OSCILLATIONS OF TOWER-LIKE STRUCTURES WITH ACCOUNT OF INERTIA AND ELASTICITY OF SOLID MEDIUM

by B.G.Korenev[1], V.A.Iljichjov[2] and L.M.Reznikov[3]

Abstract

The paper deals with the problem of taking into account the inertia of the foundation in oscillations of tower-like structures induced by the foundation displacements. A homogeneous isotropic elastic half-space serves as a model of the foundation, the material of which possesses inertia properties. The tower-like structure is presented in the form of a cantilever bar of a variable cross-section. The solution of a number of problems has been obtained in a closed form, as well as in the form of expanding by natural functions of the appropriate boundary value problem. The results of calculations made by an electronic computer are given and discussed.

Introduction

The paper deals with some problems of the design of structures for seismic effects; the design scheme of these structures may be adopted as a cantilever rod. Here come such structures as radio masts and television towers, water-pressure towers, mine head frames, etc.

The main objective of the report is to develop methods of designing tower-like structures for seismic effects using a clarified design scheme. To clarify the design scheme the authors have considered a variable section rod, damping in the rod being taken into account by the complex theory of internal friction /8/, have discussed inertia and elasticity of the medium; besides, the influence of adjoined masses and dynamic absorbers of vibration have also been considered.

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[1] Prof.B.G.Korenev, Head of Laboratory of Dynamics, the Kucherenko Central Research Institute for Building Structures

[2] V.A.Iljichjov, Scient.Worker, Sen., Laboratory of Dynamics, the Kucherenko Central Research Institute for Building Structures.

[3] L.M.Reznikov, post-graduate student, Laboratory of Dynamics, the Kucherenko Central Research Institute for Building Structures.

Solution of some important problems which are basic parts of the design is done by the method of initial parameters /5/ in a closed form with special functions.

The authors have proceeded from the ideas of the seismic effect as a stationary random process or as some deterministic effect. Semi-infinite homogeneous isotropic inertia elastic medium was assumed for the model of the ground. This model was used in the works of V.Baranov /1/, T.Sukhara /12/ for simpler design schemes of the structure. The ultimate case where the rod may be regarded as an absolutely solid body is of certain interest. This case is considered in Section 7.

The paper considers the task of a joint account of the factors mentioned above, the numerical results have been obtained by electronic computers, the use of which is facilitated by the availability of solutions in a closed form. It is clear that the description of details of solving such a complex problem, which is rather difficult within the frames of one paper, rests upon some works published previously by the authors; some of those works are developed and summarized in the present paper.

1. Consideration of the Inertia of the Ground Base

Selection of design models of the ground base in the dynamics problems was discussed in reference /5/. The problem of vibrations of a tower-like structure regarded below allows to use various models without any change. The authors have chosen a model of a homogeneous isotropic elastic semi-infinite medium - one of the best studied and, evidently, most trustworthy model in the problems of dynamics.

The usage of the model is based on the results of the classical work by G.Lamb who solved the problem of a concentrated harmonic force acting on the surface of a semi-infinite medium. This work has served as a basis for a whole set of other investigations carried out by other authors who considered the problem of vibrations of a massive body resting on the surface of the semi-infinite medium. The works by E.Reissner /13/, O.Schekhter /9/, R.Arnold, G.Bycroft and G.Warburton /10,11/, N.Borodachiov /3/ and L.Sigalov /7/ are of particular interest. In the examples given in the paper the results of /11/ and /7/ are mainly used and in the latter work computations are provided for several values of Poisson's ratio.

Let us consider that a tower-like structure rests on the medium by means of a circular rigid base of radius r_0, and that the resistance of the medium is characterized by complex coefficients C_1 and C_2 which depend on the frequency. These coefficients are equal to the ratio of a disturbing dynamic force acting on the rigid base to its displacement caused by this force and in the case of a semi-infinite medium are determined by formulae:

a) under the action of a horizontal force

$$C_1 = C_{11} + iC_{12} = -\mu r_0 \frac{f_1}{f_1^2 + f_2^2} + i\mu r_0 \frac{f_2}{f_1^2 + f_2^2}; \tag{1.1}$$

b) under the action of couples in the vertical plane:

$$C_2 = C_{21} + iC_{22} = -\mu r_0^3 \frac{f_3}{f_3^2 + f_4^2} + i\mu r_0^3 \frac{f_4}{f_3^2 + f_4^2}; \tag{1.2}$$

where f_1, f_2, f_3 and f_4 are given in references /7, 10, 11/.

In some problems discussed later it is assumed that vibrations of the structure are set up by a flat harmonic wave propagating with the velocity v. Let horizontal X and vertical Y displacements of a point of the surface of the semi-infinite medium be determined by the formulae

$$X(x,t) = x_1 \cos\left[\frac{2\pi}{L}(x+vt)+\psi\right];\quad Y(x,t) = y_1 \sin\left[\frac{2\pi}{L}(x+vt)+\psi_1\right] \tag{1.3}$$

where x_1 and y_1 are amplitudes of horizontal and vertical components of the wave; L - the length of the wave; ψ and ψ_1 are angles of retardation of phases.

2. Forced Vibrations of Variable Section Rod under Harmonic Forces.

Consider steady vibrations of a cantilever rod (Fig.1), the rigidity and linear mass of which vary according to

$$EI = EI_R\left(\frac{x}{R}\right)^{\nu+2}; \qquad m = m_R\left(\frac{x}{R}\right)^{\nu} \tag{2.1}$$

It should be noted that natural vibrations of a tapered rod ($\nu = 2$) were studied by G.Kirchhoff and N.Mononobe, the general case of an arbitrary real ν was investigated by A.Dinnik /4/ who solved it in Bessel-functions. The differential equation for the rod motion is

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right) + m\frac{\partial^2 w}{\partial t^2} = q e^{ip_0 t} \tag{2.2}$$

Substituting $w = We^{ip_0 t}$ and replacing variables /5/ we arrive at a system of two heterogeneous Bessel equations

$$L(Z) \pm Z = \mp \frac{T}{2} \tag{2.3}$$

Here

$$L = \frac{d^2}{d\beta^2} + \frac{1}{\beta}\frac{d}{d\beta} - \frac{\nu^2}{\beta^2}; \qquad \beta = 2b\sqrt{x};$$

$$b^4 = \frac{m_R P_o^2 R^2}{EI_R}; \quad Z = \beta^\nu W; \qquad T = \frac{R^\nu (2b)^{2\nu}}{m_R P_o^2 \beta^\nu} q$$

Displacements and forces are determined by the following formulae

$$W = \frac{1}{\beta^\nu} W^\circ; \quad \varphi = \frac{2b^2}{\beta^{\nu+1}} \varphi^\circ; \quad M = \frac{EI_R \beta^{\nu+2}}{R^{\nu+2} 4(2b)^{2\nu}} M^\circ; \quad Q = \frac{EI_R \beta^{\nu+1}}{R^{\nu+2} 8(2b)^{2\nu-2}} Q^\circ \tag{2.4}$$

$$W^\circ = Z; \quad \varphi^\circ = \left(\frac{d}{d\beta} - \frac{\nu}{\beta}\right) Z; \quad M^\circ = \left[L - \frac{2\nu+1}{\beta}\left(\frac{d}{d\beta} - \frac{\nu}{\beta}\right)\right] Z; \quad Q^\circ = \left(\frac{d}{d\beta} - \frac{\nu}{\beta}\right) L(Z) \tag{2.5}$$

Fundamental functions of the system (2.3) having the property of a unit matrix with respect to Z, $\frac{d}{d\beta}Z$, $L(Z)$, $\frac{d}{d\beta}L(Z)$ are found in reference /5/, and there the method of obtaining the functions X, forming a unit matrix for W°, φ°, M°, Q° at the point $\beta = \alpha$ is given.

These functions have the form:

$$\begin{aligned}
X_{1,3} &= \frac{\pi\alpha}{4}\left\{\pm Y_\nu(\beta) J_{\nu+1}(\alpha) \mp J_\nu(\beta) Y_{\nu+1}(\alpha) + \frac{2}{\pi}\left[K_\nu(\beta) I_{\nu+1}(\alpha) + I_\nu(\beta) K_{\nu+1}(\alpha)\right]\right\} \\
X_2 &= \frac{\pi\alpha}{4}\left\{-Y_\nu(\beta) J_{\nu+2}(\alpha) + J_\nu(\beta) Y_{\nu+2}(\alpha) - \frac{2}{\pi}\left[K_\nu(\beta) I_{\nu+2}(\alpha) - I_\nu(\beta) K_{\nu+2}(\alpha)\right]\right\} \\
X_4 &= \frac{\pi\alpha}{4}\left\{-Y_\nu(\beta) J_\nu(\alpha) + J_\nu(\beta) Y_\nu(\alpha) - \frac{2}{\pi}\left[K_\nu(\beta) I_\nu(\alpha) - I_\nu(\beta) K_\nu(\alpha)\right]\right\}
\end{aligned} \tag{2.6}$$

In order to take account of internal friction in the rod /8/ a complex modulus of elasticity $E_o(u + iv)$ is introduced where $u = \frac{4-\gamma^2}{4+\gamma^2}$; $v = \frac{4\gamma}{4+\gamma^2}$, the Bessel functions will therefore, have a complex argument $\beta \approx \beta_o\left(1 + \frac{\gamma^2}{4}\right)\left(1 - i\frac{\gamma}{4}\right)$ $(\gamma \ll 1)$

Expanding the Bessel function $Z_\nu(x+iy)$ into Tailor's series by power (iy) and confining to six terms of the expansion we get

$$\begin{aligned}
\operatorname{Re} Z_\nu(x+iy) &= Z_\nu(x)\left[1 - \frac{y^2}{2}(a_1 \mp 1) + \frac{y^4}{24}(a_3 \mp a_5)\right] + a_o \frac{y^2}{2x} Z_{\nu-1}(x)\left[1 - \frac{y^2}{12}(a_6 \mp 2)\right] \\
\operatorname{Im} Z_\nu(x+iy) &= -\frac{y}{x} Z_\nu(x)\left[\nu - \frac{y^2}{6}(1 + \nu \mp a_2) + \frac{y^2}{120}(a_4 \mp a_{10})\right] + \\
&\quad + a_o y Z_{\nu-1}(x)\left[1 - \frac{y^2}{6}(a_7 - 1) + \frac{y^4}{120}(a_9 \mp a_8)\right]
\end{aligned} \tag{2.7}$$

Here upper signs correspond to the functions $J_\nu(x+iy)$, $Y_\nu(x+iy)$, and the lower ones correspond to the functions $I_\nu(x+iy)$, $K_\nu(x+iy)$;

$a_0=-1$ for $K_\nu(x+iy)$; $a_0=1$ for the remaining functions

$$a_1=\frac{\nu(\nu+1)}{x^2};\ a_2=a_1(\nu+2);\ a_3=a_2(\nu+3)+1;\ a_4=a_3(\nu+4)-2;\ a_5=\frac{2\nu^2+2\nu+3}{x^2};$$

$$a_6=\frac{6(\nu^2+2)}{x^2};\ a_7=\frac{2+\nu^2}{x^2};\ a_8=\frac{7+2\nu^2}{x^2};\ a_9=\frac{24+34\nu^2+\nu^4}{x^4}+1;\ a_{10}=a_5(\nu+2)+a_6;$$

As estimations for $\nu=1\div10$; $x=1\div10$; $\gamma=0.2$ have shown, the largest error when using (2.7) is 0.03%, if to ignore the terms containing y^4, y^5, the error is not more than 0.7%.

In boundary conditions account is taken of the presence of a dynamic absorber on the upper end of the rod and horizontal displacement of the foundation together with the base within the assumptions of section 1

$$\overset{\circ}{M}=0;\quad \overset{\circ}{Q}-\chi C\overset{\circ}{W}=0 \qquad \text{at } \beta=\varepsilon \tag{2.8}$$

$$\overset{\circ}{\varphi}=D\overset{\circ}{M};\ \overset{\circ}{W}=\sigma\overset{\circ}{Q}+\zeta \qquad \text{at } \beta=\alpha \tag{2.9}$$

Here $\varepsilon=2\delta\sqrt{\tau}$; $\alpha=2\delta\sqrt{R}$; $\chi=\dfrac{R^\nu m_r\varepsilon}{2\tau^{\nu+1}m_R}$; $C=\dfrac{f_0^2+\mu_0 ip_0}{f_0^2+\mu_0 ip_0-p_0^2}$;

$$D=-\frac{EJ_R\alpha}{2R(c_{21}+ic_{22}-Bp_0^2)};\ \sigma=\frac{m_Rp_0^2\,2R}{\alpha(c_{11}+ic_{12}-\mathfrak{M}p_0^2)};\ \zeta=\frac{A\alpha^\nu}{1-\frac{\mathfrak{M}p_0^2}{c_{11}+ic_{12}}};$$

m_r, f_0, μ_0 – are the mass, partial frequency and viscous friction coefficient of the absorber;

$\mathfrak{M}$, B are the mass and inertia moment of the foundation. Assuming the rod parameters at $\beta=\alpha$ as initial ones, with account of (2.9) the expression for the mode of forced vibrations will be written as

$$Z=\overset{\circ}{M}_\alpha(DX_2+X_3)+\overset{\circ}{Q}_\alpha(\sigma X_1+X_4)+\zeta X_1+\frac{\alpha^{2\nu}}{m_Rp_0^2}\int_\alpha^\beta\frac{q(\xi)X_4(\beta,\xi)}{\xi^\nu}d\xi \tag{2.10}$$

Unknown initial parameters $\overset{\circ}{M}_\alpha$ and $\overset{\circ}{Q}_\alpha$ are determined in terms of (2.8), and the integral (2.10) in the case of a load distributed according to the power law with an integer in the exponent is easily expressed by the Bessel functions. Provided the rod has adjointed masses, they are easily taken into consideration by substituting inertia forces for their action and using the above method of initial parameters.

3. Expansion of Solution of Forced Vibrations by Modes of Free Vibrations.

Here we give in short a different solution for the problem of forced vibrations, viz. by means of expanding by the modes of free vibrations of an undamped rod V_j ; the method seems to be useful also when regarding random action. Determining the coefficients of the resistance of the medium at the exciting frequency p_o , the boundary conditions and the mode of free vibrations will be:

$$M^{\circ}=Q^{\circ}=0 \quad \text{при} \quad \beta=\varepsilon; \quad \varphi^{\circ}=DM^{\circ}, \quad W=\sigma Q^{\circ} \ \text{при} \quad \beta=\alpha \tag{3.1}$$

$$z=(\sigma X_1+X_4)Q^{\circ}_{\alpha}+DX_2+X_3 \tag{3.2}$$

where

$$D=-\frac{EI_R\alpha}{2R(C_{21}-B\omega_{oj}^2)}; \quad \sigma=\frac{m_R p_o^2 2R}{\alpha(C_{11}-\mathfrak{M}\omega_{oj}^2)}$$

Satisfying the boundary conditions on the upper end of the rod, we obtain a frequency equation and then after its solution end parameters for modes of vibrations V_j can be determined.

To the outer load let's add the response of the absorber $m_r p_o^2 C W_z$, imaginary parts of the expressions for reactive response of the medium $(-ic_{22}\Theta_R)$ and $ic_{12}(W_R-A)$, inertia load with the intensity mp_o^2A and concentrated load $(-\mathfrak{M}p_o^2A)$ where W_z and W_R are complete displacements of the upper and lower ends of the rod and Θ_R is the angle of rotation of the lower end. Presenting the relative displacements in the form

$$W-A=\sum_{j=1}^{\infty}\lambda_j V_j \tag{3.3}$$

we find the expansion coefficients

$$\lambda_j=\frac{\int_z^R(q+mp_o^2A)V_j\,dx+m_r p_o^2 C W_z V_{zj}+ic_{12}(W_R-A)V_{Rj}-ic_{22}\Theta_R\varphi_{Rj}}{[\omega_{oj}^2(u+iv)-p_o^2][\int_z^R mV_j^2\,dx+\mathfrak{M}V_{Rj}^2+B\varphi_{Rj}^2]} \tag{3.4}$$

The values of W_z, W_R and Θ_R are determined by solving a system of three equations which was obtained by substituting (3.4) into (3.3).

The integral in the denominator in (3.4) is expressed through parameters at the ends of the rod /6/

$$\int_z^R mV_j^2\,dx=\frac{m_R R}{2\alpha^{2\nu+2}}\left|\beta^2[V^{\circ 2}+2Q^{\circ}\varphi^{\circ}+M^{\circ 2}]+2\beta[(1-\nu)Q^{\circ}V^{\circ}+(1+\nu)M^{\circ}\varphi^{\circ}]\right|_{\varepsilon}^{\alpha} \tag{3.5}$$

4. Efficiency of Dynamic Absorber under Harmonic Forces

Evaluation of the application efficiency of the absorber is shown /6/ on the example of a radio-tower in the case of uniform-along-the-rod-distribution of a harmonic load setting up vibrations. To reduce the calculations it was assumed that there are no horizontal displacements of the foundation as regards the medium.

Consider a radio-tower 100 m high, 50t in weight. The linear mass and rigidity in the lower section $m_R=0{,}08\,\frac{t\,sec^2}{m^2}$, $E_oI_R=10^7\,tm^2$; periods of natural vibrations are equal to (1.38; 0.39; 0.16; 0.09) sec.

The optimization of the parameters of the absorber was done by the method of the steepest descent on the Urals-4 electronic computer in terms of the minimum values of M_R and W_τ with changing the exciting frequency in the vicinity of the lowest natural frequency of the tower vibrations.

The computation results (Table) show that even with a relatively small mass of the absorber it was possible to reduce the resonant amplitudes by 3-5 times, the efficiency of the absorber being larger than when it was located on a constant section rod of the same mass. The affect of highest harmonics is not large though it is larger than without the absorber, so the usage of different criteria of optimization of the absorber parameters (for M_R and W_τ) yields results which are in good agreement. Dissipation of the energy in the medium due to radiation has not affected the results as the frequency of vibrations appeared to be significantly lower than the frequency at which intensive radiation of elastic waves occurs; however, for more rigid structure with small internal damping the influence of the inertia of the medium can be more significant.

5. Vibration of Tower-like Structures under Seismic Effects.

For approximate description of the seismic effect the hypothesis of a stationary process /2/ will be used which enables to obtain the solution by simple calculations. The horizontal displacement of the foundation is presented in the form of a stationary random process, the correlation function and spectral density of which are equal to

$$K(\tau)=e^{-\eta_o|\tau|}\left[\cos h_o\tau+\frac{\eta_o}{h_o}\sin h_o|\tau|\right];\quad S(p_o)=\frac{2\eta_o}{\pi}\,\frac{(\eta_o^2+h_o^2)}{[(p_o^2-\eta_o^2-h_o^2)^2+4\eta_o^2h_o^2]}\tag{5.1}$$

To evaluate the dispersion of displacement or stress at any point of the rod

$$\langle s^2(\tau)\rangle=\int_{-\infty}^{\infty}|\Phi_s(ip_o)|^2\,S(p_o)\,dp_o\tag{5.2}$$

it is necessary to find the respective transfer function of the system $\Phi_s(ip_0)$ which is also a function of the coordinate x of the rod. The solution given in Section 2 may be used for the determination of $\Phi_s(ip_0)$ assuming in (2.10) q=0, A=1. The integral (5.2) is determined numerically. If to form the transfer function by expanding by modes of vibrations (Section 3), then, as is known, the dispersion $\langle S^2(\tau)\rangle$ may be presented in the form of a series

$$\langle S^2(\tau)\rangle = \sum_{i,j=1}^{\infty} \langle \lambda_j(\tau)\lambda_i(\tau)\rangle S(V_j)\cdot S(V_i) \tag{5.3}$$

where $S(V_j)$ is the distribution of stress or displacement along the rod corresponding to the mode V_j.

In this case the number of correlations of generalized coordinates being computed depend on the number of terms in (5.3) being retained and on the parameters of the spectrum of the effect. Computing the correlations gives the dispersion of stresses and displacements at any point of the rod. Correlations of generalized coordinates $\langle\lambda_j(\tau)\lambda_i(\tau)\rangle$ are determined numerically by the expression

$$\lambda_j = \frac{p_0^2 A \int_{z}^{R} m V_j\,dx + m_r p_0^2 C W_z W_{zj} + [C_{11}(p_0) - C_{11}(\tilde{p}) + iC_{12}(p_0)](W_R - A)V_{Rj} - [C_{21}(p_0) - C_{21}(\tilde{p}) + iC_{22}(p_0)]Q_R\varphi_{zi}}{[\omega_{0j}^2(u+iv) - p_0^2][\int_{z}^{R} m V_j^2\,dx + \mathcal{M} V_{Rj}^2 + B\varphi_{Rj}^2]} \tag{5.4}$$

where p_0 is the present value of the variable of integrating

$\tilde{p}$ is the fixed frequency at which the coefficients of the medium resistance are established for obtaining natural modes V_j

The limits of integration should be confined in terms of the values η_0 and h_0.

In the absence of an absorber and taking into account the fact that natural frequencies of the system are far from one another and that the value of damping is small, cross correlations of generalized coordinates ($i \neq j$) in (5.3) may be neglected. Besides, for tall structures with a foundation of a small area horizontal displacements of the foundation as regards the medium may be ignored and its rotation only may be taken into consideration.

6. Ascertainment of Efficiency of Dynamic Absorber under Random Effects.

The opinion is widely spread that the dynamic absorber may be used only in case of harmonic forces. However, by the nature of its effect on the absorber the harmonic force with unstable exciting frequency differs very little from a stationary random effect. In this connection it is of interest to investigate the behaviour of the dynamic absorber under a random excitation of the medium of the form (5.1) on an example of a more simple system (Fig.2).

If to assume for an output value the displacement of the main mass, the amplitude-frequency characteristic of the system will be

$$\Phi(ip_o)=\frac{\xi_1(-p^2+f^2\xi_2)}{(-p^2+\xi_1)(-p^2+f^2\xi_2)-\vartheta p^2 f^2\xi_2}, \tag{6.1}$$

where

$$p=\frac{p_o}{\omega_{o1}},\quad f^2=\frac{\omega_{o2}^2}{\omega_{o1}^2},\quad \xi_j=u_j+iv_j,\quad u_j=\frac{4-\gamma_j^2}{4+\gamma_j^2},\quad v_j=\frac{4\gamma_j}{4+\gamma_j^2},\quad j=1,2$$

ω_{oj} and γ_j are partial frequencies and coefficients of internal damping of the main mass and absorber;

$\vartheta=\frac{m_r}{m}$ is the ratio of masses.

The spectral density at the output (an asterisc signifies a complex conjugated value)

$$S_x=S(p_o)\Phi(ip_o)\Phi^*(ip_o)=\frac{2\eta(\eta^2+h^2)}{\pi\,\omega_{o1}}\sum_{j=1}^{3}\left(\frac{a_j}{p^2-p_j^2}+\frac{a_j^*}{p^2-p_j^{*2}}\right) \tag{6.2}$$

where

$$a_j=\frac{(p_j^2-f^2\xi_2)(p_j^2-f^2\xi_2^*)}{\prod\limits_{k=1\,(k\neq j)}^{3}(p_j^2-p_k^2)\prod\limits_{k=1}^{3}(p_j^2-p_k^{*2})},\quad \eta=\frac{\eta_o}{\omega_{o1}},\quad h=\frac{h_o}{\omega_{o1}},\quad p_3^2=(h+\eta i)^2$$

p_1^2 and p_2^2 are the roots of the denominator (6.1) determined by the solution of the square equation.

The dispersion of the displacement of the main mass is determined by integrating the expression (6.2) by means of residia, the contour of integration being taken in the upper half-plane.

$$\langle x^2(\tau)\rangle=-4\eta(\eta^2+h^2)\,\mathrm{Im}\sum_{j=1}^{3}\frac{a_j}{p_j} \tag{6.3}$$

Since the spectrum characteristics of the effect can be obtained only approximately, we shall assume that parameter η is fixed, and h has a value corresponding to the maximum of $\langle x^2(\tau)\rangle$. For a particular case at $\eta=0$ there will be a harmonic effect with unstable frequency h.

Minimized value of standard displacement σ_x at optimum parameters of the absorber are shown in Fig.(3a,b,c). As can be seen, with the increase of η the curves become closer and the efficiency of the absorber decreases; this is attributed to the growing pass-band of the system. The reduction of γ_1 and increases of ϑ lead to the increase of the efficiency of the absorber, though this result is not so significant as with the harmonic effect.

Below the formulae for selecting optimum parameters of the absorber are given

a) tuning

$$f^2=\begin{cases} 1-1{,}25\vartheta+0{,}1[-(1{,}1+9\vartheta)\gamma_1+(1{,}1+2\vartheta+3\gamma_1)\eta] & \text{at } \eta\leqslant 0{,}1+\gamma_1+\vartheta \\ 1-0{,}8\vartheta & \text{at } 0{,}1+\gamma_1+\vartheta<\eta<0{,}6 \\ 1-0{,}6\vartheta & \text{at } \eta>0{,}6 \end{cases} \tag{6.4}$$

b) damping coefficient

$$\gamma_2=\begin{cases} \sqrt{1{,}5\vartheta}\left[1-\frac{\eta}{0{,}1+3\vartheta}\left(1-0{,}75\frac{\eta}{0{,}1+3\vartheta}\right)\right] & \text{at } \eta\leqslant 0{,}13+3\vartheta \\ \sqrt{\frac{\vartheta}{1+\vartheta}} & \text{at } \eta>0{,}13+3\vartheta \end{cases} \tag{6.5}$$

The results obtained allow to conclude that a dynamic absorber may be used under random effects with a narrow band spectrum acting on a system of low damping.

7. Vibrations of Rigid Structures.

When studying vibrations of a rigid tower-like structure it may approximately be looked upon as an absolutely rigid body resting on an elastic medium; for short, such a body will be further termed a mass. The simplicity of the design scheme of the structure allows to investigate in details the interaction of the structure with the medium arriving at the numerical values required.

Consider only horizontal-rotational vibrations of the mass under the effect of a wave (1.3). The centre of gravity of the mass is on the vertical crossing the centre of the circular rigid base.

The displacement of a circular weightless rigid base under the effect of the wave passing under its foot is equated to the mean displacement of two points on the surface of the semi-infinite medium (in the absence of the rigid base) lying on the diameter ends parallel to the direction of the wave propagation:

$$x_0 = \frac{X(r,t) + X(-r,t)}{2} = x_1^0 \cos(pt + \psi); \quad p = \frac{2\pi v}{L}$$

$$\varphi_0 = \frac{Y(r,t) - Y(-r,t)}{2r_0} = \varphi_1^0 \cos(pt + \psi_1) \tag{7.1}$$

Coefficients x_1^0 and φ_1^0 depend on x_1, y_1, L, ψ and ψ_1.

The horizontal displacement of the centre of gravity of the mass $\bar{x}$ and rotation angle $\bar{\varphi}$ will be presented in the form

$$\bar{x} = x_0 + x; \qquad \bar{\varphi} = \varphi_0 + \varphi \tag{7.2}$$

Write down the differential equations defining the vibrations of the mass (Fig.4):

$$m\ddot{\bar{x}} + c_1(x - \ell\bar{\varphi}) = 0$$

$$B\ddot{\bar{\varphi}} + c_2\varphi - c_1\ell(x - \ell\bar{\varphi}) + P\ell\bar{\varphi} = 0 \tag{7.3}$$

where P is the weight of the mass; $m = \frac{P}{g}$; B is the moment of inertia of the mass about the axis crossing its gravity centre; ℓ is the height of the gravity centre above the surface of semi-medium; c_1 and c_2 are determined by (1.1).

Substituting (7.2) into (7.3) we shall arrive after some calculations, at a system of algebraic equations with complex coefficients. From this system we find Q, M and angles of retardation of phases between the displacements of the mass and Q and M, and then x and φ may be found and the displacements of the mass are determined by (7.2).

Let's take as an example vibrations of a silo block of a grain elevator, with parameters r_0 = 13.2m, P = 20000t, ℓ = 22.7m. Computations are carried out for the Poisson's ratio of the medium equal to 0.25, the results are shown in Fig.5a,b. The first resonant peak corresponds to the mode similar to rotational vibrations about the axis crossing the base of the mass, the second peak, which corresponds to the prevailing horizontal vibrations, is found only in the graph for M.

Consider an elastic-viscous ground base of Winkler type instead of a semi-infinite medium. The differential equations describing horizontal rotational vibrations of the mass resting on such a medium have the form of:

$$m\ddot{\bar{x}} + u_x(x - \ell\bar{\varphi}) + b_x(\dot{x} - \ell\dot{\bar{\varphi}}) = 0$$

$$B\ddot{\bar{\varphi}} + u_\varphi\varphi - u_x(x - \ell\bar{\varphi})\ell - P\ell\bar{\varphi} + b_\varphi\dot{\varphi} - b_x(\dot{x} - \ell\dot{\bar{\varphi}}) = 0; \tag{7.4}$$

The solution of these equations will coincide in the form with the solution of (7.3), if

$$u_x = c_{11}, \quad b_x = c_{12}, \quad u_\varphi = c_{21}, \quad b_\varphi = c_{22} \qquad (7.5)$$

Coefficients u_x, b_x, u_φ and b_φ in this case are functions of frequency. However for many structures for which partial frequencies of horizontal and rotational vibrations are far from each other these coefficients may be estimated. without gross errors in the solution, at fixed (resonance) values of frequencies: u_x and b_x at $p = p_x^{rez}$ and u_φ and b_φ at $p = p_\varphi^{rez}$. The values of p_x^{rez} and p_φ^{rez} are taken from exact solution. The results of computations according to such a scheme for the silo block of the grain elevator under consideration are presented in Fig.5a (dotted line).

For some types of non-harmonic effects on a mechanical system described by Eq.(7.4) and which is a low-frequency filter, the numerical results required may be easily obtained. Evaluation of errors of the proposed method needs, however, further analysis.

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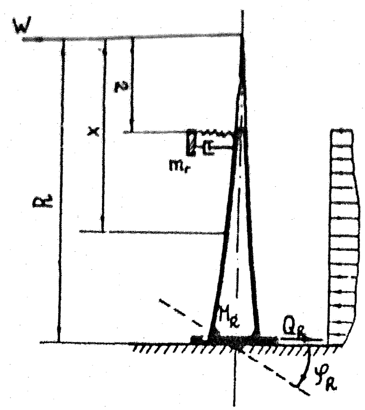

Fig. 1

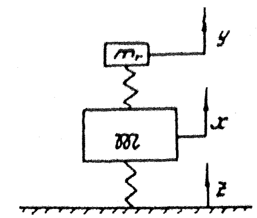

Fig. 2

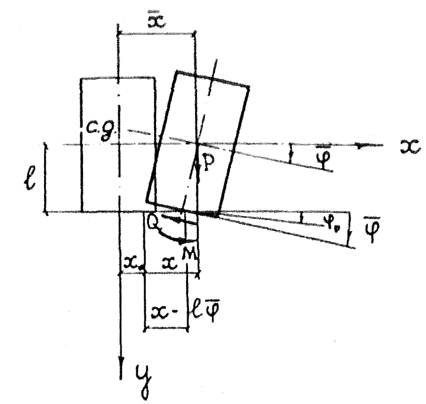

Fig. 4

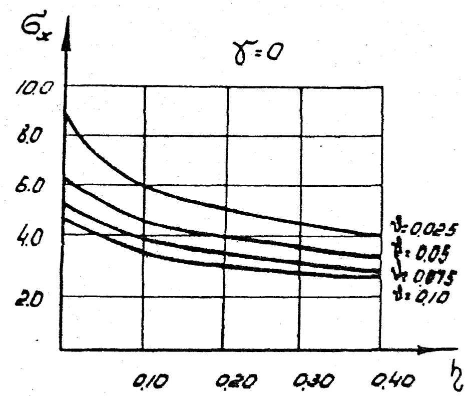

Fig. 3a

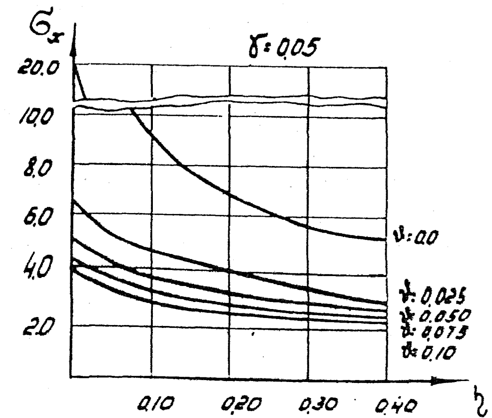

Fig. 3b

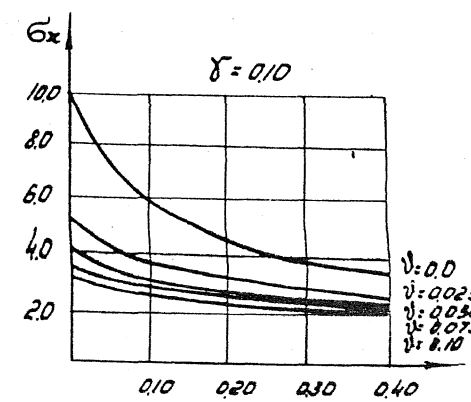

Fig. 3c

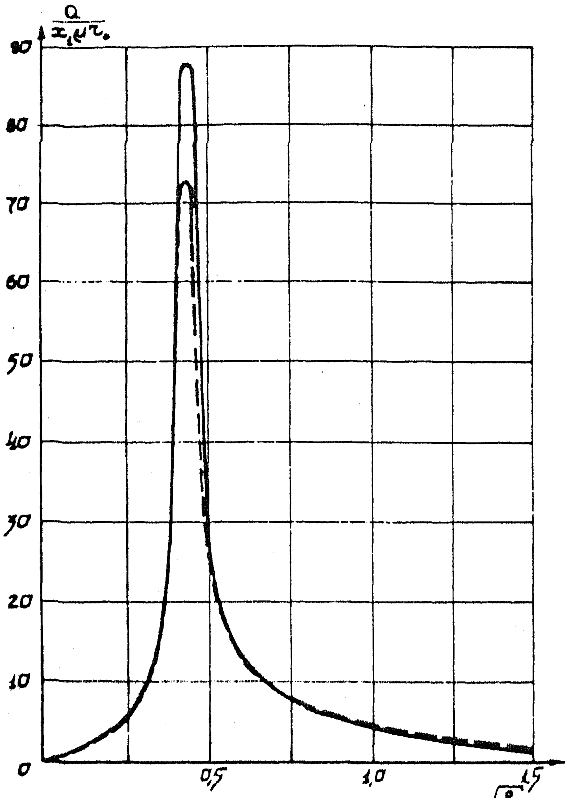

Fig. 5a

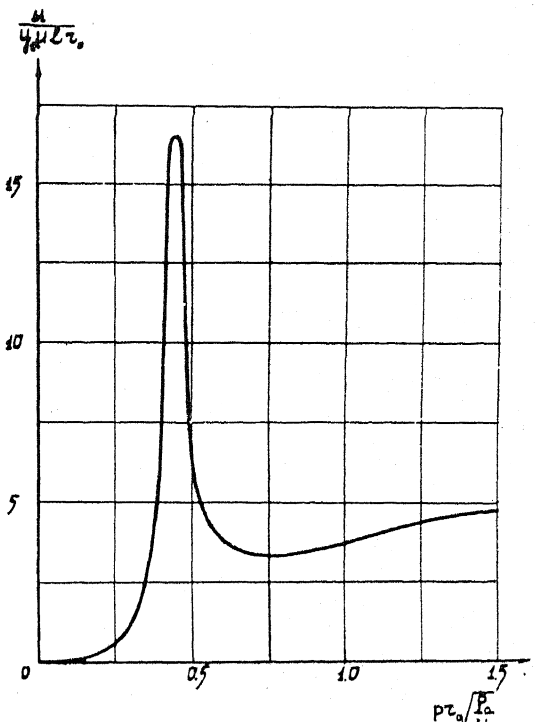

Fig. 5b

Table

Amplitude Values M_R(tm) and W_τ(cm) under Harmonic Load $q_0 = 0.003$ t/m and Optimum Parameters of Absorber.

Damping coefficient γ		Ratio of absorber mass to mass of structure				
		0	0.5%	1.0%	1.5%	2.0%
0.05	M_R	283	79	60	52	46
	W_τ	18.9	5.3	4.1	3.5	3.1
0.10	M_R	142	63	51	45	40
	W_τ	9.5	4.2	3.5	3.0	2.7
0.15	M_R	95	53	44	40	36
	W_τ	6.3	3.5	3.0	2.6	2.4