DENSIFICATION OF SAND BY VERTICAL VIBRATIONS

Robert V. Whitman (I)
Pedro Ortigosa de Pablo (II)

SYNOPSIS

During vibrations, there is rapid motion (usually expressed as acceleration) and there also are dynamic stresses. Two special sets of compression tests have been carried out to study these two variables independently: tests involving large accelerations but only very small dynamic stresses, and tests with large dynamic stresses **but** only very small accelerations. It was found that many cycles of dynamic stress always cause densification, but that acceleration alone causes *no* densification until the vertical acceleration exceeds 1g. These results are used to analyze three common problems: vibratory compaction in the laboratory and in situ, settlement of machine foundations and subsidence during earthquakes.

1. INTRODUCTION

It is commonly accepted that vibrations cause densification of sand. Vibration of a machine foundation resting on sand may cause settlement of the foundation.' The vibratory-type motion associated by earthquakes may cause subsidence. Driving of piles into a sand formation may cause settlement of nearby buildings founded upon the sand. Vibratory densification may also be beneficial, as in the use of vibratory compactors to densify a sandy soil.

However, there is very little information available to guide an engineer in deciding whether densification will occur in a given situation, and if so how much. There is also a lack of a fundamental understanding of the process of densification by vibrations. Barkan (1), one of the earliest investigators of this subject, suggested that the void ratio of an initially loose sand is, following a steady vibration, uniquely related to the peak acceleration involved in the vibration. However, this simple concept has not been substantiated by subsequent investigators (2,3,4), and as a result there is confusion as to the factors which control the amount of densification which may occur.

This paper is a first step toward identification of the fundamentals of the vibratory densification process. The paper is mainly concerned with purely vertical vibrations, although some comments will be made regarding the effect of horizontal vibrations.

I. Professor of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts, U.S.A.

II. Civil Engineer, IDIEM, University of Chile, Santiago, Chile.

2. RELATION BETWEEN STRESSES AND ACCELERATIONS

Figure 1a shows the forces acting upon an element of soil which is moving vertically. This element is cut from just below the free surface of the ground, and has unit cross-sectional area. If the acceleration a is zero, the vertical stress σ_v at any depth z equals the geostatic value: $(\sigma_v)_s = \sigma_o + \gamma z$, where σ_o is the static stress applied at ground surface and γ is the unit weight. When the element accelerates upward, the inertial force acts to make $\sigma_v > (\sigma_v)_s$, while the reverse is true when the element accelerates downward.

Now consider the special case where $\sigma_o = 0$ and where soil of depth z rests upon a rigid base which moves sinusoidally with time. The soil will be assumed to move as a rigid body.

If the peak acceleration of the base, a_p, is less than the acceleration of gravity g, the soil will move with the base. The vertical stress between the soil and the base varies sinusoidally with time as shown in Fig. 1b. Thus, if $a_p = 0.5g$, σ_v fluctuates between $1.5(\sigma_v)_s$ and $0.5(\sigma_v)_s$.

However, if $a_p > 1g$, the variation of stress with time is much more complicated, as indicated in Fig. 1c. At the point in each cycle stress where the *downward* acceleration of the base reaches 1g, the vertical stress within the soil drops to zero. Since the sand cannot sustain tension, the sand is unable to follow the subsequent motion of the base and experiences *free fall* until it impacts against the base later in the cycle. The sand and base then move upward together until separation once more occurs and the cycle is repeated. Thus the variation of stress with time consists of sharp spikes at the moments of impact, followed by gradually decreasing stress until separation occurs, and then zero stress during free fall.

While the foregoing analysis has been based upon a highly idealized model, it has served to illustrate the interdependence of acceleration and dynamic stress change. A similar analysis may be made of the case where ground experiences horizontal accelerations. Such horizontal accelerations are accompanied by shear stresses on horizontal planes. The foregoing analysis also illustrates the complexity of the motions and stresses if the vertical acceleration exceeds 1g.

Since accelerations are accompanied by dynamic stress changes, the question arises: is it the accelerations or is it the dynamic stresses which cause densification? The next two sections present results of special tests aimed at answering this question.

3. TESTS INVOLVING DYNAMIC STRESSES BUT VERY SMALL ACCELERATIONS

Tests involving repeated applications of vertical stress were carried out in the cell shown in Fig. 2. Initial stress and repeated stress increments were applied by air pressure against the flexible membrane. Vertical strains were measured by sensing the movement of the disk. This arrangement permits very accurate measurement of strains while minimizing the difficulties associated with side friction (5).

The results show that the cumulative strain from many cycles of repeated loading increases as the logarithm of the number of cycles. Figure 3 shows results for dry, uniform, fine Ottawa sand (6): the void ratio following 10,000 cycles of loading is plotted against initial void ratio. Repeated loading does by itself cause densification, especially when the initial relative density is less than 70%, but this densification is insignificant until $\sigma_{max}/\sigma_{min} \geq 5$, where σ_{min} is the minimum stress and σ_{max} the maximum stress during the cyclic loading.

The rate of stress repetition in these tests was 8 cycles per minute. The maximum acceleration in these tests was estimated to be at most 0.04g. Other results indicate that increasing the rate of cycling the stress causes a slight decrease in the cumulative strain from a given number of cycles.

In many practical problems involving vibration of the ground, there is shearing action as well as compression. Repeated shearing action may cause densification and cumulative shear strain. Repeated load triaxial tests have provided a crude basis for estimating cumulative shear strains (6).

4. TESTS WITH ACCELERATIONS BUT VERY SMALL DYNAMIC STRESSES

From the analysis in section 2, for a sample of soil 1 inch thick and with a unit weight of 100 pcf there must be a difference in stress of 0.058 psi between the top and bottom of the sample in order to produce an acceleration equal to 1g. Thus, if a thin sample is placed upon a shaking table and subjected to accelerations up to several g, only very small dynamic stresses will occur.

Tests meeting this condition were carried out by placing the cell of Fig. 2 upon a shaking table (7). It had been thought that this table prouduced sinusoidally varying vertical motion whose amplitude and frequency could be controlled so as to achieve any desired acceleration. Unfortunately, undesirable high frequency components of motion were also present, and the accelerations associated with these high frequencies often were several times the acceleration expected on the basis of the selected amplitude and frequency. By placing several layers of soft material between the table and the cell containing the soil, and by adopting other precautions, it was possible to prevent most of these unwanted high frequency accelerations from reaching the soil. The actual acceleration of the cell was measured using an accelerometer.

In these tests, the sand was first placed under a static stress σ_o by air pressure applied against the membrane. Then the peak acceleration of the table was increased in steps, permitting about 20 minutes of vibration at each step. Results of these tests are shown in Fig. 4, for 30-40 mesh dry Ottawa sand. The significant features of the results are:

(1) For a_p<1g there was little or no measurable densification.

(2) For the minimum static stress σ_o, densification started at an a_p slightly greater than 1g. Slight additional increases in a_p caused considerable densification. Finally, a terminal void ratio was reached such that further increases in a_p caused no additional densification.

(3) As σ_o increased, the critical value of a_p required to initiate densification became greater and the terminal void ratio increased.

It is not clear what mechanisms acted to cause densification when the critical values of a_p were reached. At these values, the dynamic stresses as computed in section 2 were still small compared to σ_o. Of course, the elementary theory of section 2 does not account for possible localized dynamic action within small groupings of particles. The critical value of a_p for a given σ_o appears to be rather sensitive to test conditions. Placing sand under an all-around confining stress in a triaxial cell led to critical values of a_p greater than those in Fig. 4. The critical values of a_p were greater for saturated than dry sand.

While there thus is uncertainty as to the meaning of certain aspects of the test results, two conclusions are clearly valid:

(1) When the dynamic stresses are small, no noticeable densification occurs for a_p<1g.

(2) When the dynamic stresses are small compared to σ_o, it is possible to have a_p<1g and still have no noticeable densification.

Thus, acceleration by itself, in the absence of significant dynamic stresses, cannot be regarded as a primary cause of densification.

5. BEHAVIOR DURING "STANDARD TEST"

Tests in which an open cylinder of soil is placed upon a vibrating table are often used to determine the susceptibility of a soil to vibratory compaction or to measure the maximum unit weight of a granular soil. However, even though the American Society for Testing Materials has issued a specification covering such tests (8), there has been little standardization regarding test procedures. Some shaking tables vibrate only vertically and others only horizontally, while still others give a very complex pattern of motion. Some investigators place a dead weight on top of the soil, while other investigators omit this dead weight.

For purposes of research at M.I.T., the term "standard" is applied to a test in which an open-top cylinder with a volume of about 0.1 ft.^{3} is placed upon a shaking table producing purely vertical motion (9). No surcharge, either by dead weight or air pressure is used. The soil is vibrated for 10 minutes at the desired level of acceleration, and then the shaking table is turned off so that the shaking ceases after only a few additional cycles of decaying motion. The unit weight following vibration is plotted against the nominal value of a_p calculated from the selected frequency and peak displacement. As noted before, higher frequency components of motion were present in the shaking table, and in this case attempts to prevent these higher frequencies from reaching the container and sand were only partially successful. Thus the actual peak accelerations somewhat exceeded the nominal peak accelerations.

Figure 5 shows results from tests very similar to this "standard" test (4). Depending upon the level of acceleration, three different patterns of behavior are present during such tests.

First, for $a_p<1g$, there is relatively little densification. For this condition, stress is varying as shown in Fig. 1b. The mechanism of densification is essentially the same as during tests with repeated stresses but very small accelerations. That is, the densification which does occur results from the presence of accleeration-induced dynamic stresses rather than from the presence of accelerations alone. The increase in amount of densification as the nominal peak acceleration approaches 1g may be attributed in part to the presence of high frequency motions such that the actual peak accelerations exceeds 1g.

Second, for $1g<a_p<2g$, there is considerable densification. For this condition, the stress within the soil is varying as shown in Fig. 1c. The repeated impacts of sand against the container could be heard by ear and were further revealed by accelerometers placed upon the container and embedded within the sand. The very effective densification achieved for this range of accelerations apparently resulted from (a) the intervals of free fall during which the particles were more-or-less free of each other and hence able to re-orient into denser arrangements, followed immediately by (b) impacts producing relatively large dynamic stresses which forced the particles together. The maximum unit weight achieved by a given sand during this range was found to be relatively insensitive to changes in frequency, size and geometry of the container, material of which container is made, etc. The maximum unit weight usually was achieved for $a_p \simeq 1.6g$.

Third, for $a_p>2g$, the final unit weight was often less than the maximum unit weight obtained at $a_p \simeq 1.6g$. The causes of such "over vibration" are not clear at this time. A number of factors are present in this range: greater mean free path of particles during free fall, a spalling effect when the stress wave caused by impact reaches free surface, irregular impacts, etc. The degree of overvibration has been found to be influenced greatly by test conditions, and for some conditions no overvibration is observed. The flexibility of the container appeared to influence results obtained in this rage; quite often the container magnified the small horizontal vibrations which inevitably were present in the motion of the shaking table.

When a surcharge in the form of a dead weight is used, the weight accelerates with the sand and hence the stress between the weight and the sand varies with time. Presence of such a weight means that the dynamic stress within the sand is increased, especially when $a_p > 1g$, and hence use of a surcharge in the form of a dead weight *increases* somewhat the maximum unit weight achieved during vibration. On the other hand, use of a constant surcharge applied as air pressure against a membrane *decreases* the effectiveness of vibrations in producing densification (see section 4).

6. SUMMARY OF FACTORS CONTROLLING DENSIFICATION

The foregoing results indicate that it is the dynamic stresses associated with accelerations rather than the accelerations themselves that control the amount of densification which occurs during vibration.

With regard to densification by repeated compression, such as occurs during purely vertical vibrations, the amount of densification is controlled primarily by the ratio of the maximum and minimum stress during vibration. The most effective densification occurs when the minimum stress is zero; that is, when there is free fall. A downward acceleration of at least 1g is necessary but not sufficient to produce this condition.

Although not discussed in detail in this paper, repeated shearing may cause densification.

7. IMPLICATIONS TO PRACTICAL PROBLEMS

Earthquakes: Vertical accelerations of the ground during earthquakes are generally much less than 1g. Hence such accelerations cause essentially no densification. In rare cases where the vertical accelerations may approach or reach 1g, some densification might occur. If the peak stresses associated with such accelerations could be measured or estimated, the repeated stressing condition could be reproduced in the laboratory and the expected densification measured. However, shallow earthquakes causing very large vertical accelerations generally involve only one or two cycles of motion, and hence very little densification can occur.

On the other hand, horizontal accelerations - or really the shear stresses associated with the horizontal acclerations - may produce quite significant densification and resultant subsidence.

Vibratory compaction: Conditions beneath a vibratory compactor are quite similar to those in the "standard" test described in section 5 (10). At shallow depth (up to perhaps 3 feet), the condition of free fall followed by impact occurs and very efficient densification may be achieved. At greater depth there is no free fall, but some densification may still be achieved (rather inefficiently) by repeated stressing during many passages of the compactor.

Machine foundations: A well-designed machine foundation generally experiences vertical acceleration less (usually much less) than 0.5g. Hence no settlement is to be expected from purely compressive action within the soil. However, the dynamic stresses beneath a machine foundation cause repeated shearing action, which may in turn cause densification and/or cumulative shearing strain. The settlement which might be caused by such repeated shearing action may be estimated by the stress path method (11), using strains observed in repeated load triaxial tests.

ACKNOWLEDGEMENT

This study was made possible by the M.I.T. Inter-American Program in Civil Engineering through the sponsorship of the Ford Foundation.

BIBLIOGRAPHY

1. Barkan, D. D. (1962). *Dynamics of Bases and Foundations*, McGraw-Hill, Inc.

2. Selig, E. T. (1963). "Effect of Vibration on Density of Sand," Proc. 2nd Panamerican Conference on Soil Mechanics and Foundation Engineering (Brazil), Vol. I, pp. 129-144.

3. Forssblad, L. (1965). "Investigations of Soil Compaction by Vibration," *Acta Polytechnica Scandinavica*, Stockholm, No. Ci34.

4. D'Appolonia, D. J. and E. D'Appolonia (1967). "Determination of the Maximum Density of Cohesionless Soils," Proc. 3rd Asian Conference on Soil Mechanics and Foundation Engineering (Haifa), Vol. I. pp. 266-268.

5. Whitman, R. V., E. T. Miller and P. J. Moore (1964). "Yielding and Locking of Confined Sand," Proc. ASCE, Vol. 90, No. SM4, pp. 57-8

6. Luscher, U., P. Ortigosa, K. Rocker and R. V. Whitman (1967). "Repeated Load and Vibration Tests upon Sand," Research Report R67-29, M.I.T. Department of Civil Engineering.

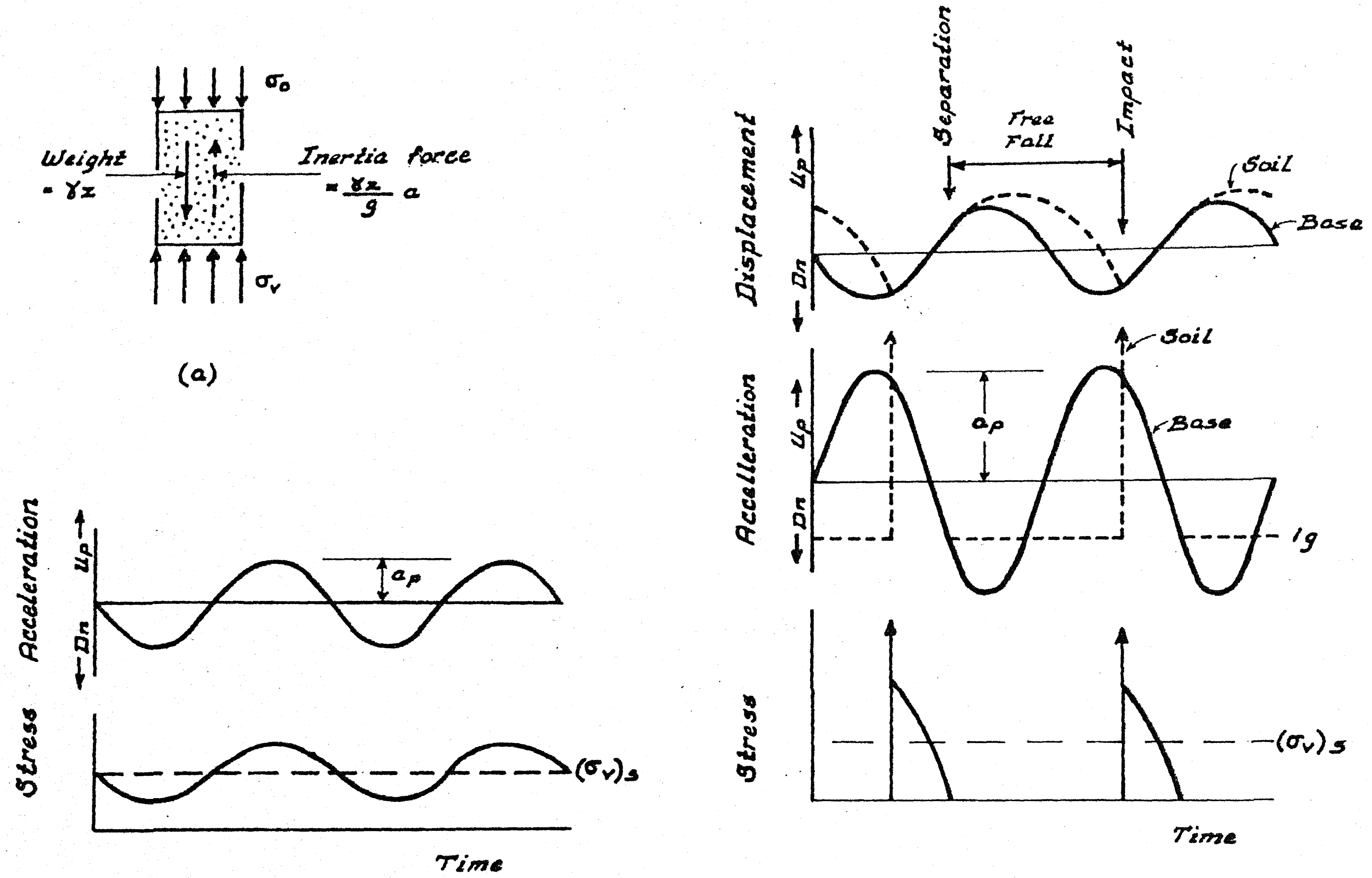
7. Ortigosa, P. (1968). "Densification of Sand by Vertical Vibrations with Almost Constant Stress," Research Report R68-2, M.I.T. Department of Civil Engineering.

8. ASTM Standard (1968). *Part II, Bituminous Materials; Soils; Skid Resistance*, Standard D-2049-64T, pp. 610-618.

9. Dobry, R. (1968). Tests at M.I.T., unpublished.

10. D'Appolonia, D. J., R. V. Whitman and E. D'Appolonia (1968). "Sand Compaction with Vibratory Rollers," ASCE Specialty Conference on Placement and Improvement of Soil to Support Structures.

11. Lambe, T. W. and R. V. Whitman (1968). *Soil Mechanics*, John Wiley & Sons.



(a)

(b) $a_p < 1g$

(c) $a_p > 1g$

FIGURE 1 RELATIONSHIP BETWEEN STRESSES AND ACCELERATIONS

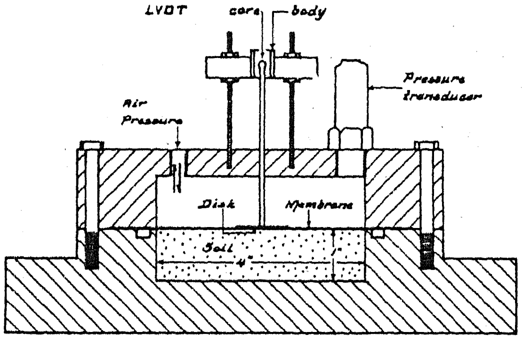


FIGURE 2 TEST CELL FOR REPEATED LOADING AND SHAKING TESTS

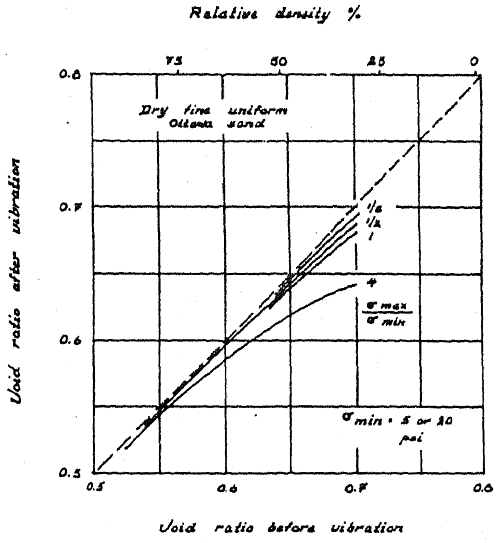


FIGURE 3 DENSIFICATION BY REPEATED STRESSES WITH SMALL ACCELERATIONS

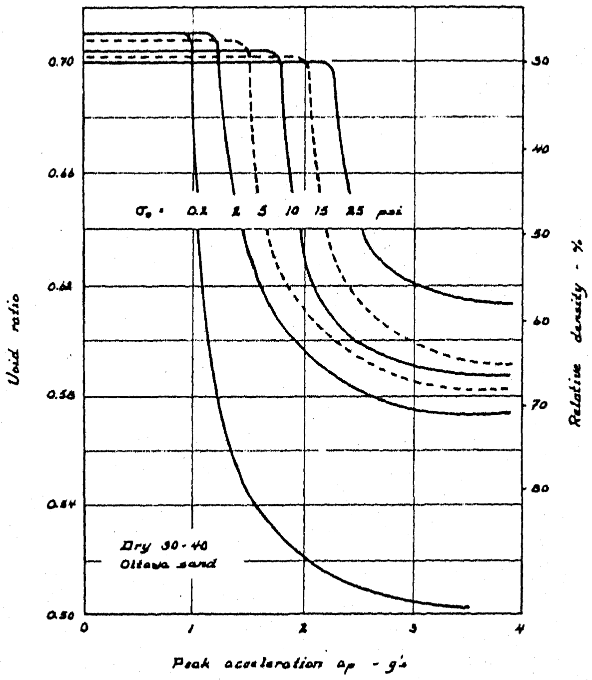


FIGURE 4 DENSIFICATION BY ACCELERATIONS WITH SMALL DYNAMIC STRESSES

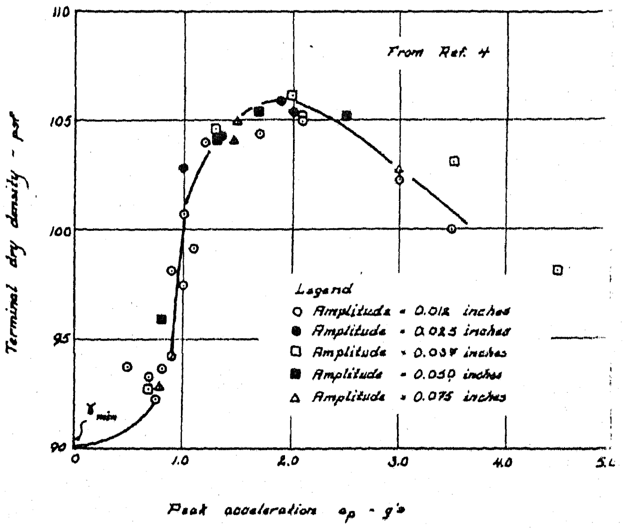


FIGURE 5 DENSIFICATION IN "STANDARD" TEST