Factors to be Considered in Calculating the Input Earthquake Force to Buildings

by

K. Matsushita, M. Izumi, Kuang-Jui Hsu and I. Sakamoto

ABSTRACT

The earthquake force obtained from data of the ground surface motion has often been regarded as the earthquake input force to buildings. For example, the Japanese Building Code contains, to some extent, this idea. Response calculations of buildings have shown the difference between the earthquake force and the input earthquake force. But this method also needs re-examination, as many examples show slight damage of structures despite big input forces estimated by this method from the same earthquake.

The authors point out the factors that should be considered and calculate their effects, and propose a simplified and practical calculation method.

The factors and their effects are as follows:

1. Acceleration records obtained at the ground-surface are not suitable as input data for response calculations, because they contain the effect of the surface soil and the boundary. Because of the non-linearity of the soil, through which the earthquake forces are transmitted, the forces are modified greatly.

2. In the design of real structures, it is necessary to consider the difference between calculated and real period of the structure, and the variations expected in the characteristics of the earthquake motions.

3. A building influences the ground motion around it. This effect is greater for buildings with a bigger mass and rigidity. When the size of the building is large, the forces acting on it are averaged to some extent, and peak values possibly obtained in a small accelerometer will not appear so sharp.

4. Earthquake force gives effects only when it is transfered into potential (strain) energy of structure, and the motion of structures without big accumulation of strain energy is not directly connected to the damage of structures.

5. Effects of wave velocities within the structure can not be neglected in many cases.

6. Absolute axes are sometimes more convenient for the analyses of the motion of structures.

In ordinary cases, effects of the factors mentioned in 1.3. - 6. have a tendency of decreasing the calculated values of input earthquake force obtained from the values of the ground-surface record, while the second item may increase it.

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Synopsis

'Input Earthquake Force to Structures' are different from the 'Existing Earthquake Forces' obtained from data of the ground surface motion. When earthquake forces are applied to a structure, they are modified by various elements, such as vibrational characteristics of structures and soils, their dimensions and their restoring force characteristics including the strength.

The factors and their effects are estimated by calculation, and compared with the features of earthquake damage in this paper. The most of these factors show a tendency of decreasing input earthquake forces to structures.

1. Introductory Remarks

The first proposal to estimate input earthquake forces to structures was made by the late Prof. Sano in 1916. He introduced the concept of 'Seismic Coefficient (K)', and he assumed a lateral force F (=kW) as the input force to a part of a structure of weight W, where k is equal to /g for the ground acceleration of (*1). In this formula, the input acceleration is assumed equal to the magnitude of the ground acceleration.

Great progress has been made since then; d (the ground accelerations) are now obtained by strong earthquake accelerographs as a function of time; structures, which were assumed as rigid bodies in Sano's formula, are replaced by simplified multi-degree of freedom systems, and responses of the structures to the earthquakes are calculated by solving the equation;

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [K]\{x\} = -[m]\{d\} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

and existing acceleration obtained at the ground surface is used directly in the calculation.

In this method, the characteristics of the grouna acceleration have a large

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effect on the response, and the selection of α, based on the mechanism and magnitude of earthquake, conditions of media of earthquake-waves and ground surrounding the structures, has gradually been considered, and random theory has sometimes been applied. But still, there are cases where the results of response calculations can not explain the feature of the earthquake damage.

In this paper, the method to estimate the input earthquake force through response calculation is re-examined, factors to be considered for the input-force estimation is discussed on the basis of calculated results. Authors also propose a simplified and practical method of calculation.

2. Factors to be Considered for Estimation of Earthquake Force

As stated in the synopsis, the input earthquake force to structures is substantially different from the existing earthquake force. For example, if a mechanism to decrease the transmission of earthquake force from the surrounding soil to the structure is installed at the boundary between the structure and the soil, the input earthquake force will be remarkably decreased in comparison with existing earthquake force. This kind of mechanism is called, 'Earthquake-Free Mechanisms'. Some of the mechanisms contain difficulties in the application to real structures, but most of them can show a remarkable effect in model tests(*2).

Factors to be considered for the estimation of input earthquake force to structures beside those already listed (like [m] , [k] , [c]) are as follows:

(1) Characteristics of Earthquake Waves Expected at a Region.

When a site for a structure to be designed is fixed, the region of epicenter of earthquakes which may give big effects to the structure is being predicted to some extent with the mechanism and magnitude of the earthquakes. The magnitude, for example, is estimated from statistical data and calculation of accumulated energy within and surrounding the region. From the predicted mechanism of the earthquakes and wave-path some of the characteristics of the earthquake wave's may be guessed. All of these factors are closely related to input earthquake forces, but they belong to seismological problems and are not discussed in this paper. In macroscopic views, the Pacific Ocean of Japan is affected by large scale earthquakes and their epicentre is far in the sea, while Japan Sea side suffers from earthquakes of small scale and of near epicentres.

(2) Effect of the Ground and Surrounding Soil of the Structure

The surrounding soil of a structure plays a quite important role in the input earthquake force to the structure as the earthquake energy is transmitted through the soil. Even data observed in an earthquake at a region have various characteristics according as the building-soil conditions of the observatories. In the most of present calculation methods, the soil is neglected and the structure is assumed to be clamped to the ground rigidly. Besides, as the input data, acceleration records obtained at the surface are directly used conventionaly. Through the soil, the characteristics of the seismic waves are modified and the energy is to some extent absorbed. Therefore it is not proper to neglect the effect of the soil in the estimation of the input force.

(3) Mass, Rigidity and Size of the Structure.

In the present calculation method, the mass ([m]) and rigidity ([k]) are considered, combined and expressed in the form of periods. The mass and rigidity mentioned here are not only the combined ones. They also effect the input earthquake force individually when the vibration of the soil and the structure is discussed seperatelly. In the structure-soil system, the second effect is contained in the calculation as interaction between the soil and structure.

Generally speaking, the energy from soft and weak ground is not easily absorbed to a structure of large mass and rigidity, and the calculated results also show these tendencies.

(4) Range in the Vibrational Characteristics of both the Earthquake Force and the Structure.

As mentioned in (1), the characteristics of effective earthquake waves can not be predicted precisely unless the epicentre, size and mechanism of the earthquake is correctly foreseen, and, consequently, some assumptions should be introduced in the calculation. Therefore, some range in the characteristics of waves must be considered. On the other hand, the vibrational characteristics of the mathematical model representing the original structure are also not very accurate. In the calculation, the effect of the secondary structural elements are apt to be neglected, and the rigidity is usually under estimated. If this is checked by tests, the rigidity obtained becomes larger than the real one, because of the difficulty in producing the large excitation corresponding to the real effective earthquakes. Concerning the uncertainties of the characteristics of seismic waves and the structure, the former is more indefinite.

(5) Energy Calculation

The problem of calculating input earthquake force is directly connected with the structural design. The degree of damage in a structure is sometimes more closely related to the strain energy than the size of the force, and, in such a case, the effect of an earthquake may be better expressed in terms of energy than in terms of force.

(6) Transmission Speed of Waves

The effect of transmission speed of energy can not be neglected, when the response of the structure is calculated. Otherwise, the displacement response is apt to give misleading results.

(7) Adoption of Absolute Axes

Though the axes of the coordinate system originated at the base of structure in the present calculation method, adoption of absolute axes seems rational including the case when the psycological effect of the vibration to the residents is discussed.

In the following chapters, articles (2)-(7) will be discussed in more detail.

3. Effect of Various Factors on Earthquake Input Force to Structures

3-1. Structures and the Soil (Corresponds to Articles (2) and (3))

When the effect of soil to input force is calculated, there are two methods that can be used.

One is to include the soil above base rock into the structure-soil system. (The very dense and hard layer as compared to the softer surface layers are referred to as bed rock hereafter.)

The other is to treat the soil as an entirely different body from the structure, considering their effects separately. The former is convenient for the response calculation, and the latter is suitable in the energy calculation. (In this paper both methods are applied, and the same tendencies are obtained, but the obtained values to some extent rely upon the used assumptions, respectively.)

Analyses through Calculation

When the response of the structure-soil system is calculated, one of the most important, but also the most difficult problem is the selection of input excitations. The recorded data usually contain the effect of the surface layers and surface vibration, and they are not recommendable for input data for calculation. Many research works have been made concerning this problem, and, among them, calculated vibration at the bed-rock from the soil data and earthquake records at the ground surface(*3) and band limited white noises(*4) are often used. Some data have obtained at the bed-rock, but they have not necessarily white noise characteristics.

On the other hand, the structure-soil models are usually obtained as the replacement of real ones by finite element (F.E.) methods, by lumped mass-spring systems and other systems. (Fig.1)

The calculated records at the surface using these kind of models give us curves where the vibration of higher order is erased. This can be avoided to some extent, when the elements in F.E. method or the sizes of masses in lumped mass systems are choosen small and the input excitation is given to the bed-rock not uniformly but with phase differences. However, the values of the phase difference needs one more assumption in the calculation. (Except a few cases, calculation is made through lumped mass-spring systems, where the stiffness matrix is formed by assuming the velocity of S waves 50 - 3000 m/sec for the imaginary braces and P waves for the imaginary colums and beams. When the force deflection relations are not very different, the calculated results by F.E. method and the mass-spring system do not show fundamental differences.) 'In the elastic response of structural systems, the effect of the mass of a structure is checked by changing the ratio to that of the soil, and the acceleration of that mass decreases except at the resonance period, but the difference is small. (Fig.2) When the plasticity of the soil is considered, the effect of the magnitude of the mass increases. The range of the soil where affected by a structure becomes smaller.

Fig.3 shows an example of relations between the ratio of input earthquake force and strength of the soil (Soil is assumed to be an idealized elasto-plastic material in this calculation). The restoring-force and displacement relation of the soil is one of the important factors that affect the input earthquake force and energy. (Fig.4) Some of the relations are studied concerning the restoring force of the dry sand and gravel against the vertical walls like walls of base floors of buildings, and one of these relations is shown in Fig.5(*5). The calculated results using this curve is not much different from those where the idealized elasto-plastic relation is used.

When the same problem is discussed as a phenomena of energy-transmission, it is convenient to combine the energy calculation with the calculation of reflection and transmission of waves or response calculation. When a block of soil is assumed in the ground, most of the energy fed to the block is transformed into strain energy and kinetic energy and then transmitted to adjacent blocks gradually. In general cases, both density and rigidity of layers increase according to their depth, and the angle of the waves to the horizontal surface increases when the waves approach to the surface from the boundary of the bed rock to the surface layers, reflected at the surface, some of the waves again come to the boundary of the bed rock and surface layer, but the critical ratio of reflection is relatively large when the waves come from a soft media to a hard one. Therefore the energy entered into the surface layers does not easily get out of the layers, and this is called' multiple reflection of waves at the surface layers'.(Fig.6)

The input energy to the structure can be calculated through the estimation of reflection and refraction of the waves. Some experimental data are available to the estimation of the ratio of the reflection and refraction to the original energy and energy consumption can be roughly calculated from soil-test results.

When a structure is built on the ground, the structure is also an adjacent body of imagined soil block, and the same steps in the calculation can be taken, though ratio of reflection and refraction differs from those between layers. The energy fed to the structure is transformed into kinetic and strain energy, and if damage produced in the structure, some portion of the input energy is consumed in the structure.

When the soil at the boundary between the structure is weak, energy is also consumed there easily, and the input energy to the structure decreases.

A simplified calculation gives the results shown in Fig.7, where input excitation to the bed rock is random like waves with the spectrum density propertional to the frequency, and where the restoring characteristics of soils are assumed to be an idealized elasto-plastic model. The ratio of energy consumption increases rapidly, when the strength of the soil decreases.

Comparison of Calculated Result to Earthquake Damage

More severe damage of structures is usually observed where the ground is weak and soft in comparison with the case of strong ground, and, consequently, the acceleration of the ground is supposed to be big.

However, the calculated result show, the tendency of a decrease in the value of maximum acceleration in the very soft ground, and the input earthquake force to a structure also decreases except in the case when the structure is very flexible and has a long natural period. Strong accelerograms obtained in an earthquake also show the similar tendensies; the maximum accelerations at the surface of the good loam are bigger than those around the surface of soft alluvium. Therefore, the damage of structures does not seem to correspond to the value of the force directly, but to the repeated feeding of energy caused by multiple-reflection of waves, and to the secondary effect of the soil, such as indifferential settlement. When the shape of land and layers are irregular, the multiple-reflection is also produced, energy can hardly be transmitted rapidly to other parts, and same kind of phenomena may be produced.

3-2 Wave-Transmission Speed

Replacement of a structure by a mass-spring system produces some difference in transmission speed of the force. The difference is small in the system where the first mode vibration is predominant, but when the structure is tall and flexible, the speed of the wave-transmission gives an effect to the response. Fig.8 shows the difference of the response of a structure between the cases where the speed of the wave is considered and neglected. Calculated result shows such a general tendency that the higher order vibration is prominent in the mode of the structure in the consideration of wave velocity. As for the maximum story-shear-forces, no fundamental difference is observed. (Fig.9)

3-3 Calculation of Input Energy and Effect of Range in Vibrational Characteristics (See Articles (3) and (5)

Energy from the boundary soil is sometimes effectively and sometimes ineffectively fed to the structure. For instance, in the resonance state, the energy is quite effectively given to the structure, increasing kinetic and strain energy within the structure. For the random excitation, some of the energy within the structure and from the boundary soil is canceled as the work done and the structure acts as if it selects the wave of a period equal to its own.

When ranges are given to both exciting waves and the structure, one convenient way may be to select the combination arbitrarily (in random selection), and to get the general tendencies by repeating the same steps for different calculation.

When the Gaussian distribution is assumed in the vibrational characteristics of the structure, and the random values are given to the phase difference of the input energy wave, and from the vibration phase of the structure in the response calculation and input wave phase, the input force and energy can be calculated where the average input energy value of the average structure (1T = 0.5) is taken as a standard. (Fig.10)

When the plasticity of the soil around the boundary is considered,the transmitted energy decreases and at the same time, the frequency characteristics of the input changes. One simplified method is to give the energy to the structure cutting the peaks at a level. The consideration of the plasticity of soil does effect the response calculation. (Fig.11)

4. Simplified and Practical Calculation Method

As shown in previous articles, various factors concern to the input earthquake force to the structure. Consideration of these factors requires a complicated calculation that includes many assumed values. This kind of study is essential and should be continued in order to catch the real input earthquake force to the structure, and, at the same time, simply and practical methods are important to design the structures.

The building is fed the earthquake energy only through the surrounding soil, from the base and walls under the ground. (Fig.4) Therefore, in the simplified calculation, the effect of the soil only around the building can be considered. The excitation can be assumed from the data near or at the surface with some modification. Then a mass-spring system shown in Fig.12, where A can be assumed as a fixed point, is obtained. The force is fed to the system near the building base. When the force given to the soil part (from the bed-rock to the load-points) is omitted, a system shown in Fig.13 is obtained.

The simplest model of the structure is the shear-frame. The equation of motion is given from the Eq.(1), where $[K]$ and $[C]$ are tri-diagonal matrices. The term $-[m]\{\alpha\}$ is caused by the transform of the axis from the absolute one to the moving one taken at the base of the structure, and the equation is in the same form to that when the system is loaded at each mass (i) by $-m_i \alpha$. α is a time variable, but during a very short time, it does not change much though the deflection changes, and naturally acceleration changes, and term corresponding to the load - $[m]\{\alpha\}$ differs as the moving axes are used. Therefore the formula (1) may be written;

$$m_r\ddot{X}_r + C_r(\dot{X}_r - \dot{X}_{r-1}) + K_r(X_r - X_{r-1}) - C_{r+1}(\dot{X}_{r+1} - \dot{X}_r) - K_{r+1}(X_{r+1} - X_r)$$

$$= -m_r(\alpha + \beta\ddot{X}_r) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where $0 \leqq \beta < 1$.

The equation (we call it Matsushita - Izumi's equation) is same to the ordinary equation of shear-frame vibration when $\beta = 0$. And when the equation is made at the stage after the structure is deformed, β becomes 1. The spring constant between the first and second masses (K_1) can include the characteristics of the surrounding soil.

The calculated results show that this formula can be used for simplified estimation of the vibration.

5. Conclusive Remarks

From the above ~~statements, the followings~~ are to be concluded:

(1) The Relation of Structure and Soil

The vibration period of structure is much affected by the mass of the structure together with the rigidity, and also, the mass of the structure compared with that of the soil gives a big influence to the input force of the earthquake. In the soft-subsoil condition, the input force of earthquake decreases greatly when the mass (=apparent density) is large. But, in hard sub soil, the amount of input force can be larger than that of the acceleration of the ground. In general cases, as the mass of structure is small, the input force of earthquake is larger than those of the acceleration of the ground.

In relation to the size of structure, when the rigidity of the foundation is perfectly large enough, the input forces with different phases are averaged, and advantageous results can be obtained for the large size structure(*6). It is efficient to decrease the input force of earthquake by building the structure with rigid and heavy foundations and basements.

(2) Considering Some Degree of Width in Relation to Structure and the Characteristics of Sub Soil.

The input force of earthquake can be increasing when considering the width of characteristics both in structure and soil. In this case, considering about 20% of increasing input force may cover 80% of all conbinations.

(3) Treatment of Energy

In treatment of input force of earthquake as an energy, it is convenient to connect the problem with response and wave-transmission calculation.

(4) The Velocity of Wave Transmission and Absolute Axes

It is necessary to consider the absolute axes and the velocity of wave transmission to catch the phenomena of vibration especially when the displacement and mode of the structure are dicussed. But, the influence of this factor to the input force estimation is small.

In summary, it can be said for the practical aseismic-design of a structure, that the ground effect is remarkable. (See Fig.12) Besides, when the basement of the building is strong, rigid and massive, the effect of the soil has the influence of decreasing the input force to half of that obtained from the ground surface-records. The secondary effect of the soft ground can not be overlooked, and the structure should be strongly supported directly or by piers or piles on rigid and strong layers.

Except special cases like resonance, the value of inertia forces produced in rigid and heavy structures in larger than that of flexible and light structures in ordinary ground conditions. But at the same time, rigid and heavy structures effectively restrain the ground motion. Besides, the input earthquake forces cannot exceed the strength of the surrounding soils even when the earthquake is very strong. Considering these effects, we can not say that flexible structure is more advantageous against earthquakes or vice versa, but there is the most suitable structural design to the given conditions including the problems of construction details, finishing, and comfort.

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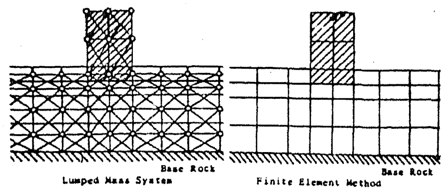

Fig. 1 Structure-Soil Models

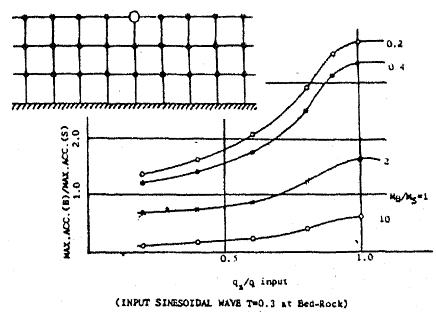

Fig. 3 Ratio of Input Acceleration to that at Soil Surface as Functions of Soil Strengthes and Mass-Ratios

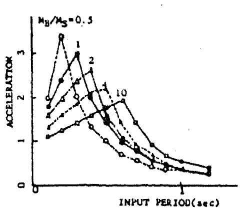

Fig. 2 Effect of Mass in Structure-Soil Model

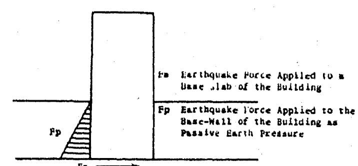

Fig. 4 Earthquake Forces Applied to a Structure

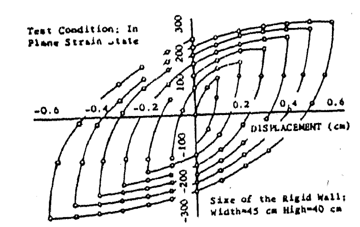

Fig. 5 Restoring Force-Displacement Relation of Gravel (Bulk Density = 1.62 g/cm^3 Relative Density = 26%)

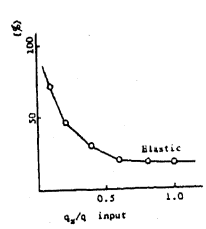

Fig. 7 Ratio of Energy Consumption Calculated for the 'Model Shown in Fig. 1 (h=0.05) Input Energy is Calculated at the 1st Layer Counting from the Base Rock, and Output Energy is that at the Surface Layer Including the Structure

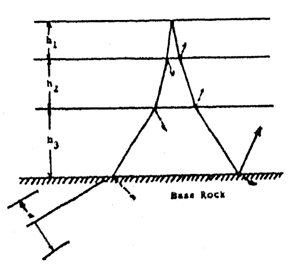

Fig. 6 Model of Reflection and Refraction in Surface Layers

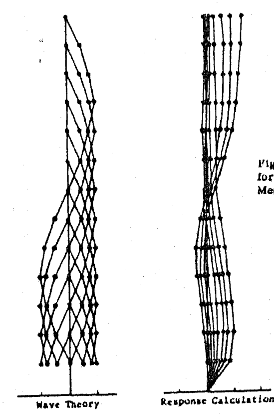

Fig. 8 Response Modes for Different Calculation Method

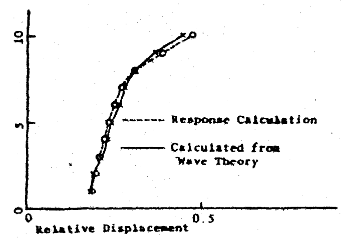

Fig. 9 Comparison of Shear Coefficients Obtained by Different Method

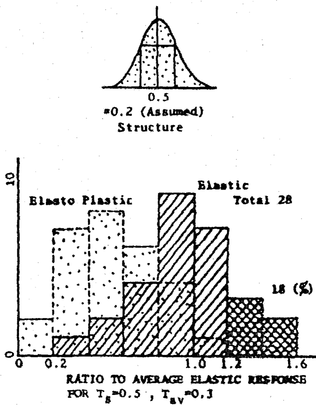

Fig. 10 Response of Structures to Band Limitted Random Wave (Spectrum density is proportional to the frequency, and T $(T_{av} + \frac{1}{2} T_{av})$ where T_{av} = 0.3)

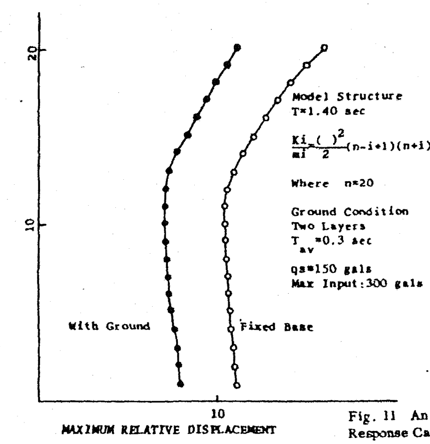

Fig. 11 An Example of Response Calculation with and without Ground Effects

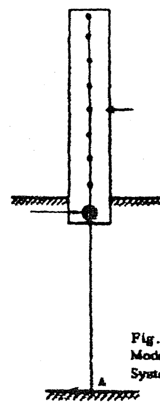

Fig. 12 Model System

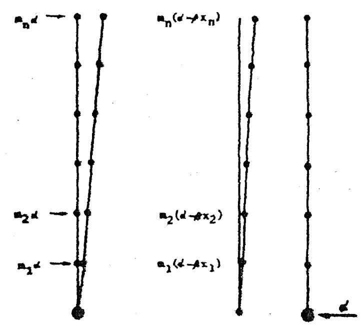

Fig. 13 Comparison of Calculation Method