EARTHQUAKE ANALYSIS OF EARTH DAMS

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SYNOPSIS

The finite element concept is utilized to develop a general two-dimensional dynamic stress analysis technique. The simpler shear beam approach for earthquake analysis of earth dams is compared with the finite element method. This comparison is based on natural frequencies and mode shapes of typical earth dam cross-sections and responses to an earthquake. The capability of the finite element method to treat earth dams with impermeable cores is demonstrated by an example. The significance of response of earth dams to vertical component of ground motion is discussed. The analysis technique is extended to include spatial variations in ground motion across the base of the dam, and their effect on dam response is investigated.

INTRODUCTION

The occurrance of frequent destructive earthquakes during the past decade and the trend towards construction of structures of unprecedented size and of novel designs has enhanced the importance of earthquake resistant structural design. The catastrophic consequences of a dam failure make it especially important that dams can safely withstand strong earthquakes. The complexities involved in earthquake analysis of dams partly account for the deficient state of knowledge in this area. In addition, experience regarding their performance during earthquakes is extremely limited[1-4]. The situation concerning buildings is quite different. Observation and careful analysis of damage to many buildings during past earthquakes has helped considerably in understanding their behavior in earthquakes.

The structural action, and consequently the concepts involved in earthquake analysis of different types of dams, are quite different. Interest is confined to earth dams in the present investigation. In general, a dam of this type is a three-dimensional non-homogeneous continuum of complicated geometry. The foundation properties are often such that they may not permit the conventional assumption of a rigid foundation[5]. In many cases, it may be important to recognize the spatial variations in earthquake ground motion along the base of the dam. The interaction between the reservoir and dam[6] during an earthquake adds to the complexity of the situation. As a result of these factors, a complete analysis of the earthquake response of such structures presents a formidable problem. The effects of foundation elasticity and those of the reservoir are ignored in the present investigation and the analysis techniques are kept within the framework of linear elasticity.

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In many practical situations the dam is rather long compared to the cross-sectional dimensions. It is, therefore, possible to simplify the problem to analysis of the two-dimensional vibration of a cross-section of the dam. The earlier investigations[7-9] treated the structure as a vertical shear beam. Results of a two-dimensional analysis[10], by finite difference methods indicated that the shear-beam-type approach is inadequate for predicting the stresses away from the central part of the cross-section. The performance of earth dams during earthquakes will be controlled primarily by the stress conditions near the dam faces; as such, the shear beam analysis may not be satisfactory.

The investigation reported herein was motivated by the need to develop satisfactory techniques for analysis of response of dam cross-sections to earthquakes. A general two-dimensional dynamic stress analysis technique[11], based on the finite element concept, is presented. The inadequacies of the shear beam approach are demonstrated by comparing results from it with those from the finite element method[12]. The application of the finite element method to analysis of earth dams with impermeable cores[13] is demonstrated by an example. Influence of the vertical component of earthquake ground motion on the response of earth dams is studied[14]. The analysis technique is extended to account for spatial variations in earthquake ground motion along the dam base, and their effect on the dam response is investigated[15].

This paper summarizes the work done at University of California, Berkeley, during the period 1964-1968; further details may be obtained in the original publications[11-15]. The entire research program has been carried out under the financial support of State of California Department of Water Resources for which the authors express their appreciation.

FINITE ELEMENT METHOD FOR EARTHQUAKE ANALYSIS OF DAMS

Finite Element Method

The finite element method and its applications to stress analysis of elastic continua has been described in a survey paper by Clough[16]; therefore, the method will only be outlined here with particular reference to its extensions for dynamic analysis.

The basic concept of the finite element method is the idealization of an elastic continuum as an assemblage of discrete elements interconnected at their nodal points. Two-dimensional elastic systems may be represented by elements of various shapes. Triangular elements have been found to be particularly convenient and are employed herein. The displacements within each element are assumed to be linear functions of the x- and y- coordinates, in order to maintain compatibility between the edges of adjacent elements. This assumption leads to constant strains and stresses over the element. Refined elements based on higher order displacement functions have been introduced[17]. However, the simple constant strain triangular element is considered to be appropriate for this study. The element stiffness matrix may be calculated once the displacement field has been assumed; it is a function of the geometric and constitutive properties of the element. The stiffness matrix [K] of the complete structural assemblage may be obtained from the individual element stiffness matrices by direct stiffness assembly procedures.

A consistent mass matrix for the constant strain triangular element may be derived[17]; it would possess the same coupling properties as the stiffness matrix does. A physical lumped mass approximation, on the other hand, leads to a diagonal mass matrix. The lumped mass approximation results in good accuracy and a considerable saving in computer storage and time. In the present study, one-third of the mass of each element is lumped at its nodal points. The diagonal mass matrix [M] for the complete structural assemblage is then easily obtained. Displacement constraints on any of the nodal points may be introduced by eliminating the corresponding rows and columns in the matrices [M] and [K].

Analysis of Earthquake Response

In the analysis to follow, it is assumed that the earthquake ground motion is same at all points on the base of the dam cross-section. Analysis including spatial variations in the ground motion will be the subject of a later section of this paper.

The equations of motion for the dam cross-section, idealized as a finite element system, subjected to earthquake ground motion (ignoring spatial variations) may be expressed in matrix form as

$$[M]\{\ddot{r}\} + [C]\{\dot{r}\} + [K]\{r\} = -\{E^x\}\ddot{u}_g^x(t) - \{E^y\}\ddot{u}_g^y(t) \qquad (1)$$

In Eq. 1, [M] and [K] are the mass and stiffness matrices obtained by the finite element procedure described above and [C] is a viscous damping matrix for the finite element system. {r} is the vector of nodal point displacements relative to the ground, and

$$\{r\}^T = \langle r_1^x \; r_1^y \; r_2^x \; r_2^y \ldots\ldots r_n^x \; r_n^y \ldots\ldots r_N^x \; r_N^y \rangle$$

where r_n^x and r_n^y are respectively the x- and y- components of displacements of the nodal point "n" and N is the number of nodal points above the base in the finite element idealization. The vectors $\{E^x\}$ and $\{E^y\}$ of Eq. 1 are defined by

$$\{E^x\}^T = \langle M_1 \; 0 \; M_2 \; 0 \ldots\ldots M_n 0 \ldots\ldots M_N 0 \rangle$$
$$\{E^y\}^T = \langle 0 \; M_1 \; 0 \; M_2 \ldots\ldots 0 \; M_n \ldots\ldots 0 \; M_N \rangle$$

where M_n is the mass lumped at nodal point "n". In Eq. 1, $\{\dot{r}\}$ and $\{\ddot{r}\}$ are respectively the nodal point velocity and acceleration vectors; $\ddot{u}_g^x(t)$ and $\ddot{u}_g^y(t)$ are respectively the horizontal and vertical components of ground acceleration.

Classical natural modes for the system of Eq. 1 exist, provided the damping matrix [C] satisfies certain restrictions[18], and may be obtained as a solution of the characteristic value problem

$$[K]\{\phi_n\} = \omega_n^2 [M]\{\phi_n\} \qquad (2)$$

ω_n is the circular frequency and $\{\phi_n\}$ the shape of the nth natural mode of vibration. The transformation

$$\{r(t)\} = [\phi]\{Y(t)\} \tag{3}$$

where $\{Y(t)\}$ is a vector of generalized coordinates and $[\phi]$ is the matrix of natural mode shapes, uncouples the equations of motion (Eq. 1). Each uncoupled equation has the form

$$\ddot{Y}_n(t) + 2\xi_n \omega_n \dot{Y}_n(t) + \omega_n^2 Y_n(t) = P_n^*(t)/M_n^* \tag{4}$$

where ξ_n is the damping ratio for the nth mode and

$$M_n^* = \{\phi_n\}^T [M]\{\phi_n\}; \quad P_n^*(t) = -\{\phi_n\}^T\{E^x\}\ddot{u}_g^x(t) - \{\phi_n\}^T\{E^y\}\ddot{u}_g^y(t) \tag{5}$$

For prescribed ground accelerations the generalized equation for the nth mode (Eq. 4) may be solved for $Y_n(t)$ by the linear acceleration method of step-by-step integration[19]. Repeating for all the natural modes leads to the vector $\{Y(t)\}$ and $\{r(t)\}$ may be calculated from Eq. 2. The stresses $\{\sigma(t)\}_p$ in element "p" at any instant of time are related to the nodal displacements $\{r(t)\}_p$ for that element by

$$\{\sigma(t)\}_p = [S]_p\{r(t)\}_p \tag{6}$$

where the stress transformation matrix $[S]_p$ takes account of the assumed linear displacement in the element as well as its constitutive properties.

A digital computer program has been developed to perform the analysis described above. The program listing and its usage is presented in a previous publication[13]. Computer programs were also developed to automatically plot on the Cal Comp plotter (i) stress contours for the dam at any selected instant of time during the earthquake and (ii) time history of stresses at any selected nodal point in the dam. Most of the results presented in this paper were originally plotted using these programs.

COMPARISON OF FINITE ELEMENT AND SHEAR BEAM ANALYSES[12]

The shear beam analysis is restricted to homogeneous symmetrical cross-sections and can account for horizontal ground motion only. However, arbitrary geometry and material non-homogeneity can be treated rationally by the finite element method; also, effects of the vertical component of ground motion can be included. Despite the general applicability of the finite element method, it is of interest to compare the results from the two methods for a case in which the shear beam method is applicable.

300 ft. high dams of symmetrical triangular cross-section involving two different side slopes: 3 on 1 and 1.5 on 1, are considered. The elastic material is assumed to be homogeneous and isotropic with the following properties: shear wave velocity, v_s = 1000 fps; unit weight, γ = 130 pcf; and Poisson's Ratio, ν = 0.45. The finite element idealization used for the dam cross-section is shown in Fig. 1A, which, allowing for the constraints at the base, provides 110 degrees of freedom.

Frequencies and Mode Shapes

The first 10 natural circular frequencies and mode shapes for the cross-section with side slopes 1-1/2 on 1, as determined from the finite element computer program referred above, are presented in Fig. 1B. The two-dimensional nature of deformations in each mode is apparent. There are two basic types of natural modes for a symmetrical dam cross-section: antisymmetrical and symmetrical modes. Of course, this classification does not apply to cross-sections with arbitrary geometry. It is apparent from Fig. 1B that only the first antisymmetrical mode resembles a pure shear distortion. There are significant vertical displacements involved in all other antisymmetrical modes and the symmetrical modes have no resemblance to the shear type modes. Frequencies and mode shapes of the cross-section with 3 on 1 side slopes are significantly different from those when the side slopes are 1-1/2 on 1; however, the mode shapes exhibit similar characteristics[12].

The shear beam analysis leads to natural frequencies and mode shapes which do not depend on the side slopes of the dam; also, the mode shapes are a function only of the vertical coordinate, i.e., the displacements at the face are the same as those at the centre line. Results for the first mode from the two methods are presented in Fig.1C. It is apparent that the displacements on the face of the dam differ significantly from those at the centre line and the shear beam solution is in considerable error. The important conclusion from this comparison is that even the first mode depends significantly on the geometry of the cross-section and the displacements have important variations across the width. Higher antisymmetrical modes exhibit these features more strongly. The fundamental frequencies given by the two methods are also presented in Fig. 1C. The finite element method results in lower frequencies; this might indicate that it is the better of the two methods. However, no definite conclusions can be drawn from this fact because lumped mass approximations yield frequencies which can be higher or lower than the exact solution. The results presented herein and previously[12] demonstrate the deficiencies in the shear beam approach and the effectiveness of the finite element method.

Earthquake Responses

The response of the two dams subjected to the North-South component of the El Centro earthquake of May 18, 1940 is determined by the finite element method, using the computer program referred to above. For these response computations, the total energy absorption capability of the structure is represented by an equivalent viscous damping of 20% of critical in each mode and the contributions of the first 15 natural modes are included. The initial (dead weight) stresses are excluded, so that the computed stresses are the dynamic stresses. The state of stress in the dam with side slopes 1-1/2 on 1 at a selected "critical" instant of time, t = 2.25 secs. is presented in Fig. 2A. Stresses in different elements reach their extreme values at different times; therefore the chosen instant represents extreme conditions only in a major part of the dam. The contour plots for shear stress indicate that there are significant deviations from constant shear stress across the width, the basic assumption of the shear beam analysis; the deviations being particularly large in the lower part of the cross-sections and for the dam with flatter side slopes[12]. It is apparent from the stress distributions that the material at the centre line is in a state of pure shear. However, the state of stress near the faces is far from pure shear and the corresponding deformations would be considerably different than the pure shear deformations assumed in the shear beam approach.

Typical results for time-history of nodal point stresses are presented in Fig. 2B. The two nodal points selected are at the 120 ft. level; one at the face and the other at the centre line. It is apparent that the time-history of stresses at the centre line nodal points are similar for both cross-sections; the stresses at centre line nodal points and face nodal points at corresponding levels are significantly different; and there are appreciable differences in the stress responses in the two cross-sections at similarly situated face nodal points.

According to the shear beam approach, the stresses at nodal points at the same height in both cross-sections are the same, thus implying that the earthquake response of the two cross-sections are identical. The stresses at points away from the centre line of the dam cross-section are not predicted satisfactorily by the shear beam approach. This discrepancy may be of significance in assessing stability of dam slopes during earthquakes.

ANALYSIS OF DAMS WITH CORES[13]

The finite element method has been employed above to determine the earthquake response of homogeneous dam cross-sections with simple geometry; and the superiority of the technique over other methods has been demonstrated. Dam cross-sections of arbitrary geometry can always be idealized as an assemblage of finite elements and therefore a rational solution may be obtained by the finite element method. The finite element method can also account for arbitrary variations of material properties over the cross-section without any additional effort because properties of elements are individually defined. Results of analysis of a dam with a central impervious core and shells of cohesionless material subjected to Taft earthquake are presented to demonstrate the above-mentioned capabilities of the finite element method.

The dam selected is 300 feet high with 2-1/2 on 1 upstream slope, 2 on 1 downstream slope; the crest width and core dimensions are shown in Fig. 3A. The finite element idealization for the structure is also shown having 340 elements and 207 nodal points with a total of 340 degrees of freedom. The properties selected for the homogeneous and isotropic core material are shear wave velocity, v_s = 700 fps; unit weight, γ = 125 pcf; and Poisson's Ratio $\nu = 0 \cdot 40$. Shear modulus for cohesionless materials is known to be approximately proportional to the cube root of confining pressure and the selected properties for the isotropic shell material are shear modulus, $G = 4 \times 10^5 \sigma^{1/3}$; unit weight, γ = 125 pcf; and Poisson's Ratio $\nu = 0.25$. The initial stresses in the dam are determined by a standard static finite element analysis using a computer program developed by Wilson[20], in which the gravity loads are applied instantaneously to the complete structure. It should be noted that this direct analysis does not account for the gradual accumulation of loads during the construction of the structure, which also could be included in the finite element procedure[21]. The response of this dam subjected simultaneously to N69W and vertical components of Taft earthquake of 1952 is determined. For these response computations, damping is assumed to be viscous in nature and 20% of critical in each natural mode and the contributions of the first 15 modes are included. The computer results include the complete time-history of stresses at all nodal points. Stress contours at a 'critical' instant time, t = 6.65 secs., are presented in Fig. 3B. These contour plots represent the complete state of stress including the initial stresses and effects of the horizontal and vertical components of earthquake ground motion. It is apparent from this analysis that the finite element method for earthquake analysis of earth dams provides a powerful tool for solution of practical problems.

IMPORTANCE OF VERTICAL COMPONENT OF EARTHQUAKE MOTIONS[14]

Conventionally, earthquake analysis of structures has involved consideration of only one horizontal component of ground motion. The neglect of the vertical component of earthquakes in analysis of building structures, although somewhat questionable, has usually been justified on the basis that such structures have considerable inherent strength in the vertical direction. The significance of the response of dams to vertical component of ground motion has not been studied previously. The development of the finite element method presented above makes it possible to investigate this aspect of the problem.

Ground Motion Characteristics

The response spectrum technique[22,23] has now become a standard approach to characterize strong motion earthquake records for purposes of estimating structural response. The spectrum intensity[24] which is defined as the area under the pseudo-relative velocity spectrum[25] between 0.1 and 2.5 seconds period is the only reliable method of estimating the intensity of ground shaking. The 20% damped spectrum intensity seems to be a good measure of the amount of damage to be expected[26]. A comparison of the pseudo-relative velocity spectra for vertical component of ground motions recorded as El Centro (1940), Olympia (1949) and Taft (1952) with the spectra for corresponding horizontal components reveals that spectra for vertical component are relatively accentuated in the short-period range and relatively reduced in the long-period range. The 20% damped spectrum intensity for vertical component of ground motions is about 20-30% of that for the corresponding horizontal components. The Taft (1952) earthquake record is selected to study the responses of an earth dam to horizontal and vertical ground accelerations and their relative importance.

Response of Earth Dams

An idealized earth dam cross-section is considered. The height is chosen as 300 feet and the cross-section is taken as a symmetrical triangle with side slopes of 2-1/2 on 1. The elastic material is assumed to be homogeneous and isotropic with the following properties: shear wave velocity, v_s = 1000 fps; unit weight, γ = 130 pcf; and Poisson's Ratio, ν = 0.45. The finite element idealization used for the cross-section is the same as the one in Fig. 1A. Viscous damping is taken to be 20% of critical in each mode. The response of this structure subjected separately to two components (N69W and vertical) of Taft earthquake record is determined by the computer program for finite element analysis. The initial stresses are excluded and the dynamic stresses for the two cases are plotted at 'critical' instants of time: t = 6.8 secs. for N69W component (Fig. 4A) and t = 9.7 secs. for vertical component (Fig. 4B).

It is apparent from these results that shear stresses in the dam due to vertical component of ground motion are a very small fraction of those caused by the horizontal component of ground motion. The largest magnitudes of principal stresses caused by vertical ground motion occur at the base on the centre line; they occur roughly at mid-height on the dam faces in case of horizontal ground motion. The magnitude of the principal stresses due to the two components are about equal. It should be noted that the maximum effects of the two components of an earthquake, in general, occur at different times.

The results discussed above are for a ground motion whose vertical component is 31% as intense as the horizontal components (based on spectrum intensity for 20% of critical damping). It is apparent from these results that the vertical component of ground motion alters the dynamic stress distribution significantly. This would be particularly so for earthquakes with unusually large vertical component.

RESPONSE TO TRAVELLING GROUND MOTION[15]

It is customary to assume in analyzing structure response that all points on the base of a structure are subjected to the same time-history of ground motion during an earthquake; i.e., the time taken for the seismic waves to travel across the base of the structure is neglected. This assumption may be reasonable if the base length is smaller than one-fourth of the wave length of those harmonics in the ground motion which are important to the response of the structure, however, it appears that such an assumption may not be valid for dams because of their large base width. It is therefore desirable to study the effect of variation in ground motion along the dam base on the earthquake response of dams. It is known that characteristics of ground acceleration recorded at different epicentral distances during the same earthquake are considerably different[27]. However, due to lack of suitable ground motion records, very little is known about differences between ground motions at two adjacent points, say a thousand feet apart, during the same earthquake. Ground motion in the epicentral region of a moderate to large earthquake is a complicated combination of various types of seismic waves with frequencies spread over a wide range; this results in dispersion of waves and it is therefore difficult to predict how the ground motion may be modified after travelling some distance. In addition, the direction of approach of the seismic waves relative to the dam axis becomes important in considering spatial variation in ground motion and the dam response problem would have to be treated as three-dimensional.

The purpose of this presentation is to indicate the importance of considering spatial variations in the ground motion in evaluating the earthquake response of earth dams. A cross-section of the dam is considered and the ground motion is idealized as a prescribed time-history of ground acceleration travelling at a particular velocity across the base. It should be noted that this idealization implies that the ground motion remains unmodified as it propagates. Analysis of this grossly simplified problem may not lead to any quantitative conclusions; however, it is appropriate for conclusions on a qualitative basis. The analysis procedure that follows is an extension of one presented by Rosenblueth[28].

Equations of Motion

The equations of motion for the dam cross-section, idealized as a finite element system, subjected to travelling ground motion may be expressed in matrix form as

$$[M^c]\{\ddot{r}_t^c\}+[C^c]\{\dot{r}_t^c\}+[K^c]\{r_t^c\}=\{0\} \tag{7}$$

In Eq. 7, $[M^c]$, $[C^c]$, and $[K^c]$ are mass, damping, and stiffness matrices of order $2(N+N_b)$ where N is the number of nodal points above the base and N_b the number of base nodal points. $\{r_t^c\}$ is the vector of (total) nodal point displacements. It is convenient to separate this vector into a quasi-static displacement vector $\{r_s^c\}$ and a dynamic displacement vector $\{r^c\}$, i.e.,

$$\{r_t^c\}=\{r_s^c\}+\{r^c\} \tag{8}$$

The quasi-static displacement vector, at any time t, represents the static displacements of all nodal points due to displacements applied at the base nodal points at time t. In partitioned form Eq. 8 is

$$\begin{Bmatrix} r_t \\ \hline r_t^b \end{Bmatrix} = \begin{Bmatrix} r_s \\ \hline r_s^b \end{Bmatrix} + \begin{Bmatrix} r \\ \hline r^b \end{Bmatrix} \tag{9}$$

where the superscript b refers to nodal points on the base. By definition

$$\{r_t^b\}=\{r_s^b\};\ \{r^b\}=\{0\} \text{ and } [K^c]\begin{Bmatrix} r_s \\ \hline r_s^b \end{Bmatrix} = \begin{Bmatrix} 0 \\ \hline R_s^b \end{Bmatrix} \tag{10}$$

Writing Eq. 7 in partitioned form

$$\begin{bmatrix} M & 0 \\ 0 & M^b \end{bmatrix}\begin{Bmatrix} \ddot{r}_t \\ \ddot{r}_t^b \end{Bmatrix}+\begin{bmatrix} C & C^b \\ (C^b)^T & C^{bb} \end{bmatrix}\begin{Bmatrix} \dot{r}_t \\ \dot{r}_t^b \end{Bmatrix}+\begin{bmatrix} K & K^b \\ (K^b)^T & K^{bb} \end{bmatrix}\begin{Bmatrix} r_t \\ r_t^b \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{11}$$

The first matrix equation, with the aid of Eqs. 9 and 10, may be expressed as

$$[M]\{\ddot{r}\}+[C]\{\dot{r}\}+[K]\{r\}=-[M]\{\ddot{r}_s\}-[C]\{\dot{r}_s\}-[C^b]\{\dot{r}_s^b\} \tag{12}$$

Eq. 12 represents 2N equations of motion for the nodal points above the base of the dam.

Analysis of Earthquake Response

The vectors $\{r_s^b\}$, $\{r_s\}$ and $\{r\}$ need to be determined to evaluate the displacement response of the dam. Considering only the horizontal component of the earthquake, the idealized ground motion may be expressed as

$$\ddot{r}^b_{s,2k-1} = \ddot{u}^x_g(t-\frac{d_k}{V});\ \ddot{r}^b_{s,2k}=0;\ k=1,2,\ldots\ldots N_b \qquad (13)$$

where time is measured from the instant the earthquake arrives at one end (first nodal point) of the base, d_k is the distance between k^{th} and the first base nodal points and v is the propagation velocity. The displacements are obtained by integrating Eq. 13 twice.

The vector $\{r_s\}$ is related to $\{r^b_s\}$ by Eq. 10 as

$$[K]\{r_s\} = -[K^b]\{r^b_s\}$$

which may be expressed as

$$[K]\{r_s\} = -\sum_{k=1}^{2N_b}\{g_k\}\, r^b_{sk} \qquad (14)$$

where $\{g_k\}$ is the k^{th} column of $[K^b]$ and represents the nodal forces resulting from applied displacement $r^b_{sk} = 1$. The vector $\{r_s\}$ may be expressed as

$$\{r_s\} = \sum_{k=1}^{N_b}\{b_{2k-1}\}\, r^b_{s,2k-1} \qquad (15)$$

where $\{b_k\}$ represents the displacements of nodal points above the base due to $r^b_{sk} = -1$; and

$$[K]\{b_k\} = -\{g_k\} \qquad (16)$$

It should be noted that the terms associated with vertical ground motion have been omitted in Eq. 15. The influence coefficients $\{b_k\}$ may be obtained by solving the set of linear equations of Eq. 16. Once $\{b_k\}$ is determined for all values of k, the displacements $\{r_s\}$ due to base displacements $\{r^b_s\}$ may be determined from Eq. 15.

The dynamic displacement vector $\{r\}$ is governed by Eq. 12, in which the excitation consists of dynamic inertia forces and forces associated with the damping in the system. These damping terms are considerably smaller than the inertia terms and may be dropped. Eq. 12, with the aid of Eq. 15, then becomes

$$[M]\{\ddot{r}\}+[C]\{\dot{r}\}+[K]\{r\} = -\sum_{k=1}^{N_b}[M]\{b_{2k-1}\}\,\ddot{r}^b_{s,2k-1} \qquad (17)$$

It should be noted that Eq. 17 is identical to Eq. 1 except for the dynamic excitation terms. In fact, Eq. 1 may be obtained as a special case of Eq. 17 when the base has only one degree of freedom. The transformation of Eq. 3 also uncouples Eq. 17 resulting in Eq. 4 in which (from Eqs. 17 and 13)

$$P^*_n(t) = -\sum_{k=1}^{N_b}\{\phi_n\}^T[M]\{b_{2k-1}\}\,\ddot{u}^x_g(t-\frac{dk}{V}) \qquad (18)$$

For prescribed ground acceleration $\ddot{u}^x_g(t)$ and velocity of propagation v, Eq. 4 may be solved for $Y_n(t)$ by the linear acceleration method. It should be noted that Eq. 4 needs to be solved only for k = 1 and the response associated with other values of k may be obtained by a shift (d_k/v) in the time scale. The nodal displacements may then be determined from Eq. 3.

Having determined $\{r^b\}$, $\{r_s\}$, and $\{r\}$ the total displacements are given by Eq. 9, from which the element stresses may be obtained by applying the stress transformation matrix of Eq. 6.

A digital computer program has been developed to perform the analysis described above. The program listing and its usage is presented in a previous publication[15].

Earthquake Responses

The selected earth dam cross-section is identical in geometry and material properties to the one employed in investigating the importance of vertical component of earthquake motion. The response of this structure subjected to N69W component of Taft earthquake record propagating at velocity v is determined by the computer program referred to above[15]. A lower limit for v may be taken equal to the shear wave velocity in the dam, which is 1000 fps in this case. The stresses (initial gravity stresses excluded) in the dam for v = 1000 fps are plotted at a 'critical' instant of time, t = 7.70 secs (zero time is assumed at that instant when the travelling ground motion reaches the left edge of the base) in Fig. 5A. The stresses for v = 5000 fps are plotted at t = 6.95 secs in Fig. 5B. The results for v = 10000 fps were found to be identical to the results of Fig. 4A, i.e., when spatial variations in ground motion were ignored.

It is apparent from these results that the response of earth dams strongly depends on the propagation velocity v. The spatial variations in the ground motion may be neglected if v is much greater than the shear wave velocity v_s for the dam. However, the spatial variations affect the response considerably when v is comparable to v_s. In particular, there is no resemblance between the stress-distribution when v = 1000 fps and that when spatial variations are neglected, i.e., $v = \infty$. Also, it is apparent that the results for v = 5000 fps (Fig. 5B) lie between the two extreme cases and somewhat resemble the results of Fig. 4A. It is important to recognize that the stresses associated with the quasi-static displacements are an important part of the total stress distribution, and may be the dominant contribution where the propagation velocity v is low. In this case large errors will result if spatial variations of the ground motion are ignored.

CONCLUDING REMARKS

The investigation reported above demonstrates that the finite element method provides a powerful tool for the earthquake analysis of dams. The finite element method, by its very nature, is ideally suited for analyzing dam cross-sections of arbitrary geometry with arbitrary variations of material properties. Effects of vertical component of ground motion can be included in the analysis without any additional effort and the method is readily extended to treat spatial variations in earthquake ground motion.

At the present stage of development, the finite element method is restricted to linearly elastic materials. However, the strains developed in a structure during a strong earthquake will be large enough to cause the material behavior to be inelastic. In an analysis including the effects of inelastic deformations, it would be necessary to follow through the entire yielding process, starting from its initiation locally. The finite element method applied in an incremental form appears to offer a promising approach to the problem. In the step-by-step or incremental procedure, the total nonlinear problem is reduced to a number of successive linear problems. Each increment is selected to be sufficiently small so that the resulting incremental stresses and deformations may be related by a linear constitutive law, which is constant during the increment but changes from one step to the next. Work is underway to select a suitable yield criterion for soils and develop a step-by-step procedure for earthquake analysis of dams, including effects of inelastic deformations.

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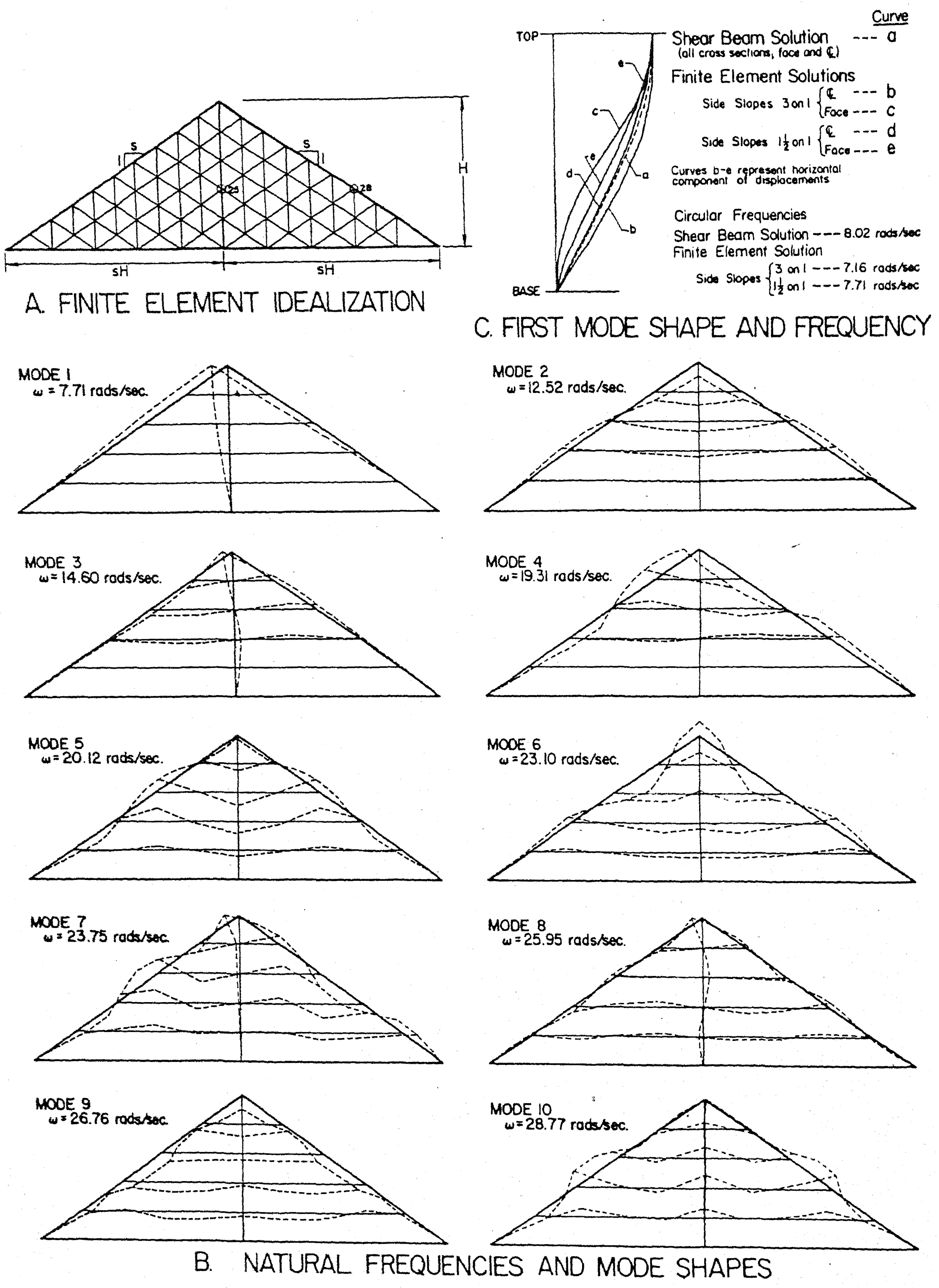
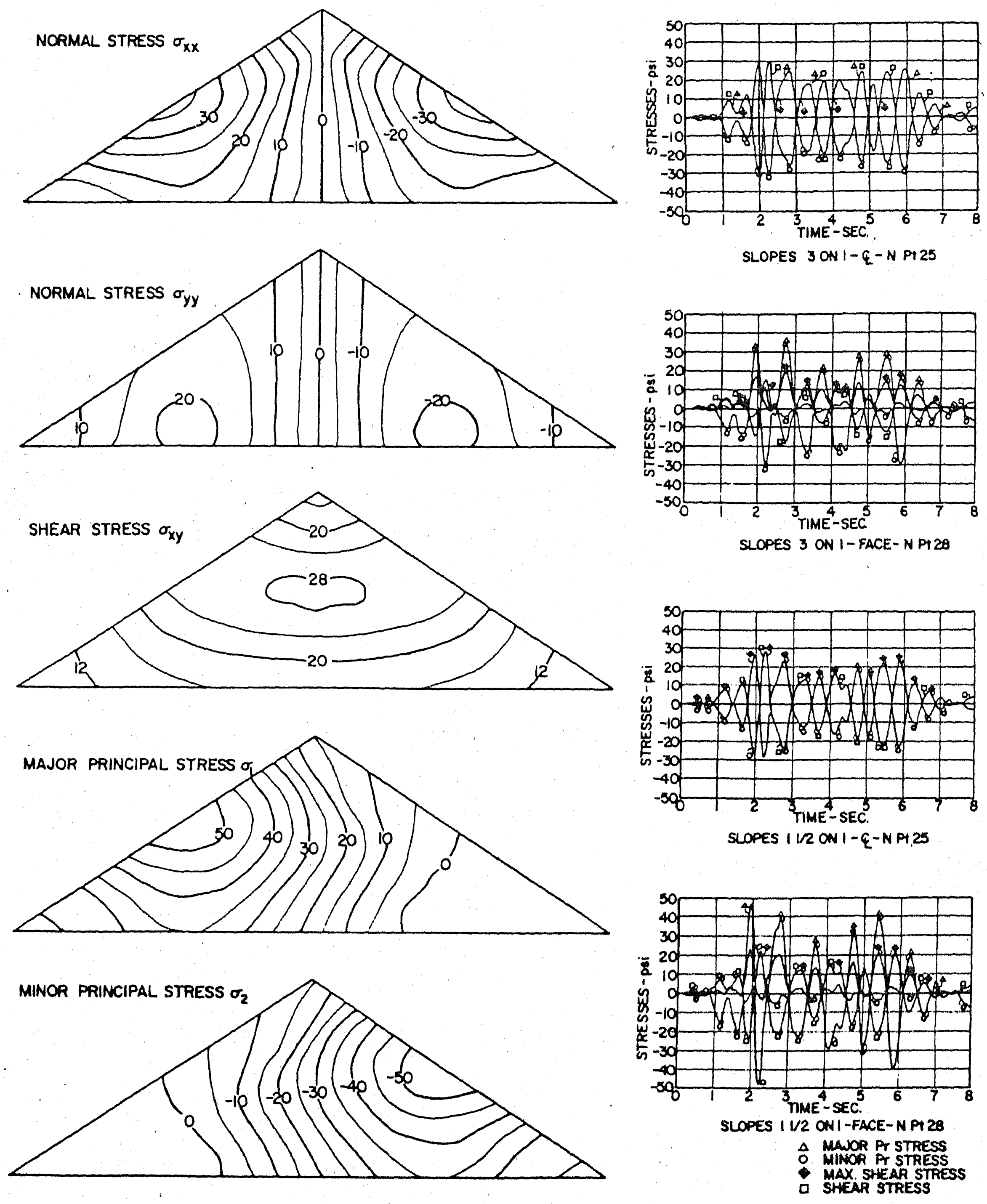


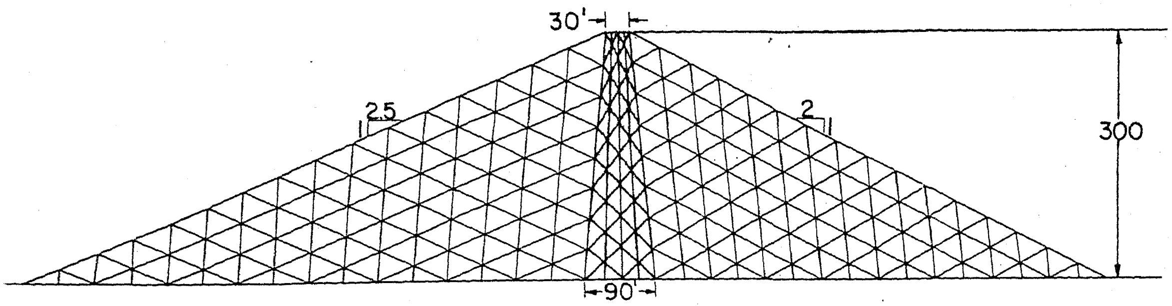
FIG. I COMPARISON OF FINITE ELEMENT AND SHEAR BEAM ANALYSIS



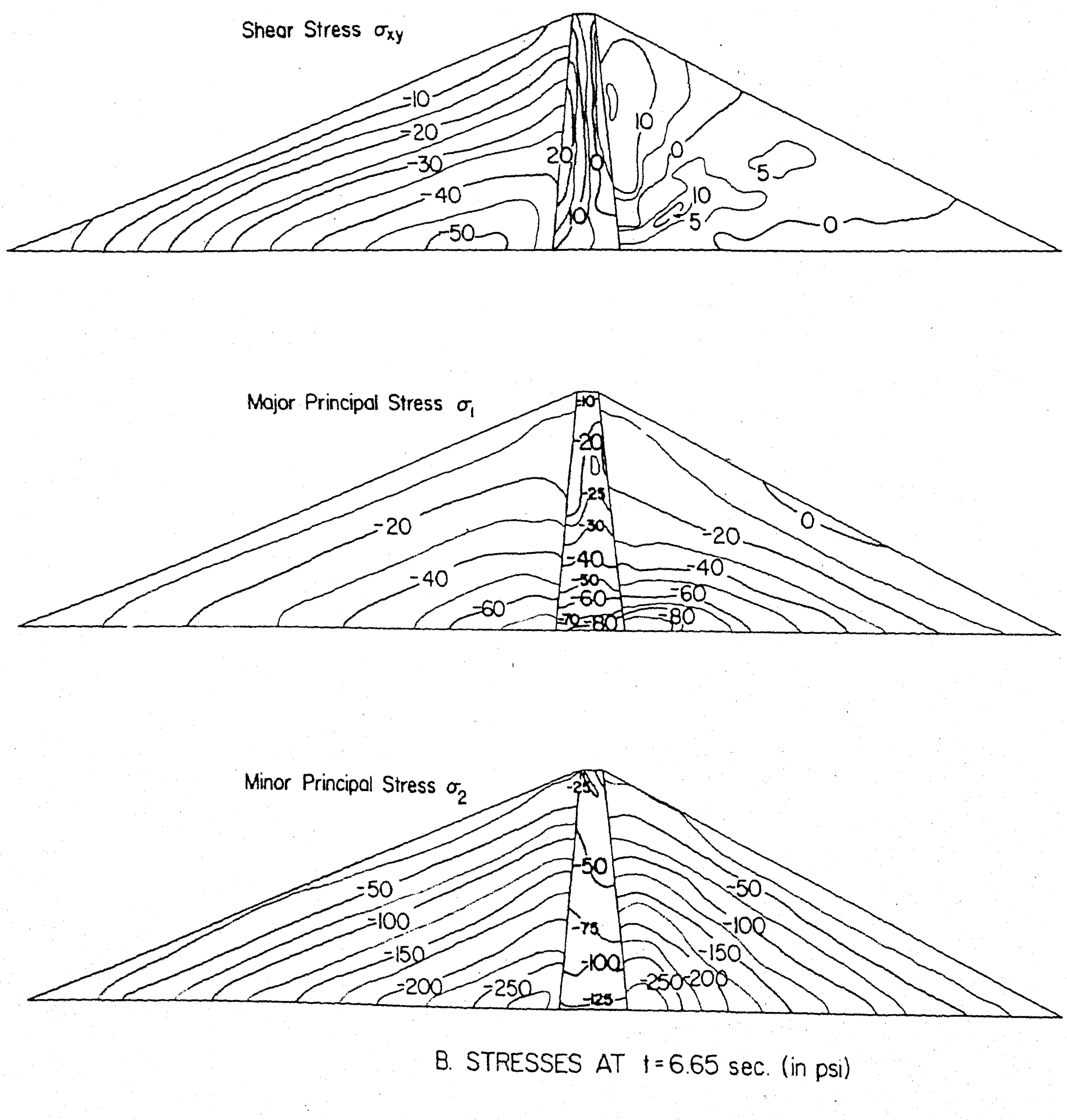
A. STRESSES AT t = 2.25 secs. (in psi)

B. STRESSES AT 120 ft. LEVEL

FIG. 2 RESPONSES TO EL CENTRO (1940) EARTHQUAKE

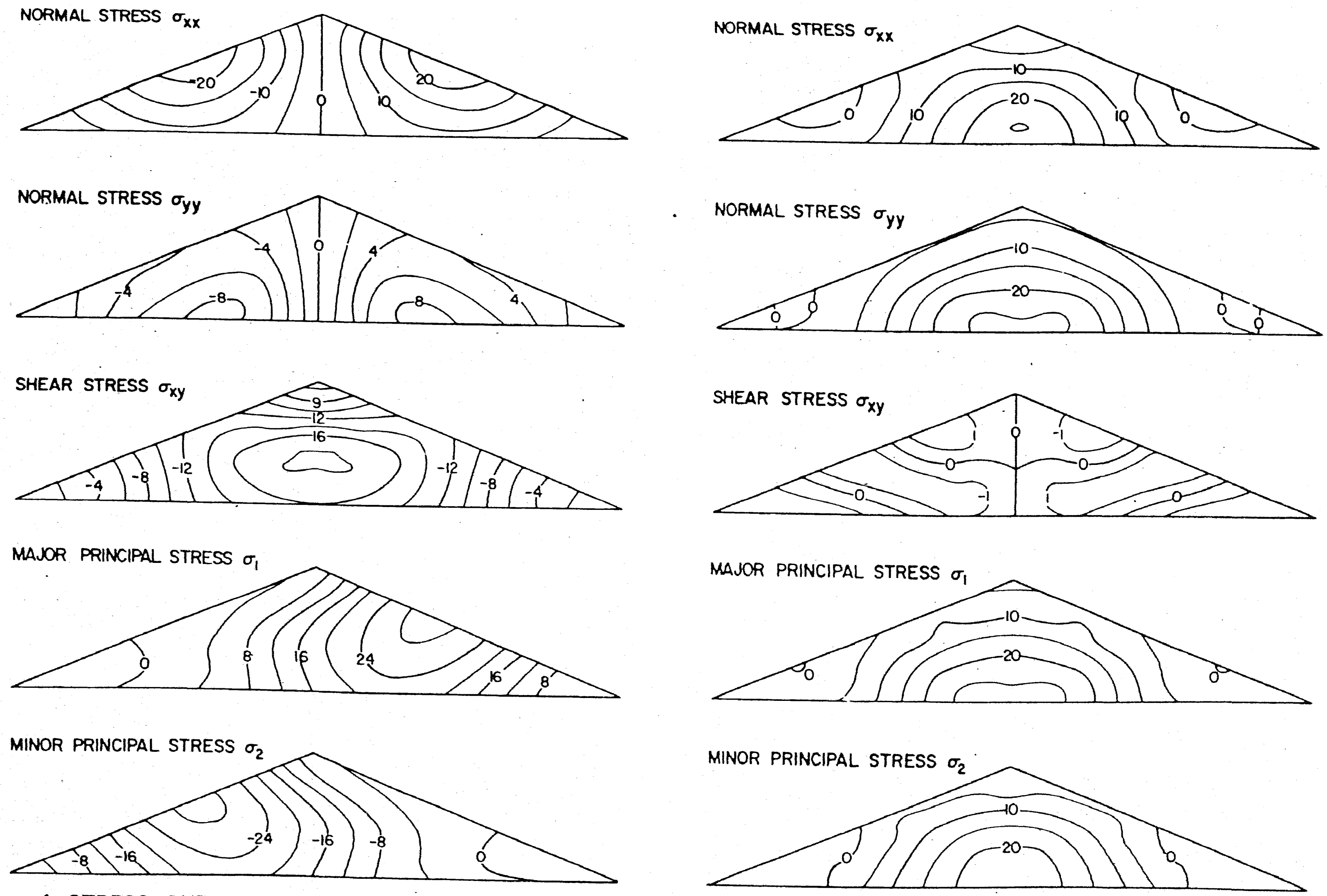


A. FINITE ELEMENT IDEALIZATION



B. STRESSES AT t = 6.65 sec. (in psi)

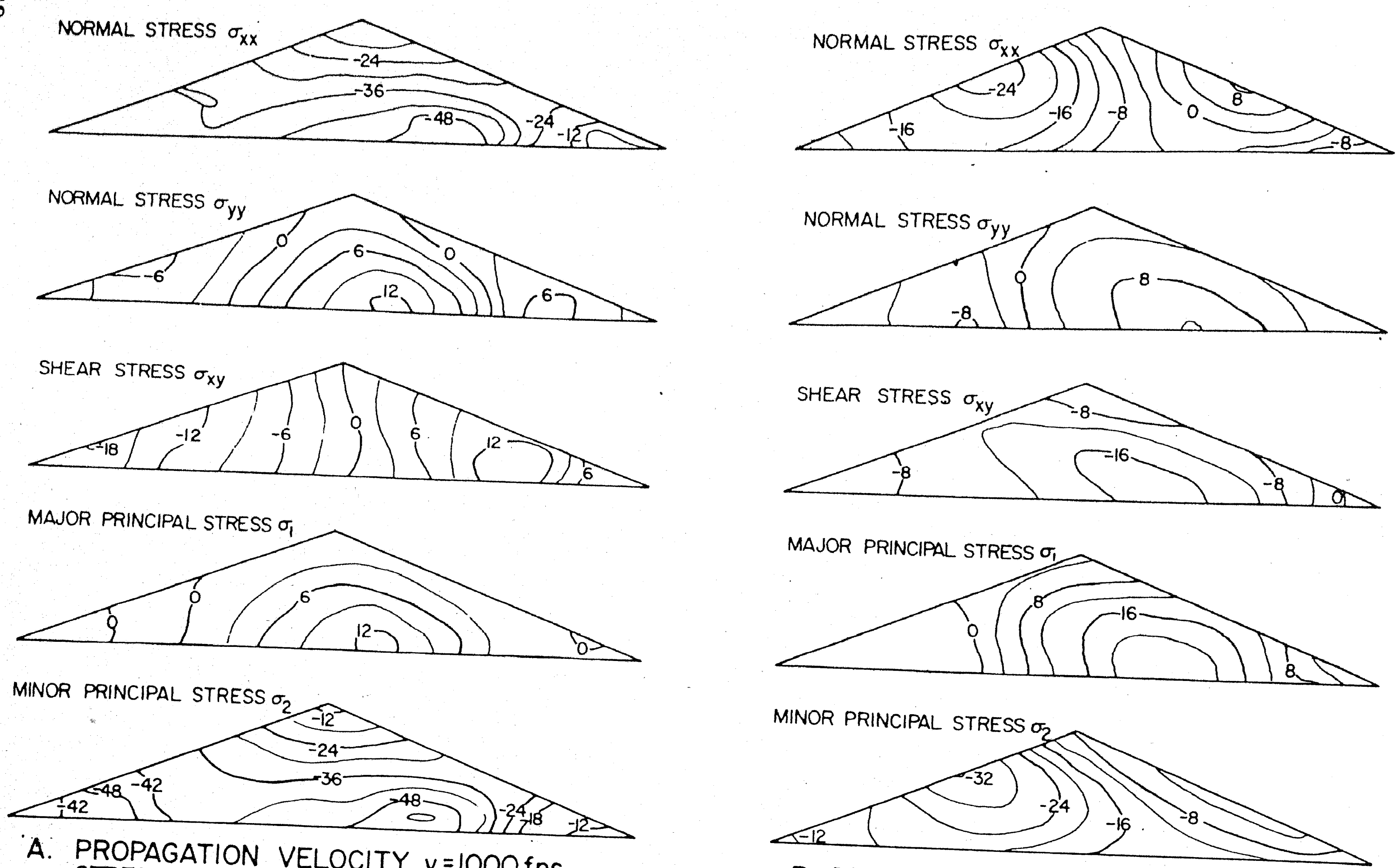
FIG. 3 RESPONSE OF DAM WITH CORE TO TAFT (1952) EARTHQUAKE



A. STRESS DUE TO N 69 W COMPONENT AT t = 6.8 sec.

B. STRESSES DUE TO VERT. COMPONENT AT t = 9.7 sec.

FIG. 4 RESPONSE TO TAFT (1952) EARTHQUAKE



A. PROPAGATION VELOCITY, v = 1000 fps
STRESSES (in psi) AT t = 7.70 secs.

B. PROPAGATION VELOCITY, v = 5000 fps
STRESSES (in psi) AT t = 6.95 secs.

FIG. 5 RESPONSE TO TRAVELLING TAFT (1952) GROUND MOTION