A PROBABILISTIC MODEL FOR SEISMIC FORCE DESIGN

by Jack R. Benjamin[1]

Introduction

The purpose of this paper is to introduce a rational probabilistic technique for use in the making of design decisions involving seismic loading. A compound Poisson model is used to model the occurrence of quakes and their Modified Mercalli intensity at a site from the historical record using Bayesian probability concepts. These Bayesian forecasts are then used in a statistical decision theory context to optimize design decisions.

Intensity and Occurrence

Divide the problem of optimum design of structures for seismic activity into two parts. The first portion is concerned with forecasting the occurrence and violence of the ground motion at a site from the historical record and the second step is the optimizing of the seismic design assuming that the probabilistic model of seismic activity is reasonably adequate.

The violence of ground motion associated with an earthquake is subjectively measured on the Modified Mercalli intensity scale with a range from I to XII in which values of six and smaller are of little concern to the designer because of the minimal damage involved.

Assume a site is under study in the San Francisco Bay region. Historical records do not provide much more than the impression that no one place in an active fault area is immune to seismic disturbances. If this assumption is acceptable in a general way, the record of MM intensities gives a measure of the maximum consequences of earthquakes for the area of interest along with the time of their occurrences.

Probability Model

The Poisson probability law is the most commonly used model in modelling the occurrence of earthquakes.[1,2,5,6,7] The model law assumes that an event is equally likely to be found in any unit time interval along the time axis, Fig. 1.

The probability of finding r events in time t if the mean rate of occurrence, μ, is known, is given by:[4]

$$p(r|t, \mu) = \frac{e^{-\mu t}(\mu t)^r}{r!} \tag{1}$$

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Table 1[3] is a summary of the seismic activity in the San Jose and San Francisco areas for two time periods roughly separating times of poor and good records. The 1836-1902 period shows the influence of poor records particularly at low intensity levels. Using the 59 year period of good records, a total of 23 earthquakes are found for all intensity levels. The mean rate of occurrence of Poisson events, μ is estimated to be:

$$\mu = \frac{23}{59} = 0.39 \text{ earthquakes per year}$$

For practical purposes use $\mu = 0.4$. The Poisson occurrence model gives the same forecast for each year. Consider a one year period and r earthquakes.

$$\mu = 0.4$$

$$t = 1$$

$$p(r|1,\ 0.4) = \frac{e^{-0.4}(0.4)^r}{r!}$$

r	p(r 1, 0.4)
0	0.67
1	0.27
2	0.05
3	0.01

If the Poisson occurrence model fits reality and the mean rate of occurrence if 0.4, the chance of no earthquakes in one year is 0.67, of one earthquake is 0.27, of two earthquakes is 0.05 and of three earthquakes is 0.01. The model includes any number of events but the probabilities of more than three such events are so small that their occurrence can be neglected.

Difficulties in application of this model arise from several sources. First, the model may not fit the real phenonema; second, the record may not be adequate to estimate the parameter of the model; and, third, occurrence by itself is insufficient. That is, design requires both occurrence and the ground motion characteristics at the site.

Let the Posson events be described by an influence function ranging discretely from VI to XII fitting the MM intensity scale. The influence function is multinomial.

$$p(k_1,\ k_2,\ \ldots\ k_r \mid n,\ p_i) = \frac{n!}{k_1!\ k_2!\ -\ k_r!}\ p_1^{k_1}\ p_2^{k_2}\ \ldots\ p_r^{k_r} \quad (2)$$

$$k_1 + k_2 + \ldots k_r = n$$

$$p_1 + p_2 + \ldots p_r = 1$$

If the record is sketchy, Bayesian forecasting procedures can be used (a diffuse prior is assumed). The resulting expressions are:

Poisson

$$p[r_o \mid t_o, r, t] = \frac{(r + r_o)}{r!r_o!} \frac{\left(\frac{t_o}{t}\right)^{r_o}}{\left(1 + \frac{t_o}{t}\right)^{r+r_o+1}} \quad (3)$$

The record is r events in time t and the forecast is r_o events in time t_o.

The multinomial expression is:

$$p[k_{1,0}, k_{2,0}, .. k_{r,0} \mid k_1, k_2, .. k_r] = \frac{n_o!(n+r-1)!(k_1+k_{1,0})!\ldots(k_r+k_{r,o})!}{k_{1,0}!k_{2,0}!..k_{r,o}!k_1!k_2!\ldots k_r!(n+n_o+r-1)!} \quad (4)$$

Observed: $k_1, k_2, \ldots k_r$ in n trials

Forecast: $k_{1,0}, k_{2,0}, \ldots k_{r,o}$ in n_o trials

Table 2 lists some of the possible events to be considered further. The large number of combinations to be considered can be reduced greatly by dropping all events having a chance of occurrence of less than, say, 0.01. The desired probabilities are found by multiplication of the probability of the number of Poisson events by the multinomial probability of finding the combination given the record. The events that must be considered are given in Table 3 with their probabilities of occurrence in a year. The Bayesian expressions were used in these calculations.

Note that the Bayesian probability of finding an intensity XII is 0.01 although no events of this type have been observed. In contrast, the classical forecast probability for this event would be zero. The Bayesian technique gives a non-zero probability for every possible event regardless of the record. Note, that the Bayesian probabilities are up-dated as more information is developed for all probabilities are conditional on the record.

Statistical Decision Theory

Statistical decision theory is concerned with the optimization of decisions under uncertainty. Major earthquakes are such rare events that it is not practical to sample their occurrence and characteristics. Designs must be finalized based on the historical record and engineering judgement.

Every structural design includes a number of design alternatives. Each such design has different characteristics. For example, seismic loads can be carried by a long period rigid framed structure or by a short period shear wall structure. In the absence of knowledge of the future loading, if any, the optimum decision is unknown and the problem comes under the techniques of statistical decision theory.

The technique is illustrated in Fig. 2 in which the engineer can take one of two actions. After taking the action, he will find the future. In this case, the future consists of the events of Table 3 with their forecast probabilities of occurrence in one year. If the engineer accepts action a_1, a flexible rigid frame design, and finds the various futures, called states, he will receive value depending on a_1, and the state actually found. If value can be expressed in terms of money and is linear with money, the decision rule is simply expressed: accept the action with the largest expected value. An expected value is defined as the product of the probability of finding a state by the value given the action and state. The expected value of a_1 is equal to the sum of the products of probabilities of states by the associated value. In the case of seismic design, the comparison of value is on the basis of expected losses so that the optimum action has the minimum expected loss.

In Fig. 2, the occurrence of events with probabilities of less than 0.01 have been omitted. Thus the total of all state probability measures for each action is 0.96. In practical problems, probability measures to 0.001 might be used. It will be found, however, that these more refined computations are difficult to justify in view of uncertainty about the value items.

Assume that the annual cost of a rigid frame structure is $1000 more than that of a shear wall structure, both designs satisfying the seismic design code. This difference might arise from the cost of architectural features to limit damage to partition walls and the use of a frame structure in a situation where adequate bearing walls are provided by the architect.

Losses from seismic activity arise from damage to structural members, non structural elements including utilities, and contents of the structure. These losses must be estimated by the engineer and constitute the major unknown. Table 4 illustrates one way of making such calculations. For each design, quake possibility, and damage source.

A possible loss is estimated along with the probability of finding this loss, P. For example, with the frame design and an intensity of X, the chance of a $1000 loss is 0.5 and of a $2000 loss is also 0.5. The total expected loss is calculated as the sum of the products of the loss magnitudes by their respective probabilities. For the frame design and intensity X, the expected loss is

$$(1000)(0.5) + (2000)(0.5) + (2000)(0.5) + (3000)(0.5) + (1000)(0.5) + (2000)(0.5) = \$5500$$

The average or expected losses show that the special features of the more expensive rigid frame design reduce the expected losses for quakes of small intensity compared to a conventional shear wall design. Note, that with intensity XII, by definition almost everything is destroyed so that this condition does not influence the design decision. This accounts for the note to neglect this item in the decision tree, Fig. 2.

Another very important factor that the engineer must consider as the maker of the decision, is the influence of such performance on his own professional practice. While most difficult to evaluate, this factor must be considered. The values entered in Fig. 2 neglect this factor.

The decision tree of Fig. 2 is now evaluated by summing the products of probability of event by cost of damage given the event. For a_1,

$$\text{Expected Loss} = (0.67)(0) + (0.16)(0) + (0.05)(500) + (0.01)(1700) + (0.01)(3500) + (0.01)(5500) + (0.02)(24{,}500) + (0.01)(\text{Neglect}) + (0.01)(500) + (0.01)(0) = \$627$$

The expected loss for a_2 is found to be $929. These figures are entered on the decision tree of Fig. 2. Now the annual cost of the rigid frame adds to its annual expected loss giving a total annual cost of $1627. It is seen that action a_2 is preferred to a_1.

The important points in this analysis are, first, the technique is rational. It affords a procedure for making logical statements about the unknown future which reflect all the engineers knowledge. While the expected losses of Table 4 may be difficult to estimate, some such procedure must be used if differing designs are being considered and one must be chosen. Second, the technique can be expanded as the state of knowledge improves. For example, if standard earthquakes are developed to represent possible loadings, it is possible to estimate damage from a given structural response to each standard ground motion. Strong subjective elements remains, but this will always be true. Finally, if in the example, the engineer chooses a_1, the more costly rigid frame, he is recognizing that factors not specifically considered in the analysis have a real expected value in favor of the rigid frame in excess of $698 per year.

Conclusion

A compound Poisson model of earthquake occurrence and MM intensity has been combined with the techniques of statistical decision theory to provide a rational format in the making of seismic design decisions. The procedures are capable of expansion and modification as our knowledge of the seismic engineering problem improves.

Acknowledgement

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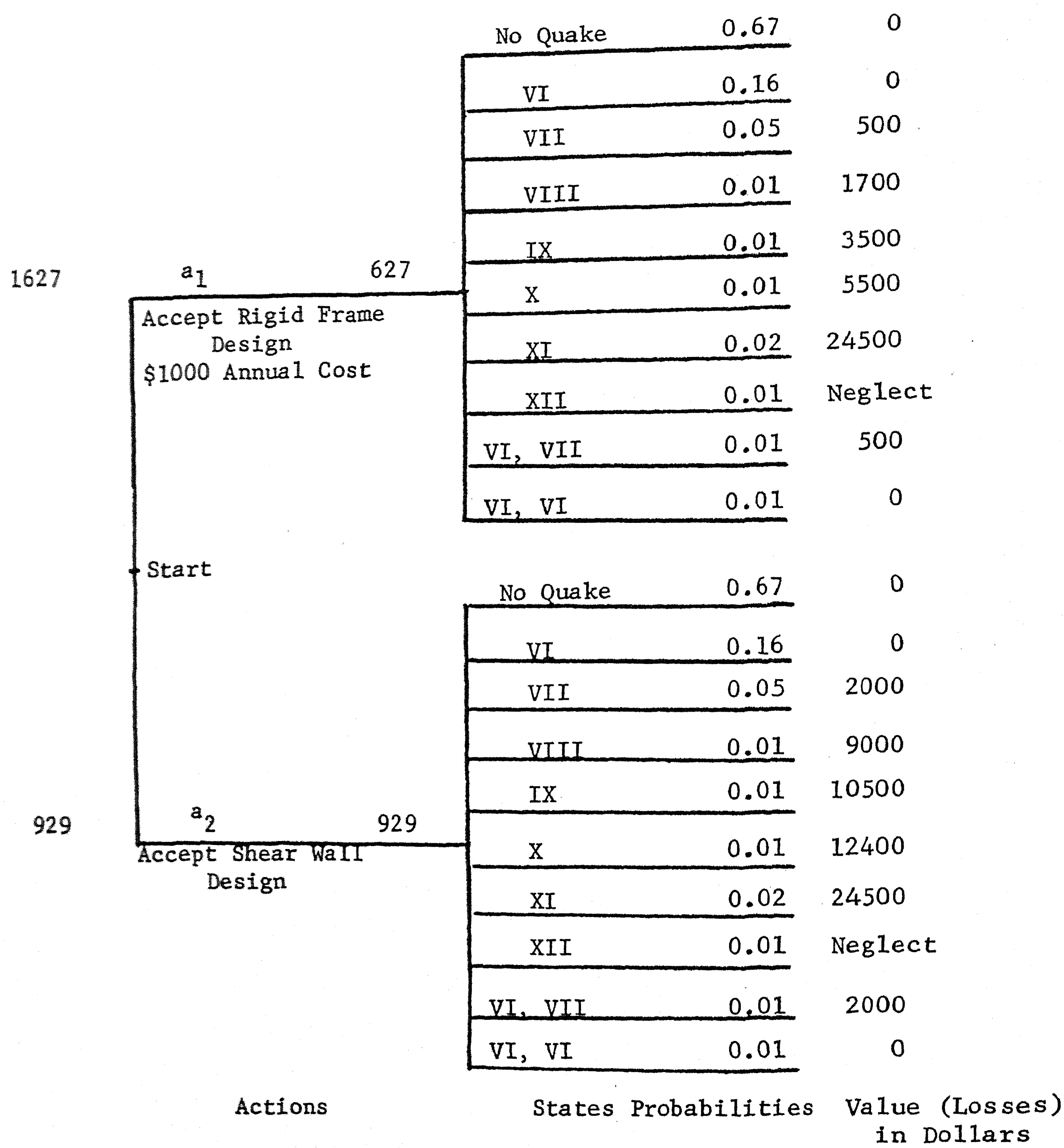

Fig. 2 Decision Tree

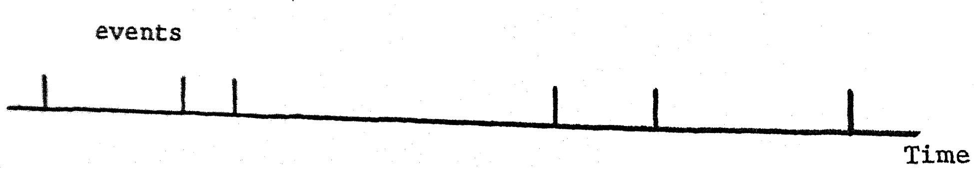

Figure 1 Simple Poisson Events

Table 1

MM Intensity (SF - San Francisco, SJ - San Jose)

Time (1)	VI SF (2)	VI SJ	VII SF (3)	VII SJ	VIII SF (4)	VIII SJ	IX SF (5)	IX SJ	X SF (6)	X SJ	XI SF (7)	XI SJ	XII SF (8)	XII SJ
1836-1902	3	1	2	1	1	1	2	1	1	1	0	0	0	0
1903-1961	5	12	1	4	0	0	0	0	0	0	1		0	0
Total	8	13	3	5	1	1	2	1	1	1	1		0	0
Total by Intensity	21		8		2		3		2		1		0	

Table 2

Possible Events In One Year

VI	VII	Intensity VIII	IX	X	XI	XII
0	0	0	0	0	0	0
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
		etc.				
2	0	0	0	0	0	0
0	2	0	0	0	0	0
0	0	2	0	0	0	0
		etc.				
3	0	0	0	0	0	0
0	3	0	0	0	0	0
0	0	3	0	0	0	0
		etc.				
1	1	0	0	0	0	0
0	1	1	0	0	0	0
0	0	1	1	0	0	0
		etc.				
1	1	1	0	0	0	0
0	1	1	0	0	0	0
		etc.				
2	1	0	0	0	0	0
1	2	0	0	0	0	0
		etc.				

Table 3

Probabilities of Events in One Year

Events	Probability
No Quake	0.67
VI	0.16
VII	0.05
VIII	0.01
IX	0.01
X	0.01
XI	0.02
XII	0.01
VI, VII	0.01
VI, VI	0.01
Other	0.04

Table 4

Design	Quake	Structural Loss	P	Non Structural Loss	P	Contents Loss	P	Total Expected Loss
Frame	VI	0	1	0	1	0	1	0
	VII	0	1	0	0.5	0	1	
				1,000	0.5			500
	VIII	0	0.9	0	0.2	0	0.5	
		1,000	0.1	1,000	0.5	1,000	0.5	
				2,000	0.3			1,700
	IX	0	0.7	1,000	0.3	1,000	0.5	
		1,000	0.3	2,000	0.7	2,000	0.5	3,500
	X	1,000	0.5	2,000	0.5	1,000	0.5	
		2,000	0.5	3,000	0.5	2,000	0.5	5,500
	XI	3,000	0.5	10,000	0.5	2,000	0.5	
		10,000	0.5	20,000	0.5	4,000	0.5	24,500
	XII	Very Large		Very Large		Very Large		Very Large
Shear Wall	VI	0	1	0	1	0	1	0
	VII	0	0.5	1,000	0.5	0	1	
		1,000	0.5	2,000	0.5			2,000
	VIII	1,000	0.5	5,000	0.8	1,000	1	
		3,000	0.5	10,000	0.2			9,000
	IX	2,000	0.5	5,000	0.7	1,000	1	
		4,000	0.5	10,000	0.3			10,500
	X	2,000	0.3	5,000	0.5	1,000	0.5	
		4,000	0.7	10,000	0.5	2,000	0.5	12,400
	XI	3,000	0.5	10,000	0.5	2,000	0.5	
		10,000	0.5	20,000	0.5	4,000	0.5	24,500
	XII	Very Large		Very Large		Very Large		Very Large