recorder
14 Analysis Control Chassis
15 Filters
16 Log Converter
17 Tape splice blanker
18 Ramp Generator and V.C. Oscillator
19 X-Y Recorder
20 Shock Spectrum Analyzer
21 Log Converter
22 Amplifier
23 Oscillograph

Fig. 1

BLOCK DIAGRAM OF THE SEISMIC UNIT

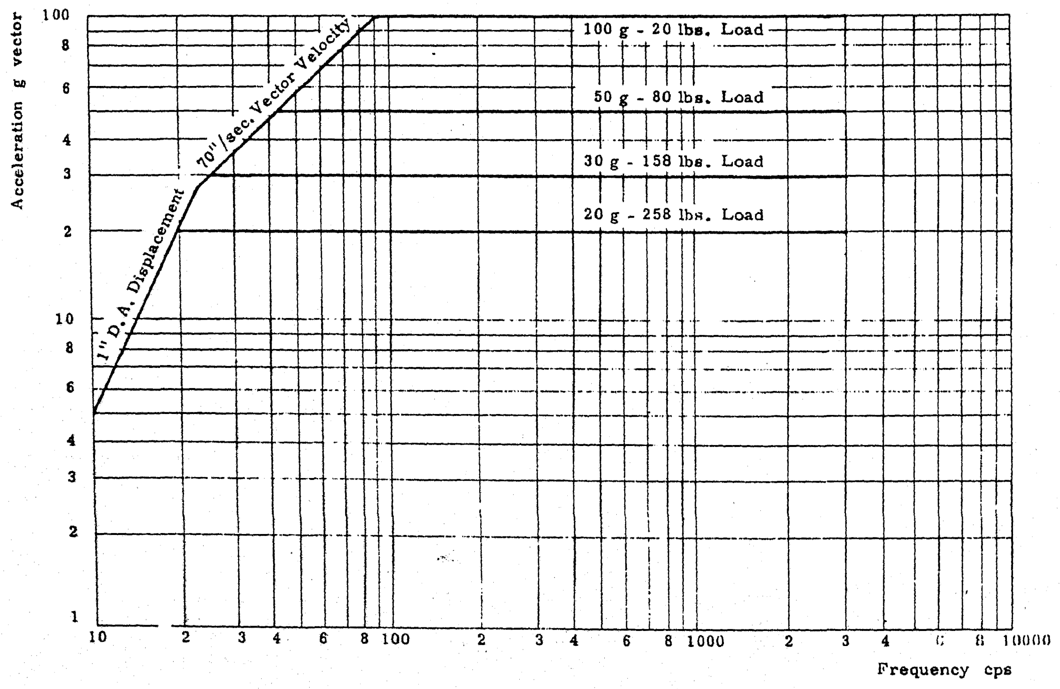
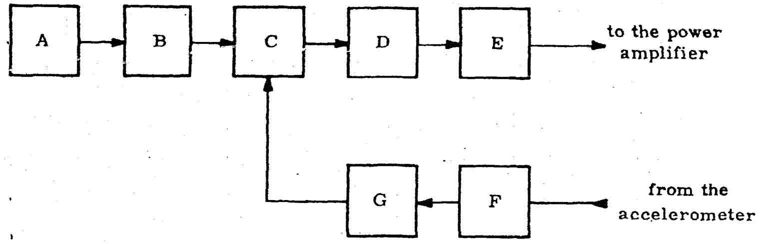


Fig. 2

EXCITER-AMPLIFIER PERFORMANCE



A	Noise Generator
B	Input Signal Converter
C	Random Equalizer
D	Mixer
E	Signal Lev. l Control
F	Input Signal Converter
G	Analyzer

Fig. 3

BLOCK DIAGRAM OF RANDOM EQUALIZER

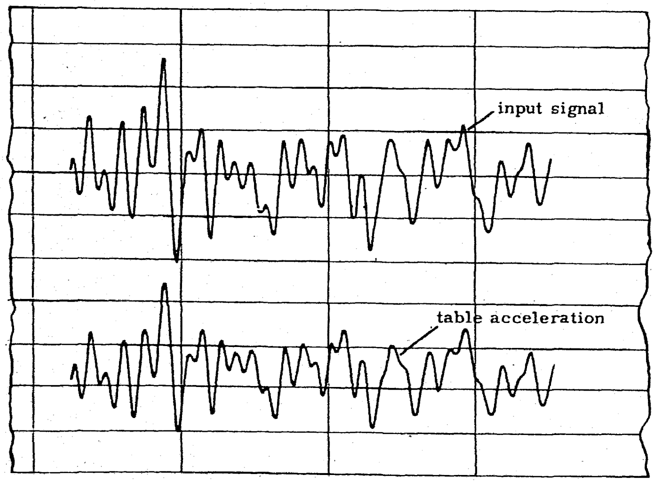


Fig. 4

COMPARISON BETWEEN THE SHAKE TABLE ACCELERATION AND INPUT SIGNAL WAVE SHAPES

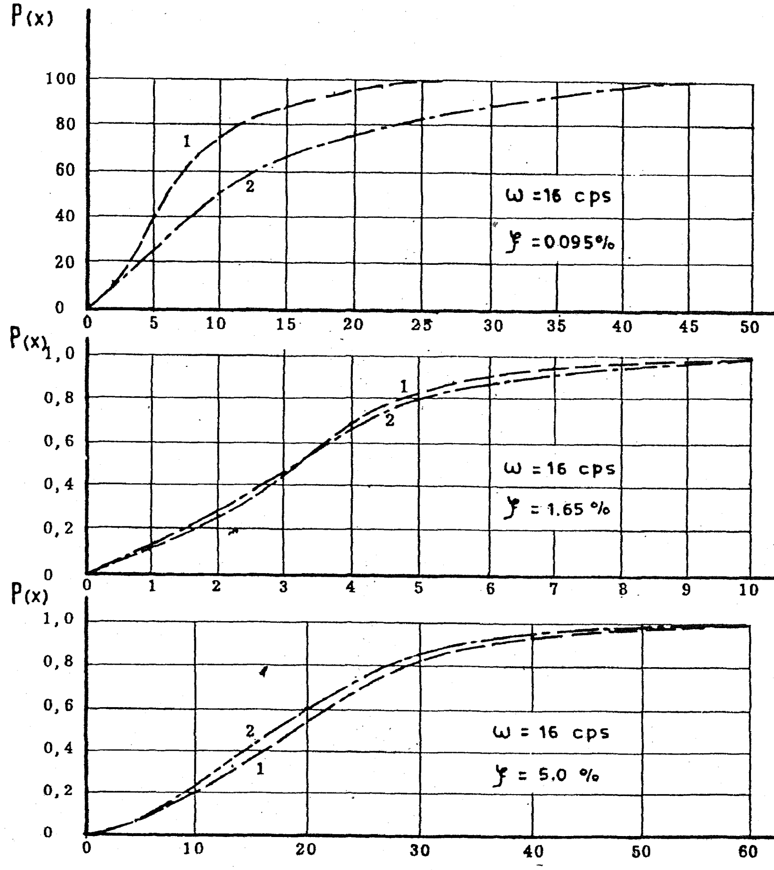


Fig. 5 a

Fig. 5 b

Fig. 5 c

1 average curve for short excitations (duration 3 seconds)

2 stationary excitation curve (duration 1 minute)

PEAK VALUE PROBABILITY DISTRIBUTION OF A ONE-DEGREE-OF-FREEDOM OSCILLATOR RESPONSE

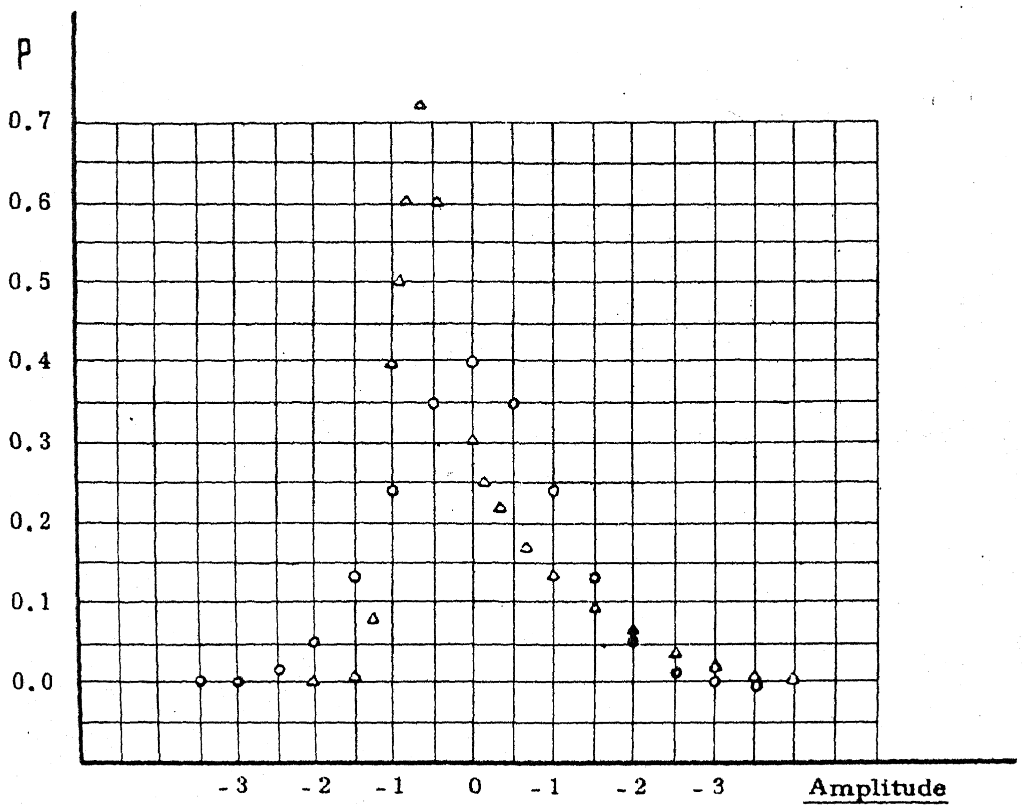


Fig. 6a

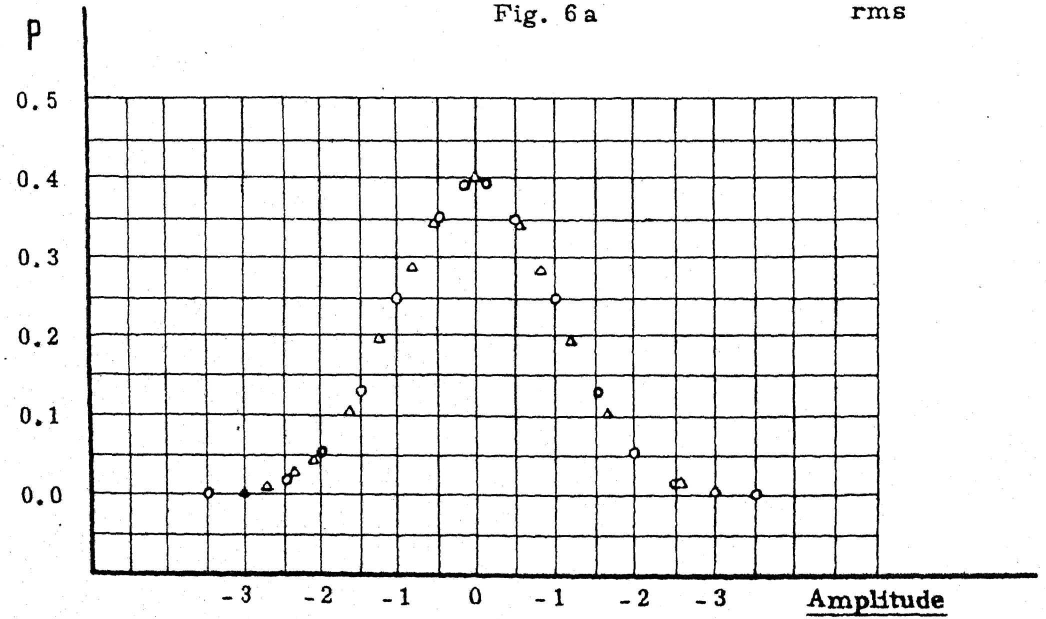


Fig. 6b

△ = measured density　　　　o = gaussian density

PROBABILITY DENSITY CURVES OF THE INSTANTANEOUS VALUES OF INPUT SIGNAL (Fig. 6a) AND OUTPUT SIGNAL (Fig. 6b) OF A 100 cps BAND WIDTH FILTER