SOME EFFECTS OF SUBSTRUCTURE AND ADJACENT SOIL INTERACTION ON THE SEISMIC RESPONSE OF BUILDINGS

By J. Kazuo Minami[1] and Joji Sakurai[2]

SYNOPSIS Some results obtained from investigations on the effects of substructure and adjacent soil interaction on the seismic response of buildings are presented. Seismic response for 126 cases has been obtained for the basic model subjected to the El Centro and Taft earthquake accelerations with the aid of an analog computer. Vibrational characteristics of a more detailed 14-mass model have also been studied with the aid of a digital computer and trends obtained are discussed. Results obtained so far indicate that soil-substructure interaction may be favorable or unfavorable, depending on the parameters that relate to the soil, substructure, and superstructure characteristics.

1. INTRODUCTION

The Japanese National Standard Building Laws were revised in July 1963 to permit buildings higher than 31 meters (100 feet) to be built. The Review Board for High-Rise Buildings has been created to check the adequacy of the structural analysis and design for seismic and wind loads in addition to the normal vertical loads for buildings that exceed 45 meters in height. Some 36 buildings have been reviewed and approved to date (June 1968) ranging from 10 to 40 storeys above the ground level. The average number of storeys above ground is 17.86 for the main portion, 1.60 storey penthouse above the roof level, and 3.02 storeys under the ground, making an average total of 22.48 storeys from the bottom of the foundation to the top of the penthouse for the 36 buildings. The average depth from the ground level to the supporting soil layer is 20.77 meters, the minimum depth being 8.8 meters for Shizuoka City, 30 meters maximum for Osaka, and approximately 20 meters for the Tokyo area for the buildings reviewed. Buildings are either supported directly or indirectly by piers on the bearing soil layer.

It is evident from the above that large important buildings have basement storeys of considerable depth in the ground. Buildings that do not have substructures or adequate depth foundations have fared badly in past earthquakes. This has been demonstrated dramatically in the settlement, inclination, and overturing of 4-storey reinforced concrete apartment buildings and other structures at the time of the Niigata earthquake of June 16, 1964. On the other hand, buildings with adequate foundation depths or those supported by piles or protected by sheet piles around their perimeter suffered only minor settlements or no damage.

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1) Professor and 2) Research Associate, Department of Architecture, School of Science and Engineering, Waseda University, Tokyo, Japan

In the seismic analysis of the above buildings, the earthquake accelerations were assumed to act at the ground level in 18 cases, at the first basement level in 15 cases, at the base of the fifth basement in 1 case, and at the second floor level above the ground in 2 cases. In all cases, the underground structure is considered to be rigid and the effects of the substructure and adjacent soil interaction on the seismic response of these buildings are not taken into account. It is believed that the soil adjacent to the substructure should have some effects on the seismic response of these buildings. Whether these effects are favorable, unfavorable, or both were unknown but the problem is of great importance to earthquake engineering.

This paper presents some of the results obtained from exploratory investigations using a basic model subjected to El Centro and Taft earthquake wave forms in obtaining the seismic response which takes into consideration certain parameters relating to the soil, the substructure, and the superstructure. A more detailed 14-mass model has also been studied to investigate the vibrational characteristics with the substructure and adjacent soil parameters taken into account.

2. BASIC CONCEPTS AND MODELIZATION

The prototype building and its modelization are shown in Fig. 1. The actual building is shown in (a) and multi-mass model in (b). In the basic model shown in Fig. 2, the usual weightless springs connect the concentrated masses of the building. The soil masses adjacent to the substructure on both sides (as a two dimensional problem) act through springs that are activated in compression only and inactivated when the soil and the substructure are not in contact, considering the plastic behavior of soils that have some cohesion which is usually the case in the field. It is assumed that the building is adequately supported vertically, either by direct contact with the bearing stratum or indirectly by piers that reach the bearing soil layer. Rotation of the building at the bottom of the foundation is not considered, although this effect may be taken into account without much difficulty.

In the basic model, shown in Fig. 2, the superstructure portion is represented by a single mass, the substructure portion by a single mass, and the adjacent soil by two equal masses acting through special horizontal springs.

Studies by Kanai et al indicate that periods of vibration of hard and soft soils vary and that the predominent periods of vibration of hard and soft soils are in the neighborhood of 0.3 sec and 0.6 sec, respectively. The stress-strain relationships for the soil adjacent to the substructure have been varied as shown in Fig. 3 for hard soils (curves A), soft soils (curves C), and intermediate elastic soils (curve B). Although the change from the elastic to the plastic range is gradual in the actual case, bilinear relations shown in the figure have been assumed for ease in computer analysis.

3. INVESTIGATIONS WITH THE BASIC MODEL

The weight of the building has been computed on the basis of 0.5 tons per square meter, the stiffness determined so that the fundamental period of vibration T_1 (sec), is expressed by the relation, $T_b = (N/4) \times 0.1\pi$ wherein N is the number of storeys above the ground. The values of T_b range from 0.1π to 1.0π. The mode shape is such that it is 1.00 at the top level and 0.05 at the ground level in the fundamental mode. The damping coefficient of 0.03 (3%) has been assumed for the building. The weight of the substructure has been computed on the assumed unit weight of 1.2 tons per cubic meter. The unit weight of the adjacent soil has been assumed as 1.6 tons per cubic meter and the mass of the soil has been assumed as twice the weight of the substructure portion of the building and this mass acts on both sides of the substructure as shown in the figure.

The periods of vibration have been assumed as 0.1π (0.314) sec and 0.2π (0.628) sec for the hard and soft soils, respectively, with damping coefficient values of 0.10 and 0.25 corresponding to hard and soft soils.

In Fig. 3, curves A represent 1 centimeter displacement at 200 and 100 t/sq m pressures, respectively. For curves C, the pressure of 50 t/sq m produce deformations of 2 cm and 4 cm, respectively. Curve B represents an intermediate soil stiffness that is linear and is mainly of academic interest. Considering the fact that soils are not isotropic, and elastic only compressionally, it has been assumed that soils do not respond to tension in the engineering sense. Soil stiffness ratio of hard soil is 8 to 16 times that of the soft soil.

In the basic model, m_u represents the mass of the upper half of the superstructure above the ground level; m_G consists of the mass of the bottom half of the superstructure, plus the upper half of the substructure; and m_s represents the mass of the adjacent soil on each side of the substructure which has been taken as twice the mass of the upper half portion of the substructure. m_s acts on each side of m_G through special horizontal springs that act only in compression. The vertical springs connecting the building masses, k_u, represent shear rigidities and k_f represents the shear rigidity of the substructure. The shear rigidities of the vertical springs representing the soil adjacent to the basement portion are determined from the predominent periods of vibration for hard and soft soil types. The axial stiffness of springs, k_s, is determined from the values of lateral soil reaction coefficients. The springs representing soil stiffness, k_{s1} and k_{s2}, have bilinear characteristics when compressed but assume a zero value in tension. The subscript numbers, 1 and 2, represent left and right side of the substructure respectively.

El Centro 1940 NS component and the Taft 1952 EW component of the acceleration, the latter normalized to a maximum acceleration of 330 gal, have been used as inputs at the base of the foundation.

The equation of motion for the basic model system is expressed by Eq. (1)

$$\begin{bmatrix} m_u & & & \\ & m_G & & \\ & & m_s & \\ & & & m_s \end{bmatrix} \begin{Bmatrix} \ddot{U}_u+\ddot{U}_0 \\ \ddot{U}_G+\ddot{U}_0 \\ \ddot{U}_{e1}+\ddot{U}_0 \\ \ddot{U}_{e2}+\ddot{U}_0 \end{Bmatrix} + \begin{bmatrix} C_u & -C_u & & \\ -C_u & C_u+C_f & & \\ & & C_{e1} & \\ & & & C_{e2} \end{bmatrix} \begin{Bmatrix} \dot{U}_u \\ \dot{U}_G \\ \dot{U}_{e1} \\ \dot{U}_{e2} \end{Bmatrix} +$$

$$+ \begin{bmatrix} k_u & -k_u & & \\ -k_u & k_u+k_f+k_{s1}+k_{s2} & -k_{s1} & -k_{s2} \\ & -k_{s1} & k_{s1}+k_{e1} & \\ & -k_{s2} & & k_{s2}+k_{e2} \end{bmatrix} \begin{Bmatrix} U_u \\ U_G \\ U_{e1} \\ U_{e2} \end{Bmatrix} = \begin{Bmatrix} \\ 0 \\ \\ \end{Bmatrix} \qquad (1)$$

The solution of this equation of motion was obtained by Hitachi ALS-1000 analog computer in the Electronic Computer Center of Waseda University. Analog computation circuit (block diagram) is shown in Fig. 4.

The various constants used in the basic model have been computed by using the following relationships.

$$w^2 = (2\pi/T_b)^2$$

$$k_u = w^2\, m_u/0.95 \qquad (2)$$

$$k_G = w^2\, (m_u + 0.05 m_G)/0.05 \text{ and}$$

$$k_{e1} = k_{e2} = m_s\, (2\pi/T_s)^2 \qquad (3)$$

wherein T_b = 0.1π, 0.2π, 0.4π, 0.5π, 0.6π, 0.8π, and 1.0π sec for the building;

T_s = 0.1π (hard soil) and 0.2π (soft soil) sec

The values of the constants used in the computations are tabulated in Table 1.

The seismic response for 7 different values for the natural period of vibration of the building, for 2 values of predominent period of vibration for the adjacent soil with 2 values of damping coefficients for both the hard and soft soil types, and 2 earthquake accelerations (112 cases) were obtained in addition to the standard seismic response for the building only for two earthquake motions (14 cases), making a total of 126 case studies with the basic model.

Typical samples of the response curves for the El Centro 1940 earthquake NS acceleration component input are shown in Fig. 5 for differing building and soil vibration periods, soil types, soil spring constants and damping coefficients.

The top curve is the 1940 El Centro earthquake, NS acceleration component which is used as the input. The second curve represents the relative displacement of the mass of upper portion of the superstructure m_u, the third curve that of the mass m_G; the fourth curve the sum of the displacements, u_u and u_G, referred to the foundation; the fifth curve the displacement of the soil on the left side of the substructure (the right side soil displacements have been found to differ only slightly from those on the left side); and the sixth curve, the compressive displacements between the substructure mass, m_G, and the soil mass, m_S, which are proportional to the stresses acting on the contact surface. When the two masses cease to be in compression, there is no stress as explained previously.

The set of curves at the bottom of these figures shows the seismic response of the building only without consideration of the soil-substructure interaction.

From these response data, the relationships between the ratios of maximum relative displacement of buildings with and without the soil-substructure interaction for buildings of different fundamental periods of vibration have been measured and computed for the superstructure and substructure, respectively. In Fig. 6, the effects of the soil around the basement portion for the superstructure are indicated. The effects on the substructure response are of less engineering significance than for the superstructure.

Some of the effects of substructure and adjacent soil interaction on the seismic response of buildings that emerged from the present investigations are listed below.

For the case of hard soils (k_S = 200 and 100 tons/cm/sq m): (Fig. 8)

1. When the fundamental period of the building is in the range of (0.1 - 0.2)π sec, the soil-substructure interaction has the unfavorable effect in producing large relative displacements and shears.

2. When the fundamental period becomes 0.4π sec or longer, the soil-substructure interaction has the favorable effect by producing smaller displacements and shears than for the building alone without the soil interaction consideration.

3. The effect of the soil-substructure interaction changes from unfavorable to favorable at a certain value of the fundamental period of the building, approximately 0.3π sec in the case of hard soils. The fundamental period of the building corresponding to this change occurs is referred to as the "transmutation period". There is a value of the fundamental period of the building when the relative maximum displacement becomes large and this value of the period may be called the "ill-displacement period"; and there is another value of the fundamental period that produces a large seismic shear and the period that corresponds to this condition may be termed the "ill shear period" for the building. On the other hand, there is a certain fundamental period of the building when both the relative displacement and the seismic shear assume minimum values and the period that produces this condition may rightfully be called the "optimum period" for the building for the conditions relating to the earthquake motion, soil, substructure, and superstructure characteristics.

4. Two values of damping coefficient for the hard soil, namely 10% and 25%, were used but there appears to be no significant effects in the seismic response due to changing the values of damping coefficients. This problem requires further research in the field of dynamic soil mechanics.

For the case of soft soils (k_s = 25 and 12.5 t/cm/sq m): (Fig. 9)

5. When the natural period of the building is in the range of (0.2 -0.4)π sec, the soil-substructure interaction produces unfavorable seismic response with respect to displacements and seismic shear.

6. When the natural period of the building is longer than 0.5π sec, the soil-substructure interaction produces favorable seismic response but the effects are not prominent.

7. The influence of different values of the damping coefficient was found to be not significant which was also noted for the case of hard soils.

8. The seismic response to the 1952 Taft earthquake EW acceleration component gave results similar to the El Centro earthquake response, except for the natural period in the vicinity of 0.5π sec which may be considered as a special peculiarity of this earthquake that has been noticed by other investigators.

The following general conclusions seem to be justified:

9. When the building is rigid (T_b = (0.1 -0.2)π sec) and surrounded by hard soil, the substructure and the adjacent soil tend to vibrate together and earthquake energy is fed into the building more than dissipated out from it and it is believed that this results in unfavorable seismic response.

10. Favorable seismic response is obtained for buildings when the natural periods are greater than the "transmutation period", this value being influenced by the earthquake motion, soil, substructure and building characteristics. In this case, the earthquake energy accumulated by the building is dissipated out from it in amounts greater than the input energy into the building.

4. INVESTIGATIONS WITH A MULTI-MASS MODEL

In order to find out the differences in the vibrational characteristics between the soil-structure system as compared with the building system only, a more detailed 14-mass model was designed and investigated.

In this model, the building is represented by 5 concentrated masses, m_{b1} - m_{b5} and the adjacent soil by 9 concentrated masses, m_{s1} - m_{s9}, as shown in Fig. 1 (b). The fundamental periods of vibration of the building were assumed as (0.1, 0.2, 0.4, 0.5, 0.6, 0.8 and 1.0)π sec to correspond with the periods used in the study with the basic model; the spring constants were determined by making the first mode vibrational shape vary linearly from 1.0 at m_{b1} and 0.05 at m_{b5} levels. For the adjacent soil, the vertical spring constants were determined from the predominant periods of vibration of 0.1π sec for the hard soil, and 0.2π sec for the soft soil, the same values that were used in the basic model studies. The horizontal spring constants were computed for soil reactions of 200 $t/m^2/cm$ (hard soil) and 25 $t/m^2/cm$, (soft soil) respectively. The conditions assumed are believed to be evident from Fig. 1 (b).

Designating the stiffness of the member ij by k_{ij}, the joint stiffness matrix is expressed by eq. (4).

$$\begin{bmatrix} k_{ij} & -k_{ij} \\ -k_{ij} & k_{ij} \end{bmatrix} \qquad (4)$$

The stiffness matrix (k) for the whole system is obtained by summation of the stiffness values of each member. The normal mode values may be obtained from the relation expressed by eq. (5).

$$[m]^{-1}\,[k]\,\{u\} = w^2\{u\} \qquad (5)$$

wherein

$[m]^{-1}$ is the inverse mass matrix
$[k]$ is the stiffness matrix and
$\{u\}$ is the eigen vector

When the masses in the system are equal, the inverse mass and stiffness matrices form a symmetrical square matrix and the Jacobian method may be used for calculating the normal mode values. The values of the masses and stiffness constants for the building and the soil for the 14-mass model, used in digital computer calculations, are indicated in Table 2. In calculating the mass of the building for a given fundamental period, the same procedure was followed as for the case with the basic model. The Nippon Electric NEAC-2203 digital computer in the Electronic Computation Center of Waseda University was used in the computations.

The results of the analysis on the vibrational characteristics of the multi-mass model are indicated in Figs. 12-14. Figs. 12 and 13 show the variation in the values of the ratio, B_{bs}/B_b, relating to the modal participation factors for the hard soil and soft soil conditions, respectively. Fig. 14 indicates the variation in the values of the period ratio, T_{bs}/T_b, corresponding to the fundamental periods of the buildings. The upper set of curves is for the hard soil and the lower set for the soft soil for the first three modes.

The results obtained to date indicate the following trends:

1. With regard to the normal modes of vibration, when the fundamental period of the building is in the range of approximately 0.3 - 0.6 sec (0.1π - 0.2π sec) the first mode vibrational shape for the superstructure tends to be larger in the middle and lower portions for the soil-structure system, in comparison with the building-only system. When the fundamental period of the building is greater than approximately.

1.2 secs (0.4π sec), this tendency is reversed. This relationship is evident for the first mode and second mode vibration shapes.

2. With regard to the natural period, when the fundamental period is in the range of 0.1π - 0.2π sec for the building, the natural period of the soil-structure system is longer than that for the building-only system and this difference becomes more pronounced as the fundamental period of the building becomes shorter. On the other hand, when the fundamental period of the building is in the range of 0.4π sec or more, this tendency is reversed and the natural period of the soil-structure system becomes shorter than for the building-only system. This relationship is found to be the same for the 1st, 2nd, and 3rd modes.

3. With regard to the modal participation factor, a similar comparison reveals the fact that its value is greater for the soil-structure system than for the building-only system and the difference between the soil-structure and building-only systems becomes greater for short natural periods. As the fundamental period increases, the difference in the values becomes smaller and there is practically no difference when the period reaches 1.0π which corresponds roughly to a 40 storey building. The modal participation factors are found to have maximum values for fundamental periods in the vicinity of 0.2π sec in the 2nd and 3rd modes.

4. The tendencies noted above for the vibrational characteristics of the multi-mass system are in good agreement with the results on the seismic response obtained from the exploratory investigation using the basic model.

5. SUMMARY

The results of investigations to study the problem of the effects of substructure and adjoining soil interaction on the seismic response of buildings seem to indicate the propriety of designing rigid buildings having fundamental periods in the range of 0.3 - 0.6 sec for high seismic shears that results from soil-substructure interaction.

For buildings of intermediate rigidity (or flexibility) with fundamental periods in the 1.2 - 1.8 sec range, soil-structure interaction effects are less pronounced than for the rigid buildings. For given soil and substructure conditions, there appears to exist a fundamental period for the building which for this value of the period shows the same seismic response whether the soil-substructure interaction is taken into consideration or not. The transition period from the unfavorable shorter periods to the favorable longer periods range has been named the "transmutation period" by the authors. The optimum period of the building for given soil-substructure interaction conditions exists in the range of period longer than the transmutation period and the ill-periods for displacements and seismic shears exist in the range of period shorter than the transmutation period. A building designed for the optimum period for soil-substructure conditions would undergo minimum displacements and be subjected to the smallest seismic shears.

For high flexible buildings with fundamental periods exceeding, say 2.4 sec, soil-substructure interaction need not be considered because buildings designed on the basis of no such interaction seem to be on the side of safety.

Finally, it should be kept in mind that in the basic model studies, El Centro and Taft earthquake wave forms were used as the input earthquake accelerations and the seismic response results are the outputs from them. Other earthquake wave forms may modify considerably the trends and tendencies found from the present investigations.

It is even conceivable that buildings that have satisfactory characteristics for near strong earthquakes may demonstrate undesirable displacement response to long epicentral distance earthquakes with long period seismic waves. Also, rigid buildings resting even on good firm ground may be subjected to unexpectedly large seismic shears that may cause fatal structural damage unless the buildings possess the ability to undergo plastic deformations from the sudden application of seismic shears.

A recent work by Hashiba and Whitman (1968) on soil-structure interaction during earthquakes seems to support some of the findings given in this paper.

Much work still remains to be done with earthquake motions of different types and the simulated earthquake motions developed by Jennings, Housner and Tsai (1968) are expected to be helpful when applied to this problem.

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BIBLIOGRAPHY

Hashiba, T. and Robert V. Whitman: Soil-Structure Interaction During Earthquakes, Soils and Foundations, Vol. 8, No. 2 (June 1968) Jap. Soc. of Soil Mech. & Foundation Engineering, Tokyo

Jennings, P. C., G. W. Housner and N. C. Tsai: Simulated Earthquake Motions, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, California (April 1968)

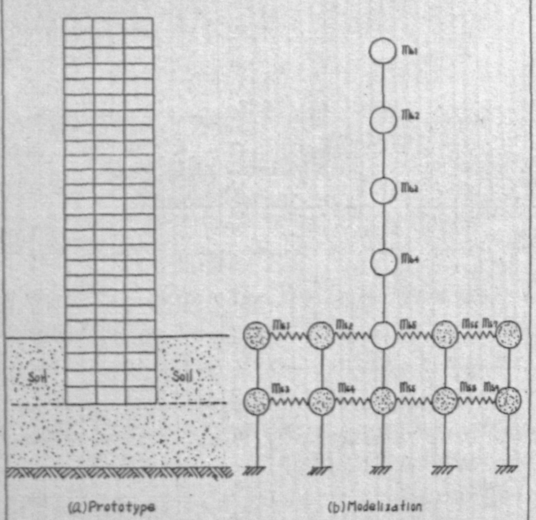


Fig.1 Prototype and Modelization

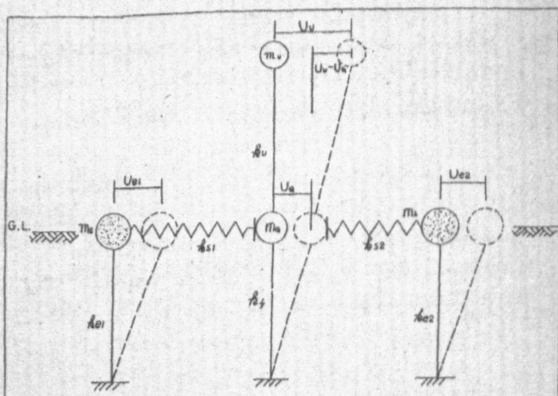


Fig.2 Basic Model

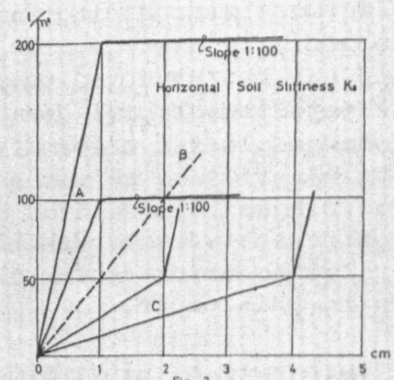


Fig. 3

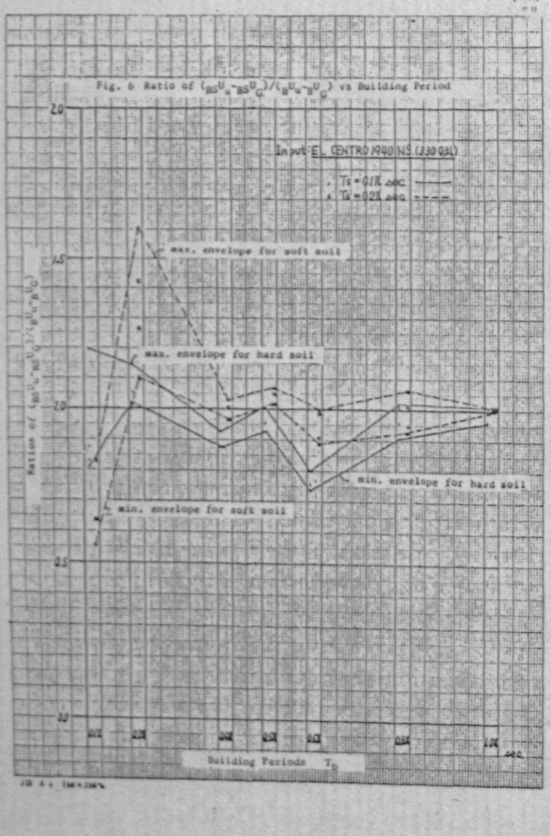


Fig. 6 Ratio of $({}_{BS}U_u - {}_{BS}U_G)/({}_{B}U_u - {}_{B}U_G)$ vs Building Period

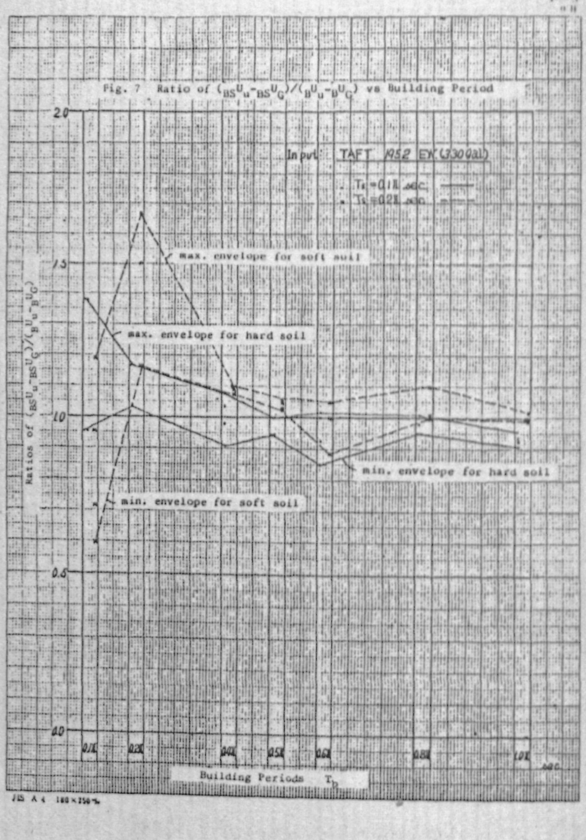
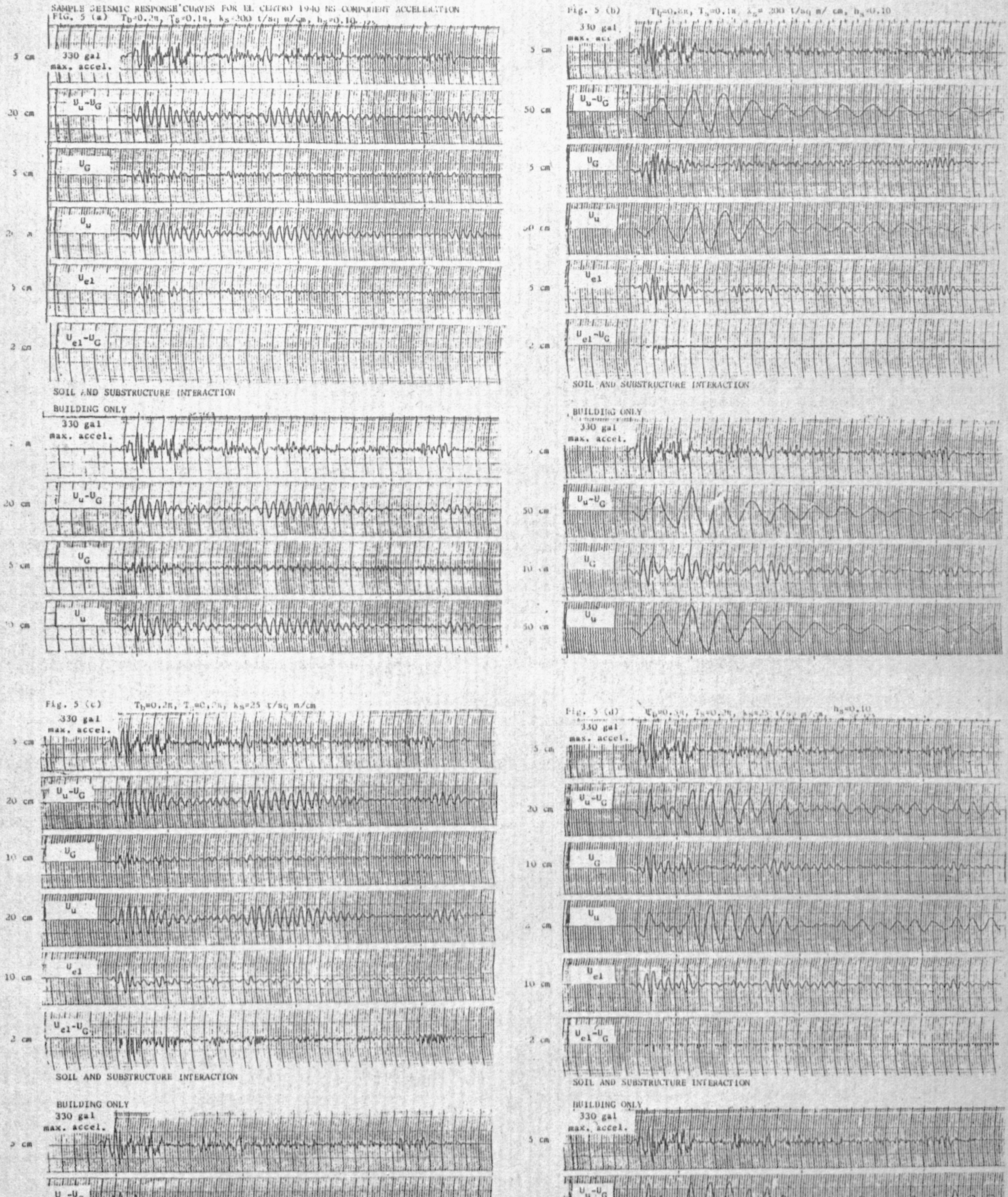


Fig. 7 Ratio of $({}_{BS}U_u - {}_{BS}U_G)/({}_{B}U_u - {}_{B}U_G)$ vs Building Period

SAMPLE SEISMIC RESPONSE CURVES FOR EL CENTRO 1940 NS COMPONENT ACCELERATION



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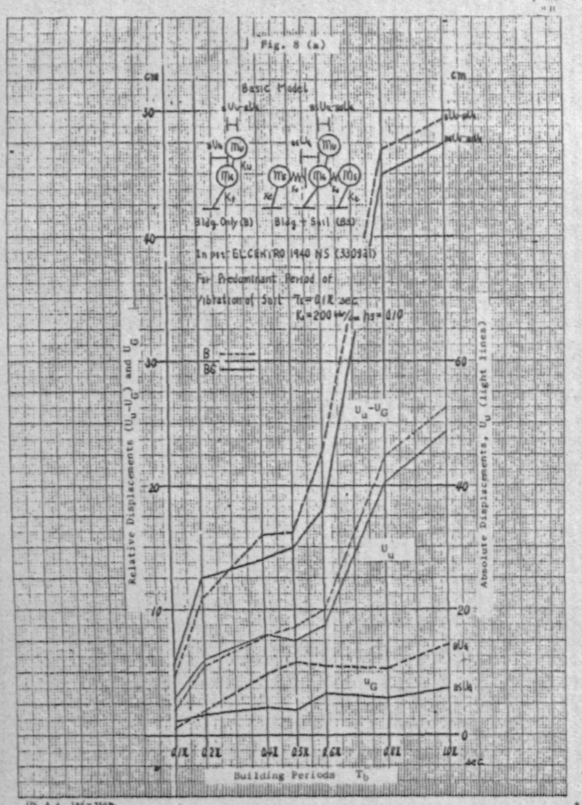


Fig. 8 (a)
Basic Model
Bldg. Only (B)
Bldg. + Soil (BS)
Input: EL CENTRO 1940 NS (330gal)
For Predominant Period of
Vibration of Soil Ts=0.1 sec
B
BS
Relative Displacements (U_u-U_G) and U_G
Absolute Displacements, U_u (light lines)
U_u-U_G
U_u
U_G
Building Periods T_b

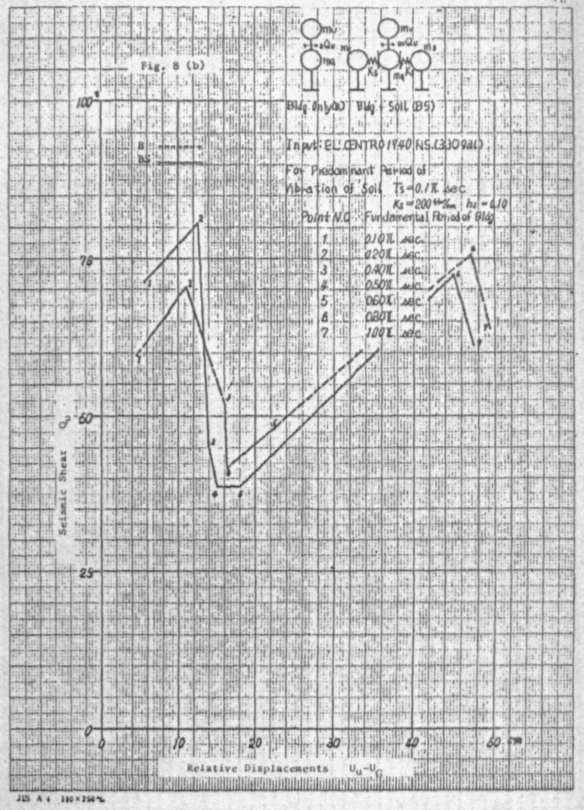


Fig. 8 (b)
Bldg. Only (B)
Bldg. + Soil (BS)
B
BS
Input: EL CENTRO 1940 NS (330gal)
For Predominant Period of
Vibration of Soil Ts=0.1 sec
Point NO. Fundamental Period of Bldg.
Seismic Shear Q_B
Relative Displacements U_u-U_G

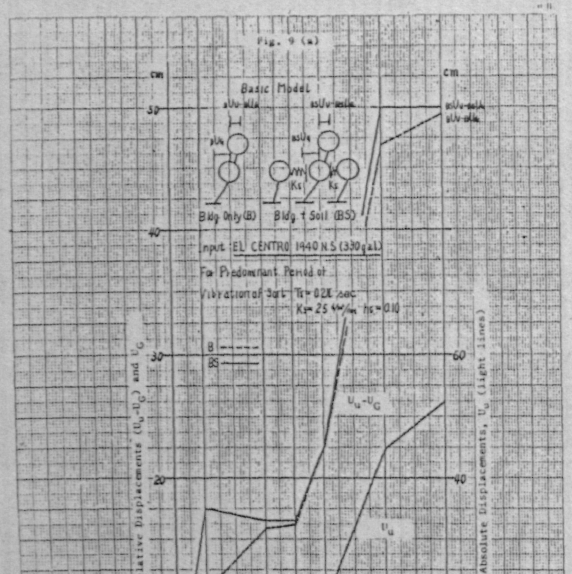


Fig. 9 (a)
Basic Model
Bldg. Only (B)
Bldg. + Soil (BS)
Input: EL CENTRO 1940 NS (330gal)
For Predominant Period of
Vibration of Soil Ts=0.2 sec
B
BS
Relative Displacements (U_u-U_G) and U_G
Absolute Displacements, U_u (light lines)
U_u-U_G
U_u
U_G
Building Periods T_b

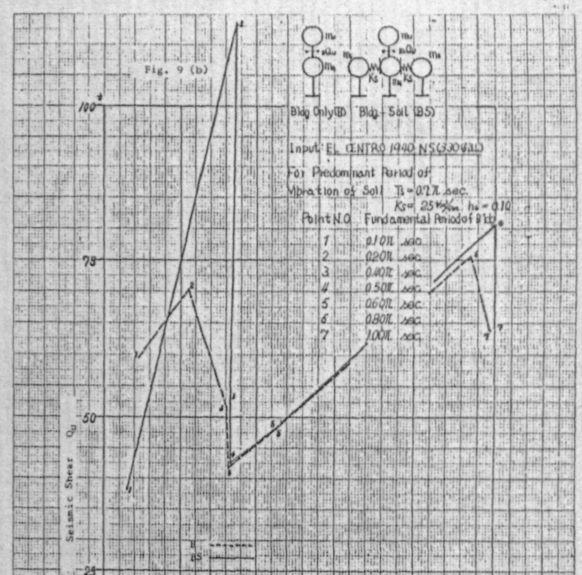


Fig. 9 (b)
Bldg. Only (B)
Bldg. + Soil (BS)
Input: EL CENTRO 1940 NS (330gal)
For Predominant Period of
Vibration of Soil Ts=0.2 sec.
Point NO. Fundamental Period of Bldg.
B
BS
Seismic Shear Q_B
Relative Displacements U_u-U_G

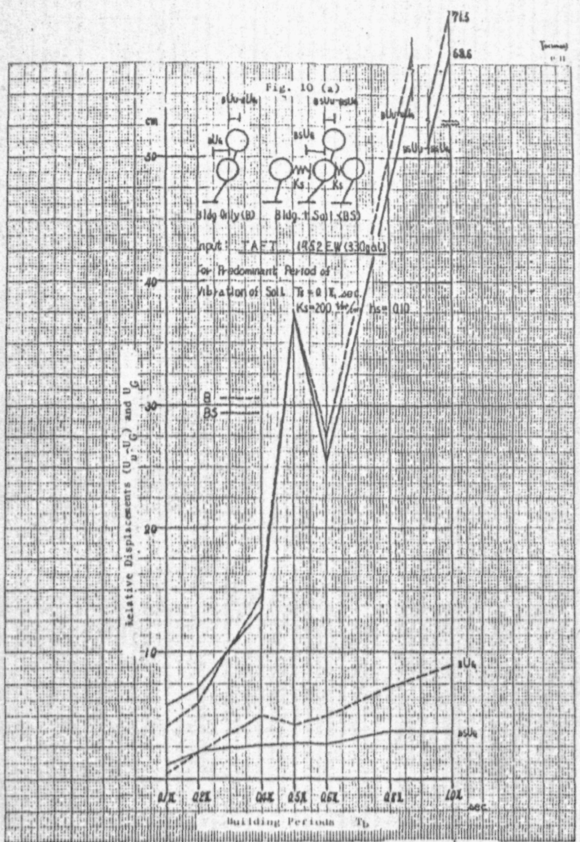


Fig. 10 (a)

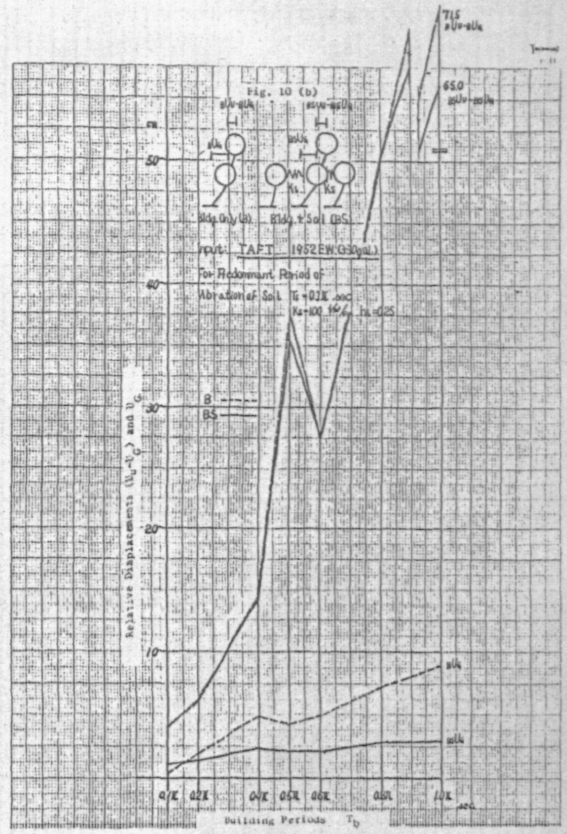


Fig. 10 (b)

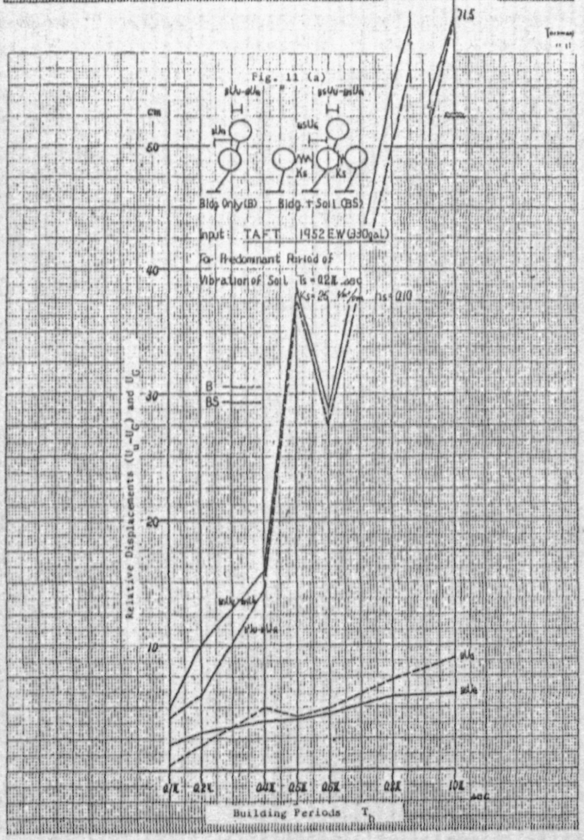
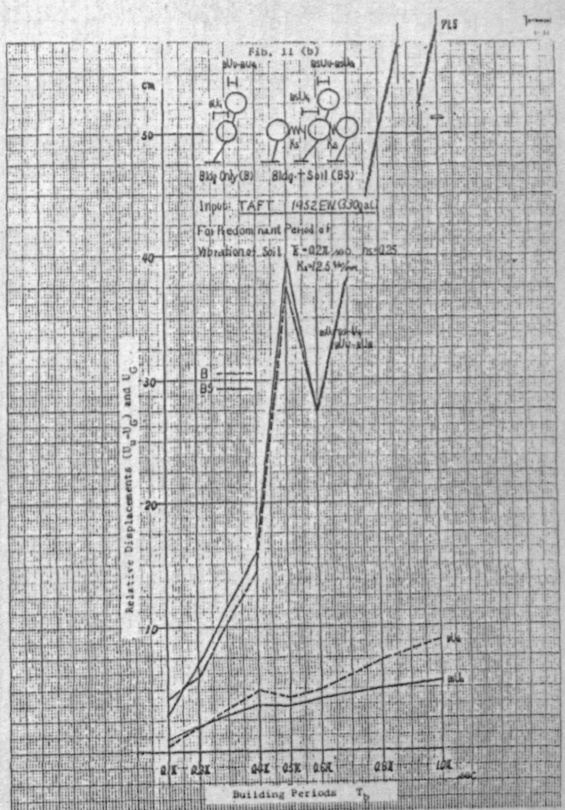


Fig. 11 (a)



Fib. 11 (b)

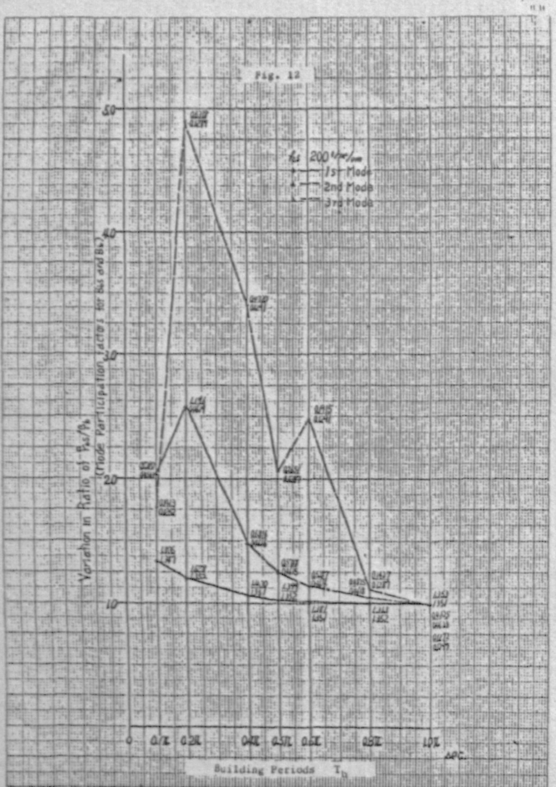


Fig. 12
1st Mode
2nd Mode
3rd Mode
Building Periods T_b

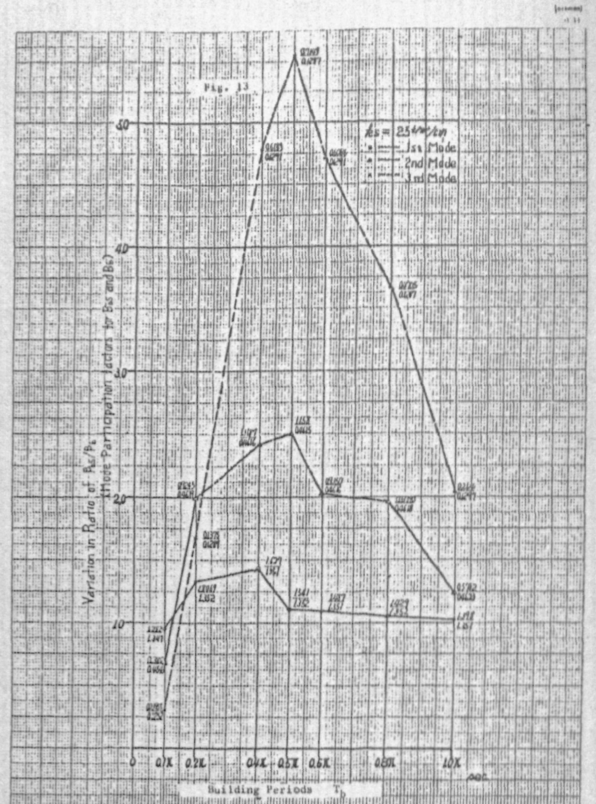


Fig. 13
1st Mode
2nd Mode
3rd Mode
Building Periods T_b

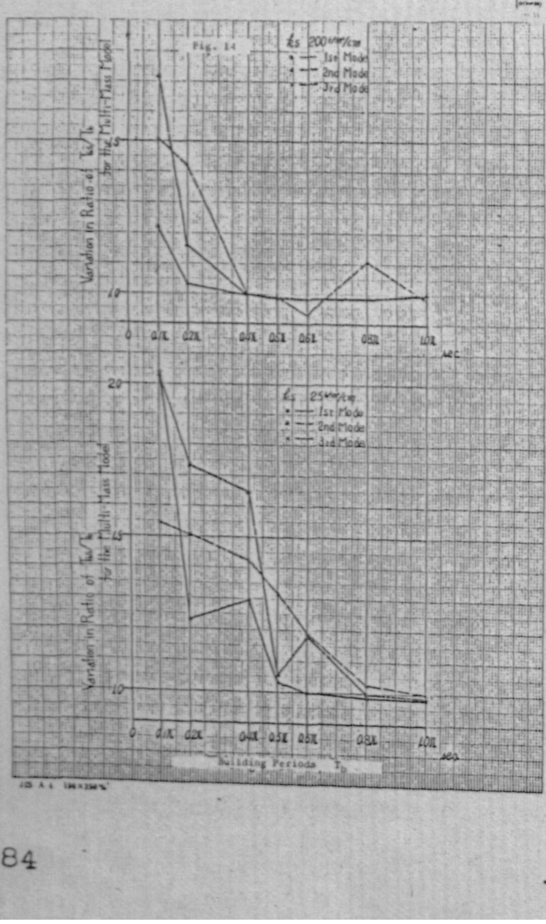


Fig. 14
1st Mode
2nd Mode
3rd Mode
Building Periods T_b

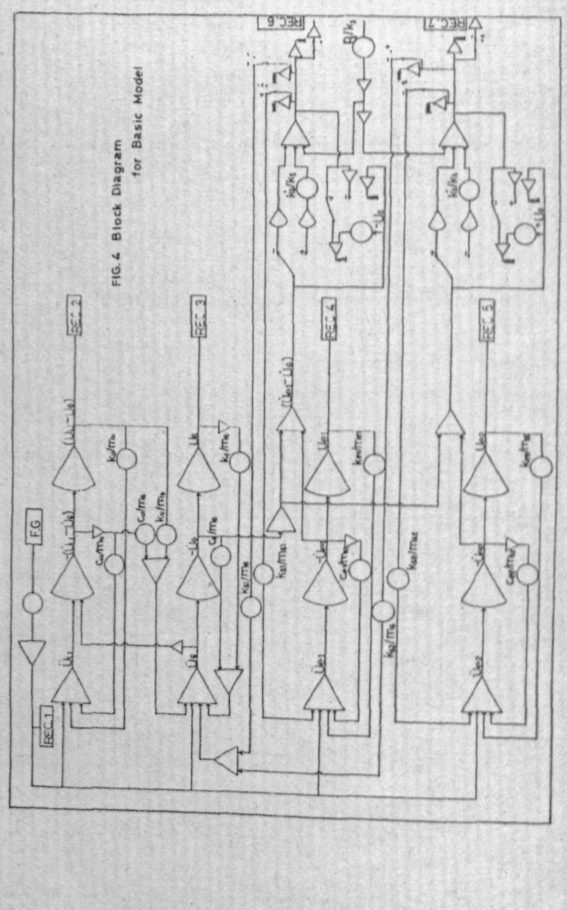


FIG.4 Block Diagram
for Basic Model
F.G
REC.1
REC.2
REC.3
REC.4
REC.5
REC.6
REC.7

Table 1 Constants for the Basic Model

		Fundamental Period of Building (sec)																											
		0.1 π				0.2 π				0.4 π				0.5 π				0.6 π				0.8 π				1.0 π			
Building Constants	ω_b (rad/sec.)	20.0				10.0				5.0				4.0				3.33				2.5				2.0			
	m_u ($t \cdot s^2/cm$)	0.0302				0.0631				0.123				0.153				0.184				0.245				0.306			
	m_k ($t \cdot s^2/cm$)	0.308				0.339				0.401				0.431				0.462				0.523				0.584			
	k_u (t/cm)	12.7				6.45				3.24				2.58				2.15				1.61				1.29			
	k_f (t/cm)	365.0				156.5				62.5				56.0				46.0				33.8				26.8			
Soil Constants	m_s ($t \cdot s^2/cm$)	0.556				0.556				0.556				0.556				0.556				0.556				0.556			
	ω_s (rad/s)	20 (T_s=0.1π)		10 (T_s=0.2π)		20 (T_s=0.1π)		10 (T_s=0.2π)		20 (T_s=0.1π)		10 (T_s=0.2π)		20 (T_s=0.1π)		10 (T_s=0.2π)		20 (T_s=0.1π)		10 (T_s=0.2π)		20 (T_s=0.1π)		10 (T_s=0.2π)		20 (T_s=0.1π)		10 (T_s=0.2π)	
	k_e (t/cm)	222	222	55.6	55.6	222	222	55.6	55.6	222	222	55.6	55.6	222	222	55.6	55.6	222	222	55.6	55.6	222	222	55.6	55.6	222	222	55.6	55.6
	k_s (t/cm)	3000	1500	375	187.5	3000	1500	375	187.5	3000	1500	375	187.5	3000	1500	375	187.5	3000	1500	375	187.5	3000	1500	375	187.5	3000	1500	375	187.5

Table 2 Constants for the Multi-mass Model

Stories (Period)	Masses ($t \cdot s^2/cm$) $m_{b1} \cdots m_{b5} = m_{s1} \cdots m_{s4} = m$	k_{b1} (t/cm)	k_{b2} (t/cm)	k_{b3} (t/cm)	k_{b4} (t/cm)	k_{b5} (t/cm)	k_{eu} (t/cm)	k_{ed} (t/cm)	k_s (t/cm)
4 (0.1π)	0.0151	25.7	45.3	58.8	61.1	318	12.8 3.2	18.1 4.5	1500 187.5
8 (0.2π)	0.0245	10.3	18.2	23.6	26.6	128.9	19.6 4.9	29.4 7.35	1500 187.5
16 (0.4π)	0.0490	5.17	9.12	11.8	13.3	64.4	39.2 9.8	58.8 14.7	1500 187.5
20 (0.5π)	0.0612	4.13	7.29	9.46	10.6	51.5	49 12.25	73.5 18.37	1500 187.5
24 (0.6π)	0.0734	3.44	6.06	7.86	8.84	42.8	58.7 14.7	88.2 22.05	1500 187.5
32 (0.8π)	0.0980	2.58	4.56	5.92	6.65	32.25	78.4 19.6	117.6 29.4	1500 187.5
40 (1.0π)	0.1224	2.05	3.56	4.69	5.27	25.6	97 24.25	145.5 36.6	1500 187.5