ELASTIC SOIL-STRUCTURE INTERACTION

by

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SYNOPSIS

The soil under a two-dimensional building frame is idealized by rectangular finite elements. The natural frequencies and corresponding mode shapes of the idealized lumped-mass system are determined, and the influence of the soil on the free vibration characteristics is studied. The idealized system is, then, subjected to Earthquake Accelerograms, and time-histories of displacements, accelerations and forces are obtained.

The results of the dynamic analyses indicate the importance of soil-structure interaction in determining the seismic response of the frame when founded on soft soil. An indication is also obtained of the development of significant dynamic moments in bearing piles used to support the frame.

INTRODUCTION

The seismic response of a structure under earthquakes is sometimes determined by applying recorded accelerograms to the base of a mathematical or analog model of the structure. However it is known that the extent and properties of the soil between the foundation and the bed-rock significantly influences the ground motion which may be felt at a specific location. Recently Fleming, Screwvala and Kondner (1) have suggested that the Design Earthquake should be applied to the rock layer below the structure, and the stiffness of the soil, and the mass of the soil that moves with the structure should be included in the determination of the seismic response. A similar view-point has been adopted by Finn and Khanna (2) in a study of the seismic response of earth dams under various foundation conditions. A significant feature of this study (2) has been the use of Finite Elements, which allow a consistent representation of the stiffness and mass of the structural system, in the idealization.

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In this paper, the foundation under a two-dimensional building frame is idealized by rectangular isotropic finite elements. Various soil conditions are considered, including the presence of bearing piles, idealized as one-dimensional finite elements (i.e., frame members). Then the elastic dynamic response of the structural system, including displacements, accelerations and member forces is determined by the method outlined in the next section. Also the maximum base shear obtained by Earthquake Input is compared to that obtained by the Response Spectrum Technique (3).

METHOD OF ANALYSIS

The equations of motion of a multi-degree lumped mass system may be written as

$$[M] \{\ddot{d}\} + [C] \{\dot{d}\} + [K] \{d\} = \{F(t)\} \qquad (1)$$

where $[M]$ is the mass matrix, $[C]$ the viscous damping matrix, $[K]$ the stiffnes matrix, $\{F(t)\}$ the dynamic load vector, and $\{d\}$, $\{\dot{d}\}$, $\{\ddot{d}\}$ are the displacement, velocity, and acceleration vectors in the structure coordinate system.

These equations are transformed to normal coordinates by first solving the eigenvalue problem

$$-W_n^2 [M] \{\phi_n\} + [K] \{\phi_n\} = 0 \qquad (2)$$

for the natural frequencies of undamped free vibration $\{W_n\}$, and the corresponding mode shapes $[\phi]$.

Now the normal or modal coordinates, Y, are related to the structure coordinates by

$$\{d\} = [\phi] \{Y\} \qquad (3)$$

The equations of motion can, therefore, be reduced to n normal mode equations

$$\ddot{Y}_n + 2 \lambda_n W_n \dot{Y}_n + W_n^2 Y_n = \frac{P_n^*(t)}{M_n^*} \qquad (4)$$

in which $M_n^* = \{\phi_n\}^T [M] \{\phi_n\}$ where the superscript T signifies transpose,

$P_n^* = \{\phi_n\}^T \{F(t)\}$, and λ_n is % critical damping in n^{th} mode.

It is assumed that the damping matrix has the orthogonality property

$$\{\phi_m\} [C] \{\phi_n\} = 0 \quad m \neq n \qquad (5)$$

The normal equations are solved for the modal displacements Y_n and accelerations $\ddot{Y}_n$, using the step-by-step matrix analysis method described by Wilson and Clough (4). Then, using equations (3) the structure displacements $\{d\}$ and, by a similar transformation, the structure accelerations $\{\ddot{d}\}$ are obtained at discrete intervals of time. From the structure displacements the element forces for the ith element are determined by

$$\{f_i\} = [k_i]\{\delta_i\} \tag{6}$$

where $\{f_i\}$, $[k_i]$, $\{\delta_i\}$ are the element forces, stiffness matrix and displacements respectively.

ANALYSIS OF PLANE FRAME

The structure investigated in this paper is shown in Figure 1. It is a three storey steel frame hinged at the base. The section properties, and lumped masses are shown in Figure 1. Three degrees of freedom are permitted at each joint of the plane frame. However, for the dynamic analysis, it is assumed that the rotary inertial forces are negligible, so that an implicit relation is available between the rotations and translations at the joints allowing a reduction in the order of the stiffness matrix.

The soil under the building frame is idealized by rectangular plane strain finite elements 20 feet wide. This assumption is based on the idea that the frame is a part of an infinitely long building with 20 feet bays, and is subjected to earthquake in the transverse direction causing only plane strain deformation.

The area of the soil acting with the frame is assumed to be between the bed-rock and the side boundaries for a shallow foundation with 30 feet depth, and a deep foundation with 100 feet depth (figure 2). The mass of the soil in a rectangular element is lumped equally to the four nodes. Two sets of elastic properties are used to characterise the soil as hard or soft as shown in figure 1. These values are considered to be representative of dense silty clay with sand (or sand and gravel), and clay (or loess) respectively by Barkan (5), and Timoshenko and Goodier (6). In a variant of the deep and soft foundation, steel bearing piles fixed at the base are inserted in the structural system by representing them as frame members as shown in figure 2.

The idealized soil-structure system is subjected to 0-10 seconds of the N-S component of the El Centro 1940 earthquake, which is a relatively high frequency record, and the 10-20 seconds interval of the N79° - 14'E acceleration component of the Alameda Park, Mexico City, 1962 earthquake, which is a low frequency earthquake. The Alameda Park record was scaled up so as to give the same peak value of the acceleration as El Centro (i.e., 0.32g). Damping was assumed to be 5% of critical in each mode.

DISCUSSION OF RESULTS

The natural frequencies of undamped free vibrations corresponding to the first three modes of the idealized system are shown in Table 1 for the various cases considered. Note that for the range of the soil elastic modulus and the extent of the participating soil considered, the fundamental frequency of vibration of the frame is hardly influenced by the presence of t soil, although a very slow trend towards reduction of the frequency with reduction in the soil modulus is in evidence. It may also be seen from the fundamental modes of vibration, for the case of soft soil, and the shallow and deep foundations considered, shown in figures 3 and 4, that normalized soil displacements corresponding to this mode are very small.

For shallow foundation and E_s = 40 ksi, even the second mode has a frequency within 0.5% of the rigid case indicating that for soil modulus greater then 40 ksi, and soil participation in the vibration corresponding to or less than this case, the dynamic response of the frame would not be significantly influenced by the presence of the soil. This is borne out by the closeness of maximum response values for column shear, top displacement and acceleration to the Alameda Park Earthquake for the rigid case and for shallow foundation (E_s = 40 ksi) shown in Table 2. Note however that reductions in soil modulus, and increase in depth of soil, which in effect reduce the soil stiffness, affect the second and higher modes significantly. Thus, it may be seen from Table 1 that for the deep foundation and soft soil, the second mode frequency is less than a third of the corresponding frequency when the foundation is assumed rigid. The increase in soil participation in this mode may also be noted in the mode-shapes corresponding to the second mode, for the shallow and deep foundations with soft soil, shown in figures 5 and 6 where the normalized soil displacements are fairly large although the frame still moves in the 'whipping mode' as is usual for the rigid case.

The effect of variations in the soil modulus, and extent of participating soil, on the modes of vibration higher than the second mode is considerable, as may be seen from the third mode frequencies given in Table 1. It should be pointed out here that the modes of vibration in these higher modes are not necessarily similar to each other for different soil modulii, and extent of participating soil.

The maximum response of the idealized system to the Alameda Park 1962, and El Centro 1940 earthquakes are shown in Table 2 with respect to base shear in a column, and horizontal accelerations and displacements at the top of the frame. It was found that these maxima occurred either at the same instant or close to each other for a specific earthquake input.

It is interesting to note that the maximum probable column shear determined separately by the Response Spectrum method using the Idealized 1940 El Centro Response Spectrum (3), having a displacement bound of 13 in., a velocity bound of 28 inches per second, and an acceleration bound of 1 g, is about 35% less than the maximum response obtained by applying the actual El Centro 1940 earthquake to the frame with shallow foundation (E_s = 40 ksi) which as has been previously noted corresponds closely to the rigid case. This variation occurs mainly due to the use of the idealized polygonal spectrum. Note also that the maximum response of the frame to the Alameda Park Earthquake for the same shallow foundation is about 50% larger than for El Centro. This is due to the fact that the predominant frequency in the Alameda Park record in the latter half of the interval considered is closer to the fundamental frequency of the frame, and points to the necessity of choosing the Design Earthquake with care.

Note, also, that for the deep foundation and soft soil (E_s = 10 ksi) the maximum column base shear is about 150% greater than the rigid case for Alameda Park input, and about 170% greater than the rigid case for El Centro input. This shows that the elastic modulus and the extent of soil can significantly influence the response of the frame, and such an influence will not necessarily alleviate the seismic forces in comparison to the rigid case, as in sometimes assumed.

A time-history of the base column shear when subjected to Alameda Park earthquake is shown in figure 7 for soft soil, and shallow and deep foundations, and the amplification in the latter case is obvious.

Finally, figure 8 shows the mid-point bending moment in the bearing piles which were used to support the frame in the case of soft soil, and deep foundation. The piles were assumed to be fixed at the base, and hinged at the top. While the effect of the piles is not significant because of their negligible effect on the soil mass and stiffness (Tables 1, 2) the effect of the soil movements on the pile is quite significant, as may be seen from the 120 kip feet maximum moment generated in the pile.

CONCLUSIONS

1. This study has shown that Finite Elements may be used advantageously for including the soil in a study of seismic response of structures.

2. For the plane frame, within the range of elastic modulus, and extent of participating soil considered, the second and higher modes of vibration are significantly influenced by the soil.

3. The presence of soft and deep soil significantly influences the seismic response of the frame. While it is admitted that the amplifications observed in this study pertain to elastic frame response, and are based on assumed elastic soil behavior, nevertheless the results suggest that significant amplifications can occur due to actual soil-structure interaction.

4. For the frame considered here, the base shear determined by using an idealized polygonal response spectrum differs significantly from the elastic base shear determined by Earthquake Accelerogram input.

5. The seismic response of the frame, under the soil conditions idealized herein, can vary significantly with the input of different Earthquake Accelerograms scaled to have the same peak value.

6. Significant bending moments can occur in bearing piles fixed at the base due to seismic movements of the soil through which they are driven.

ACKNOWLEDGEMENTS

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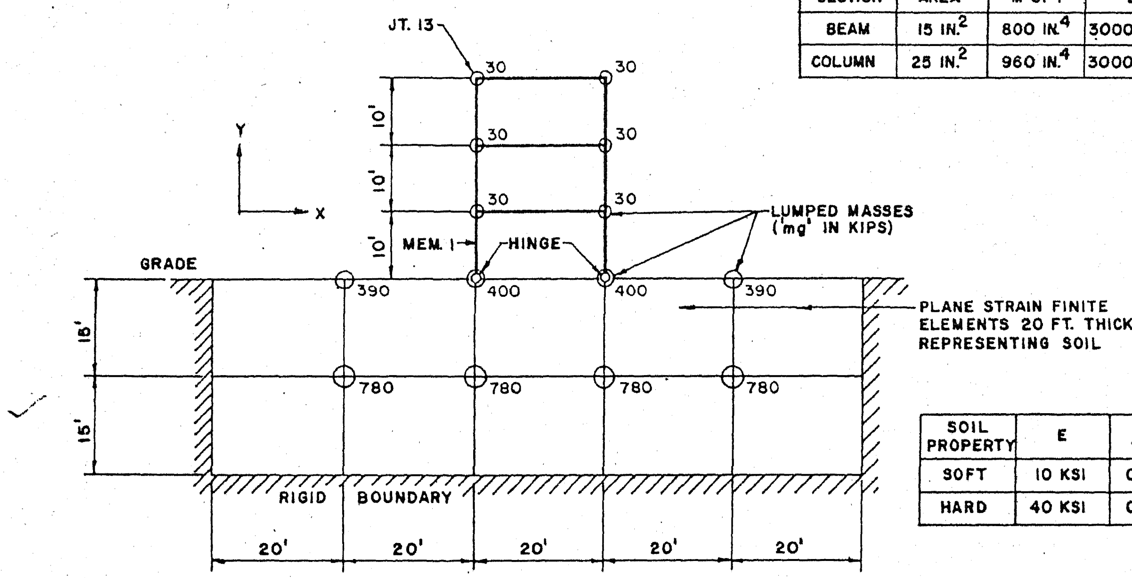


SECTION	AREA	M OF I	E
BEAM	15 IN.^{2}	800 IN.^{4}	30000 KSI
COLUMN	25 IN.^{2}	960 IN.^{4}	30000 KSI

SOIL PROPERTY	E	μ	γ
SOFT	10 KSI	0.4	130 pcf
HARD	40 KSI	0.3	130 pcf

FIG. 1 FRAME ON SHALLOW FOUNDATION

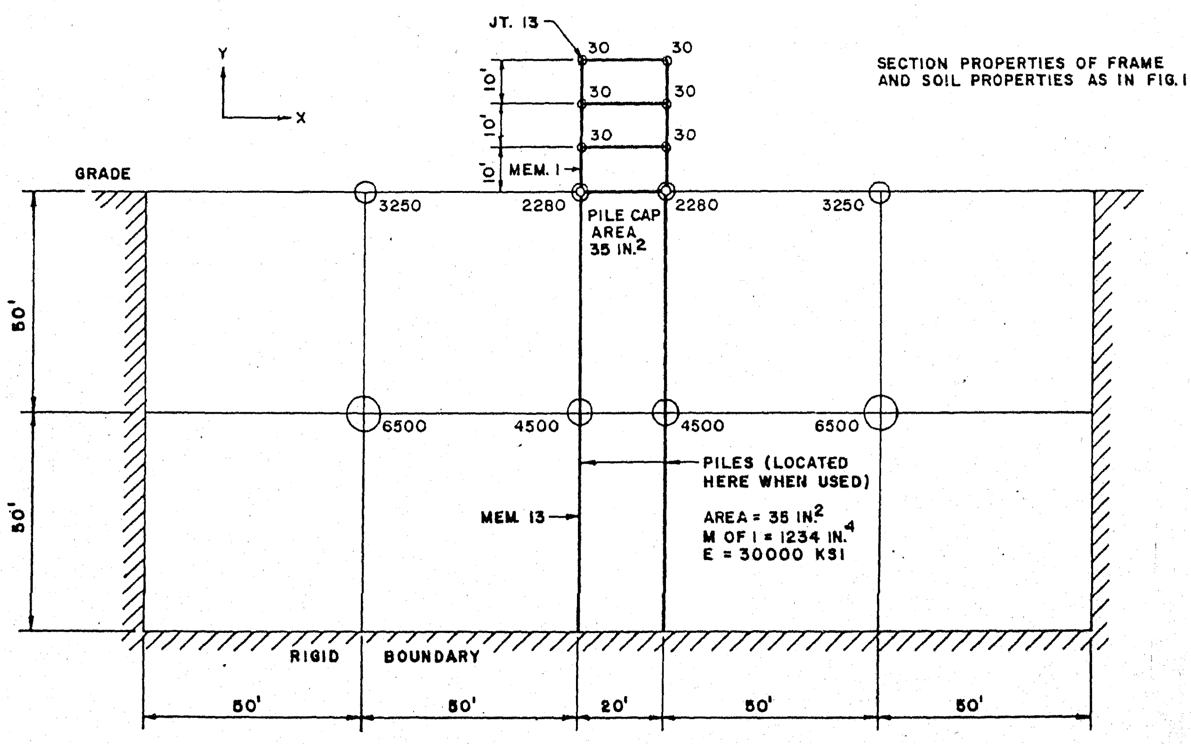
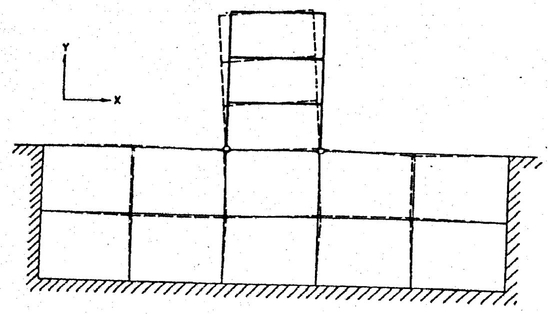
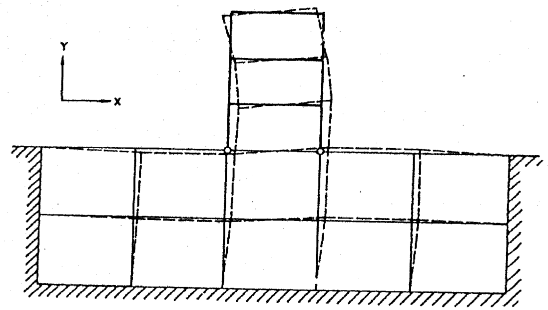


FIG. 2 FRAME ON DEEP FOUNDATION



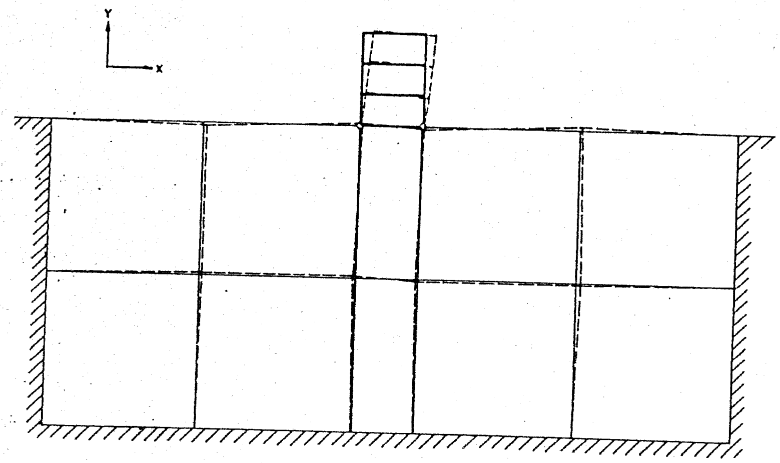
FREQUENCY 1.22 C.P.S.

FIG. 3 FUNDAMENTAL MODE IN SOFT SOIL SHALLOW FOUNDATION



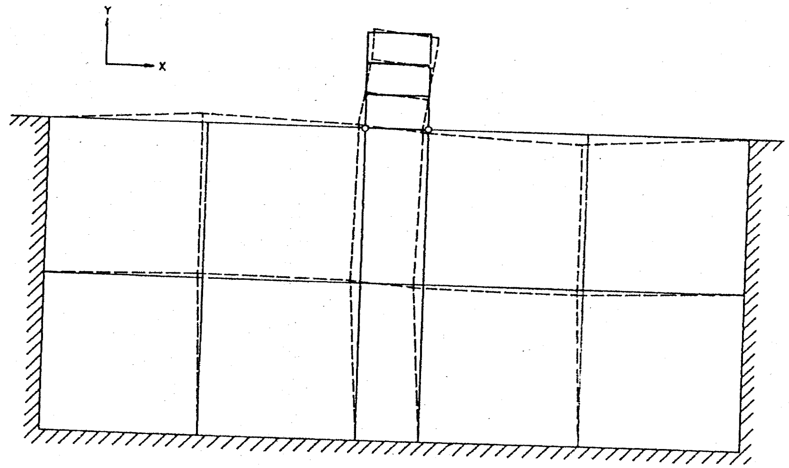
FREQUENCY 4.17 C.P.S.

FIG. 5 SECOND MODE IN SOFT SOIL SHALLOW FOUNDATION



FREQUENCY 1.21 C.P.S.

FIG. 4 FUNDAMENTAL MODE IN SOFT SOIL DEEP FOUNDATION



FREQUENCY 1.63 C.P.S.

FIG. 6 SECOND MODE IN SOFT SOIL DEEP FOUNDATION

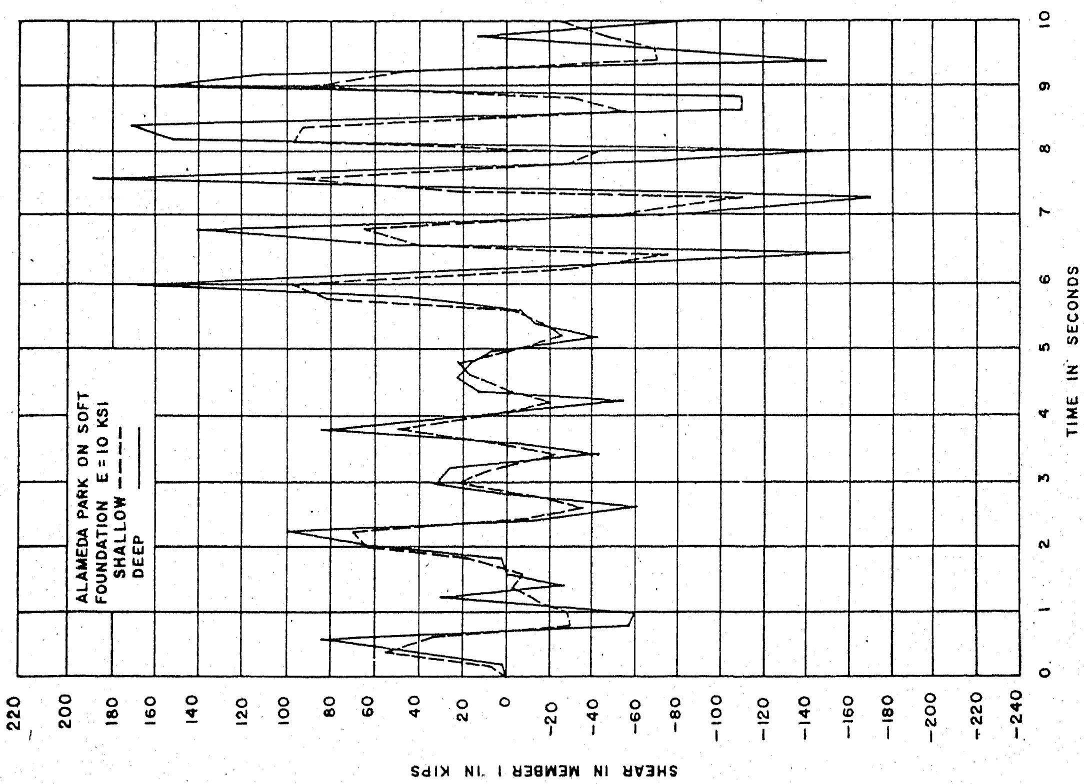


FIG. 7 SHEAR IN COLUMN

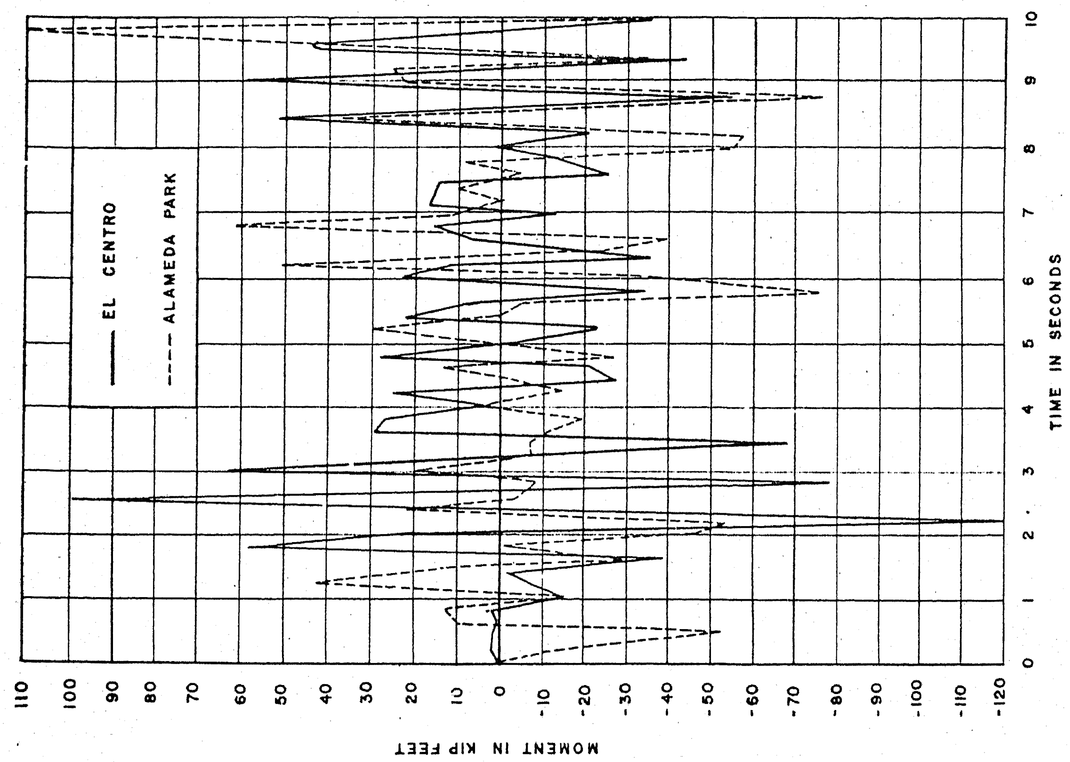


FIG. 8 BENDING MOMENT IN PILE IN SOFT SOIL

TABLE 1

NATURAL FREQUENCIES OF IDEALIZED SYSTEM

TYPE OF FOUNDATION	MODE 1 C.P.S.	MODE 2 C.P.S.	MODE 3 C.P.S.
RIGID	1.24	5.25	11.85
SHALLOW (E_s = 40 KSI)	1.24	5.23	8.33
SHALLOW (E_s = 10 KSI)	1.22	4.17	5.02
DEEP (E_s = 40 KSI)	1.24	3.22	3.31
DEEP (E_s = 10 KSI)	1.21	1.63	1.85
DEEP (E_s = 10 KSI WITH PILES)	1.22	1.64	1.96

TABLE 2

MAXIMUM RESPONSE OF IDEALIZED SYSTEM

TO EL CENTRO 1940 AND SCALED ALAMEDA PARK 1962 EARTHQUAKE

TYPE OF FOUNDATION	SHEAR IN* COLUMN 1 KIPS		X-ACCN AT JT. 13 FT/SEC^2		X-DISP AT JT. 13 FT.	
	EL CENTRO	ALAMEDA PK.	EL CENTRO	ALAMEDA PK.	EL CENTRO	ALAMEDA PK.
RIGID	-	110	-	19.1	-	0.33
SHALLOW (E_s = 40 KSI)	75	110	28.1	18.7	0.24	0.32
SHALLOW (E_s = 10 KSI)	98	110	45.2	19.9	0.27	0.34
DEEP (E_s = 40 KSI)	110	120	37.4	22.9	0.30	0.37
DEEP (E_s = 10 KSI)	200	190	41.0	37.6	0.51	0.53
DEEP (E_s = 10 KSI WITH PILES) **	210	190	-	-	-	-

* By Response Spectrum Method using Idealized 1940 El Centro Response Spectrum, the maximum probable shear (based on r.m.s. value) was 49 kips for rigid foundation.

** In this case the maximum pile bending moment was found to be 120 kip feet.