SEISMIC FORCES AND OVERTURNING MOMENTS IN BUILDINGS, TOWERS AND CHIMNEYS

by

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ABSTRACT

A rational preliminary design procedure for seismic loading should be reliable enough so that only a final check on the designed structure should be required. Most current code provisions are dependent almost exclusively on the fundamental period of the structure. An initial attempt to develop a more rational design approach, taking into account all the significant variables, is presented.

Numerical results for idealized shear-beam and flexural beam buildings are presented for the distribution of effects along the height of the structure. The results are presented in terms of an acceleration factor, which, when multiplied by the story mass and the base design acceleration, yields seismic forces corresponding to a consistent distribution of maximum probable shears and a moment reduction factor which yields a reasonable bound on the overturning moments.

It is shown that the seismic lateral loads used for design must reflect both the type of framing of the structure and the fundamental frequency of the building, inasmuch as the latter determines which branch of the response spectrum governs the response of the structure.

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SYNOPSIS

An initial attempt to develop a rational design procedure for seismic lateral loading is presented. The factors affecting seismic design are discussed, and results of analyses on idealized buildings are presented. It is shown that seismic lateral loads used for design must reflect both the type of framing of the structure and the fundamental frequency of the building. The overturning moment reduction factors also depend on both of these factors.

GLOSSARY OF TERMS

a_x, a_i = acceleration coefficients at levels x and i, respectively

C_j = modal contribution factor for mode j

D_a = spectral acceleration bound

D_d = spectral displacement bound

D_v = spectral velocity bound

F_x, F_i = lateral seismic forces at levels x and i, respectively

g = acceleration of gravity

h_x, h_i = height above base at levels x and i, respectively

J_x, J_i = overturning moment coefficients at levels x and i, respectively

M = overturning moment at base of structure

$\overline{M}_i$ = probable overturning moment at story i

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$\widetilde{M}_i$ = cantilever moment caused by lateral seismic forces at story i

m_i = mass at story i

n = number of stories

T = fundamental period of structure

V = shear at base of structure

$\overline{V}_i$ = probable shear at story i

W = total weight of structure

w_i = weight at story i

$x_i^{(j)}$ = modal response at story i in mode j

$\overline{X}_i$ = probable combined response

ω_j = circular frequency of mode j

Whenever applicable, the symbols introduced in this paper are identified by an overbar to distinguish them from the corresponding symbols used in Reference (1).

INTRODUCTION

The designer of a building or other structure subject to seismic inputs must make major decisions concerning the magnitude and distribution of seismic forces to be resisted by the structure without the benefit of any exact analysis at the preliminary design stage, and often does not perform the required analyses even at the final checking stage. Any preliminary design procedure used should be reliable enough so that only a final check should be required after the structure has been completely designed. Ideally, from the designer's standpoint, this check should be performed only for the most unusual structures.

The current codes and regulations for seismic design, e.g., References (1) and (2), generally present two types of information: those dealing with the magnitude of the seismic effects, e.g., the base shear V and base overturning moment M; and those defining the distribution of the effects along the height of the building, i.e., lateral forces F_x and moment coefficients J_x. The code expressions for V and M are primarily dependent on the fundamental period, T, of the structure. To a lesser extent, the code formulae for F_x and J_x seem also to be derived on the basis of first or fundamental mode response of an idealized shear beam, modified slightly on the basis of empirical and analytical evidence.

This study was undertaken to investigate the importance of other variables which have an effect on the seismic response. The aim of this study is to provide a rational basis for the preliminary design of structures subject to seismic loading, taking into account all of the significant variables. Primary emphasis is given to the distribution of effects along the height of the structure.

FACTORS AFFECTING SEISMIC DESIGN

The three factors of primary importance in determining the seismic response of a structure are the type of framing, the mass distribution, and the relative contribution of the higher modes of vibration.

The type of framing affects both the shape of the fundamental mode and thus the distribution of seismic forces corresponding to that mode, as well as the separation of the successive higher frequencies. An exhaustive study of the behavior of actual structures is clearly outside the scope of this paper. Sufficiently reliable results can be obtained by studying two simplified extremal types: an idealized shear beam which may be used to represent a moment-resistant frame and an idealized flexural beam which represents quite accurately a chimney or a building with substantial shear walls. For illustrative purposes, results from an "exact" analysis of a moment-resistant frame, including the effect of joint rotations and member elongations, are also presented.

The effect of the mass distribution is similar to that of the type of framing, in that it affects both the shape of the fundamental mode and thereby the magnitude and distribution of the first-mode responses, as well as the separation of the higher modes.

The relative contribution of the higher modes can most easily be taken into account by considering a response spectrum which can be represented by three straight-line segments:

(i) a constant-displacement branch, corresponding to very low frequencies (less than 0.1 to 0.3 cps);

(ii) a constant-velocity branch, corresponding to intermediate frequencies; and

(iii) a constant-acceleration branch, corresponding to high frequencies (above 2 cps).

On the constant-displacement branch, all modes have equal contribution factors, i.e., spectral displacement values, in the total response, whereas for the constant-velocity and constant-acceleration branches, the contribution factors are proportional to $1/\omega_j$ and $1/\omega_j^2$, respectively, where ω_j is the circular frequency of mode j. Thus, the constant-displacement branch yields the largest contribution of the higher modes, and the constant-acceleration branch the smallest, with the constant-velocity branch giving intermediate values.

It should be noted that even for structures with very low fundamental frequencies, only the frequency corresponding to the fundamental mode will fall on the constant-displacement branch, the higher modes being on the velocity or acceleration branches. For most tall buildings, several modes may fall on the constant-velocity branch, while for very stiff buildings or chimneys, all the modes will be on the constant-acceleration branch.

In order to accentuate the effect of the higher modes, in this study all the modes are assumed to be on only one of the three branches of the spectrum. Thus, the results presented below for the displacement branch are not significant themselves, as they grossly overestimate the contribution of the higher modes.

METHOD OF ANALYSIS

The method of analysis used in this study is based on the modal summation technique. Only lateral vibrations are considered. The masses are assumed to be lumped at the floor lines. For the idealized shear and flexural beams, the lateral stiffness matrix was generated directly, whereas for the "exact" analysis, the effective lateral stiffness matrix, after the elimination of all degrees of freedom other than the horizontal displacement of the centerline joints of the structure, was obtained from an extension of the STRESS program. For each structure, the natural frequencies and mode shapes for all modes of vibration were evaluated; the former were normalized to a constant fundamental period and the latter to a unit base input displacement. Next, the accelerations, shears and moments corresponding to each mode of vibration were evaluated separately. Finally, the probable value of the total response, calculated as the square root of the sum of the squares of the individual modal contributions for each response of interest, was obtained according to the formula:

$$\overline{X}_i = \sqrt{\sum_{j=1}^{n} [c_j \; x_i^{(j)}]^2} \qquad (1)$$

where $\overline{X}_i$ = probable combined response (shear or moment) at story i

$x_i^{(j)}$ = modal response at story i in mode j

n = number of stories

c_j = modal contribution factor for mode j

= D_d for constant-displacement branch

= D_v/ω_j for constant-velocity branch

$= D_a/\omega_j^2$ for constant-acceleration branch

ω_j = circular frequency of mode j

and D_d, D_v and D_a are the specified spectral bounding values of displacement, velocity and acceleration, respectively.

FORMAT OF RESULTS

In order to obtain dimensionless parameters which can be used most directly in design, and at the same time compared most meaningfully with the expressions for F_x and J_x in Reference (1), the results of the analyses were reduced to two types of factors.

For the purpose of calculating the lateral seismic forces which produce a consistent distribution of maximum probable shears, the results are presented in terms of an acceleration coefficient, $\bar{a}$, which, when multiplied by the story mass, m, and the base lateral design acceleration, Vg/W, yields the lateral seismic forces, $\bar{F}$. The significance of the acceleration coefficients is illustrated schematically in the top three diagrams of Fig. 1, and may be clarified further by reference to the code expression for F_x [Eq.(14-5) of Ref. (1)] when F_t equals zero:

$$F_x = \frac{V w_x h_x}{\sum_{i=1}^{n} w_i h_i} \quad (2a)$$

$$= \frac{V}{W} w_x \frac{h_x}{h_n/2} \quad (2b)$$

when the mass is uniformly distributed. Writing w_x as $m_x g$, Eq. (2b) can be written as

$$F_x = \frac{Vg}{W} m_x a_x \quad (3)$$

where

$$a_x = \frac{h_x}{h_n/2} \quad (4)$$

is a dimensionless acceleration factor. It can be seen that for a uniformly distributed mass, the code provides for a linearly varying acceleration, with ordinate of 0 at the base and 2 at the top.

For the purpose of calculating the overturning moments, the results are presented in terms of a moment coefficient, $\bar{J}$, by which one multiplies the static cantilever moment due to the lateral seismic forces in order to obtain a reasonable bound on the overturning moment. The significance of the moment coefficients is illustrated in the bottom diagrams of Fig. 2. These coefficients are directly comparable to the expression for J_x, Eq. (14-10) of Ref. (1):

$$J_x = J + (1-J)\left(\frac{h_x}{h_n}\right)^3 \tag{5}$$

The two factors used can be formally derived from the calculated results as follows:

Define:

w_i = weight at story i

W = total weight of structure

$\overline{V}_i$ = probable shear at story i

$\overline{V}$ = probable base shear = $\overline{V}_1$

$\overline{M}_i$ = probable overturning moment at story i

$\overline{F}_i$ = lateral seismic force corresponding to $\overline{V}_i$

= $\overline{V}_i - \overline{V}_{i+1}$ for i = n-1, n-2, ..., 1

= $\overline{V}_n$ for i = n

$\tilde{M}_i$ = cantilever moment caused by the lateral seismic forces, $\overline{F}_i$, at story i

Then, the acceleration factor is given by:

$$\bar{a}_i = \frac{\overline{F}_i/w_i}{\overline{V}/W} \tag{6}$$

and the moment factor by:

$$\overline{J}_i = \frac{\overline{M}_i}{\tilde{M}_i} \tag{7}$$

RESULTS OF CALCULATIONS

Calculations were performed for 10, 20, and 40-story shear-beam and flexural beam structures and for one 10-story actual structure. For the idealized shear-beam and flexural beam structures, four cases were investigated, as illustrated in Fig. 2. In Case A, both the stiffness and mass are uniformly distributed. In Case B, the stiffness varies linearly from the base to the top, the mass being uniform. In Case C, the stiffness is uniform and the mass distribution varies linearly, whereas in Case D both the stiffness and the mass vary linearly. In all the linear variations, the quantity at the top is one-third of the base value.

The actual building analyzed was a two-bay, 10-story symmetrical structure, with the following parameters:

Story spacing = 10 feet

Bay spacing = 20 feet

Column moment of inertia = 4,480 in^4

Column area = 75 in^2

Girder moment of inertia = 8,950 in^4

Girder area = 40 in^4

The member properties and the mass distribution were assumed to be constant throughout the height.

The results of the calculations, in terms of the acceleration factors, $\bar{a}_i$, and the moment factors, $\bar{J}_i$, are presented for the 20-story shear-beam and flexural beam buildings in Figs. 3 and 4, respectively. In Fig. 5, results are shown for the actual structure, with the corresponding shear-beam and flexural beam buildings (Case A). Results for the other cases studied are not presented, as they exhibit essentially identical trends.

It should be emphasized again that in this study all the modal response factors are assumed to fall on the same branch of the response spectrum, as identified in the figures.

INTERPRETATION OF RESULTS

As a preliminary step to the interpretation of the results, it should be recalled that when the fundamental mode is the only one excited, the seismic accelerations are directly proportional to the displacements, i.e., the fundamental mode shape. Furthermore, in this case, the same set of seismic forces produces both the shears and overturning moments, so that the J-factor is identically equal to unity over the height of the building.

When the higher modes enter into the probable maximum response, but their effect is small, the above statement is still essentially valid. This case is encountered when all modes fall on the constant-acceleration branch, i.e., when the fundamental period of the building is of the order of 0.5 seconds or less. As can be seen from the figures, the acceleration diagram still closely resembles the first mode shape, and can be approximated for practical design purposes by a triangle. For the structures presented, the maximum value of the top acceleration factor, $\bar{a}_n$, was 2.28 for the shear-beam buildings and 3.96 for the flexural beam buildings. Also, it can be seen that the J-factor is essentially equal to 1.0, regardless of the type of structure or mass and stiffness distribution.

For fundamental periods longer than 0.5 seconds, i.e., when the constant-velocity branch of the spectrum governs, the contribution of the higher modes is significantly increased. As can be seen from the figures, this effect is manifested by a sizable increase of the seismic accelerations near the top of the structure. For shear-beam buildings, the accelerations can be approximated by a trapezoidal distribution, with ordinates of 0.5 at the bottom to the order of 1.5 at the top, plus a concentrated intensity of the order of 20 to 30 percent of the total lateral load applied at the top. The corresponding J-factors vary parabolically from 1.0 at the top to approximately 0.9 at the base. For flexural buildings, such as chimneys and shear walls, the effect is much more pronounced. A consistent set of seismic design forces requires that 80 to 85 percent of the lateral load be applied at the top of the structure, with the remainder applied near the base. Because of the large concentration of forces at the top, the J-values drop very rapidly from 1.0 to about 0.2 in the upper half of the structure, and then remain constant in the bottom half.

In the case of the constant-displacement branch governing, essentially all of the seismic load is concentrated at the top of the structure. It should be noted that both the acceleration and moment factor curves for the shear beam on the constant-displacement branch are nearly the same as the corresponding curves for the flexural beam on the constant-velocity branch. Even though the results based on the constant-displacement branch are unrealistic, as described earlier, in that they are based on equal contribution factors for all modes, it is apparent that for structures with a very long fundamental period the overturning moments may be several times as great as those given by the code, and that the effect of these moments on foundation rocking and column axial forces must be carefully considered.

As mentioned above, the results of analyses of 10 and 40-story structures are not presented. It should be pointed out, however, that the contribution of the higher modes converges more rapidly for the overturning moments than for the shears. While this trend is of little practical significance for the constant-acceleration branch for both types of structures and the constant-velocity branch of shear-beam buildings, it is quite noticeable for the constant-velocity branch of flexural beam buildings and the constant-displacement branch of both types. Typically, the J-factors for the constant-displacement branch of the 40-story shear-beam buildings is approximately 1.42 times less, and for the 10-story buildings 1.43 times more than the values shown for the 20-story building.

The comparisons shown in Fig. 5 clearly indicate that the seismic accelerations for an actual framed structure are very similar to, and only slightly larger than, the corresponding accelerations for an idealized shear-beam building, even when all "flexural" effects such as joint rotations and column distortions are taken into account. Similarly, the J-factors for the actual structure are somewhat less than those for the shear-beam building, but considerably higher than those for the idealized flexural beam building.

CONCLUSIONS AND RECOMMENDATIONS

This study may be considered to be an initial step toward the development of a more rational procedure for preliminary seismic design. It was shown that all the design quantities of interest can be represented by two sets of coefficients which are closely related to the expressions for F_x and J_x in the U. S. Code [Ref. (1)]. It is also clear that the expressions for F_x and J_x of the present U. S. Code represent a step in the right direction compared to previous codes, i.e., the inclusion of F_t and the formula for J_x.

In order to achieve the degree of generality and rationality desired, however, three further steps must be undertaken:

1. A more exhaustive parameter study of height variation and stiffness and mass distribution must be performed, and the data for actual structures related to the idealized buildings used in this study.

2. A more inclusive formula for the distribution of lateral seismic forces, F_x, (or the acceleration factors, $\bar{a}_i$) must be derived, which takes into account the basic behavior of the structure. A distinct possibility is to define a K-factor based on type of framing in a manner similar to that now used for the determination of the magnitude of the base shear.

3. A more realistic formula for the overturning moment factors, J_x, must be derived. For example, for all structures with a fundamental period less than 0.5 seconds, changing the numerator of the present expression for J from 0.5 to 0.65 would insure that the moment coefficient J_x would be equal to 1.0 throughout the height of the structure. However, to develop appropriate factors for longer period structures would require further study.

BIBLIOGRAPHY

(1) Uniform Building Code, 1967 Edition, International Conference of Building Officials, Pasadena, 1967, Sect. 2314, pp. 118-127.

(2) Recommended Lateral Force Requirements and Commentary, Seismology Committee, Structural Engineers Association of California, San Francisco, 1967.

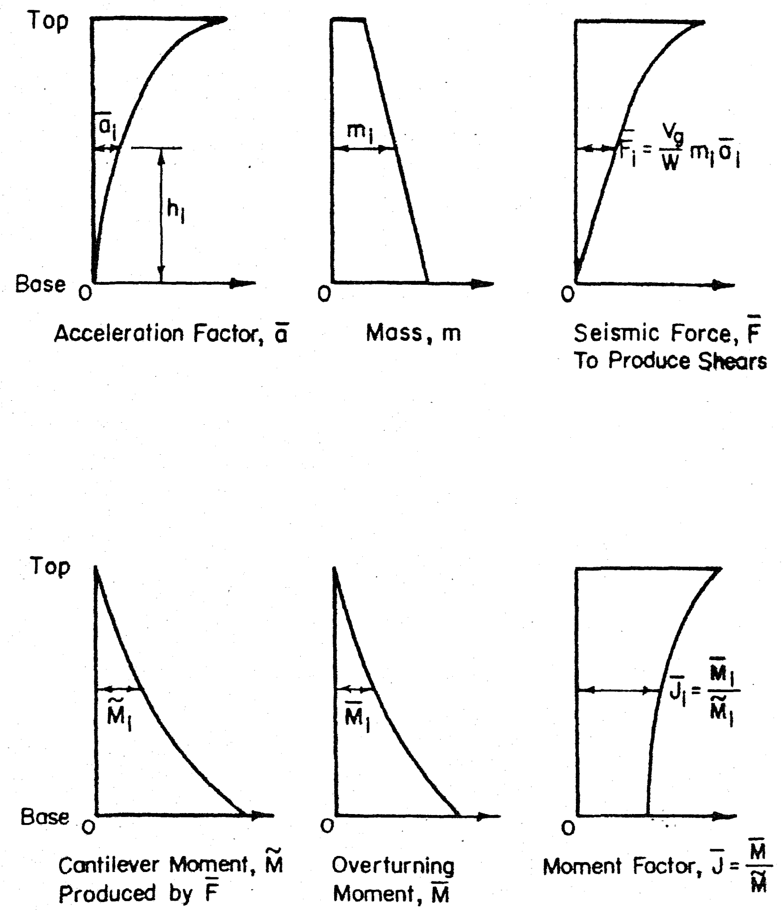

FIG. I ILLUSTRATION OF PARAMETERS USED

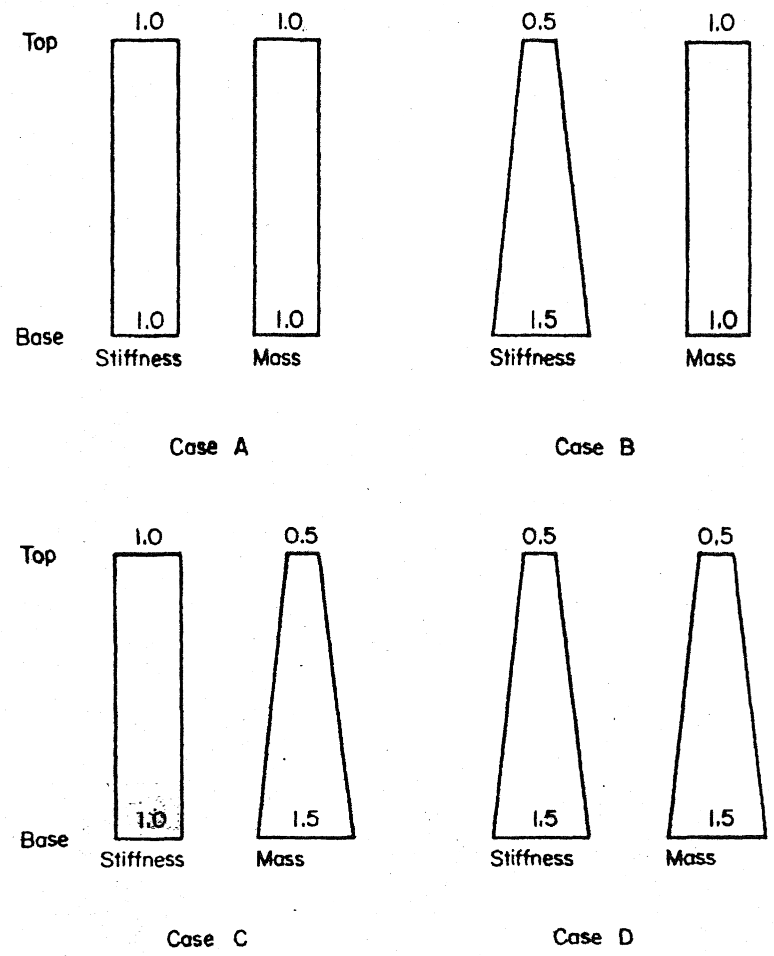

FIG. 2 SUMMARY OF STIFFNESS AND MASS DISTRIBUTIONS STUDIED

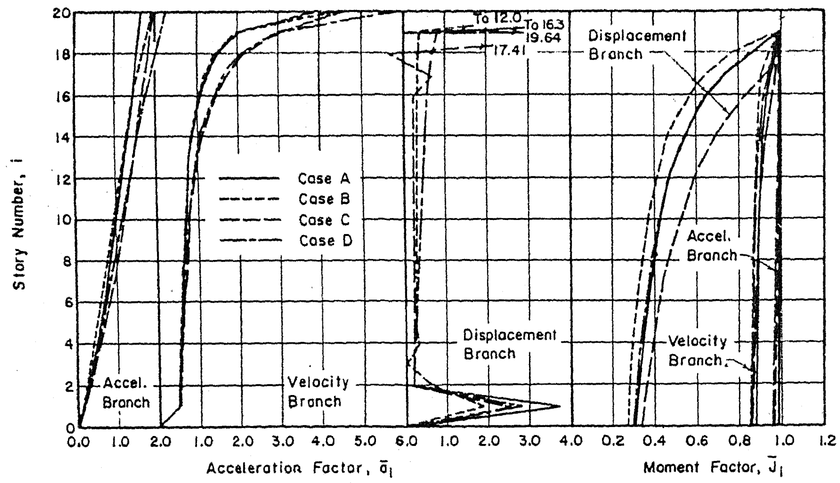

FIG. 3 ACCELERATION AND MOMENT FACTORS, 20-STORY SHEAR-BEAM BUILDINGS

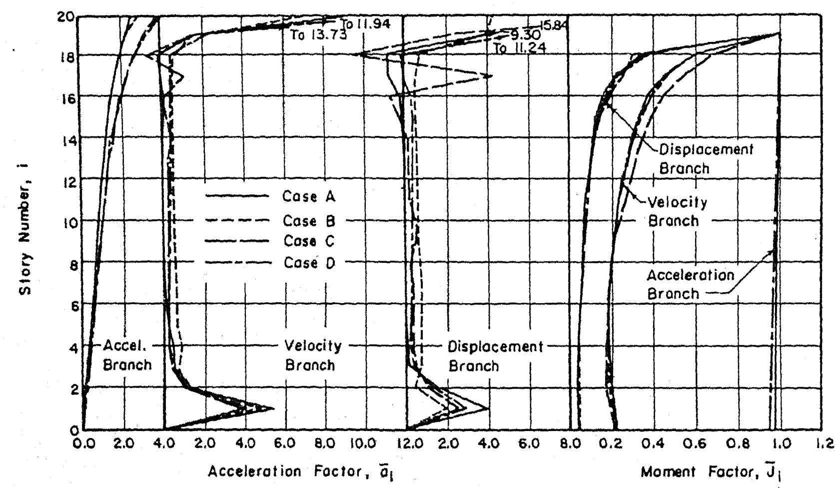

FIG. 4 ACCELERATION AND MOMENT FACTORS, 20-STORY FLEXURAL-BEAM BUILDINGS

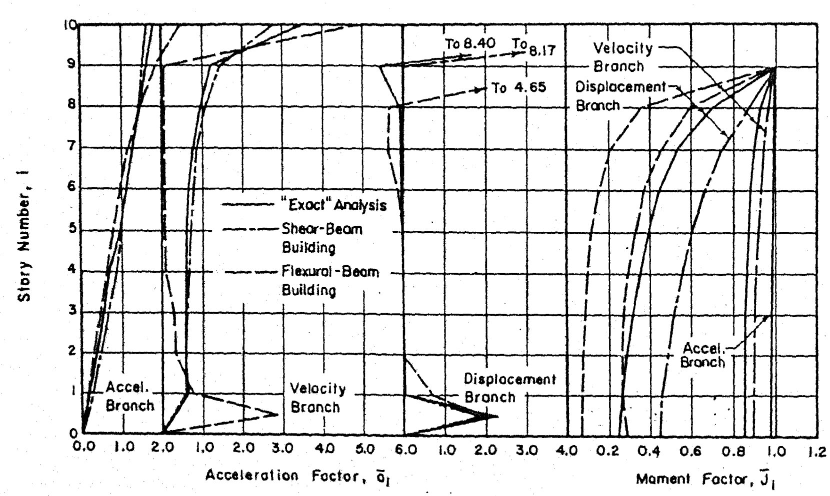

FIG. 5 ACCELERATION AND MOMENT FACTORS, 10-STORY STRUCTURES