EARTH PRESSURE DISTRIBUTION BEHIND RETAINING WALL DURING EARTHQUAKE

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ABSTRACT

The theoretical method for the determination of dynamic lateral earth pressure is a modification of Coulomb's theory and its distribution is assumed to be hydrostatic. This assumption is not valid even for static case for rough walls. Also experimental evidence has shown it to be non-hydrostatic.

In this paper, distribution of dynamic lateral earth pressure has been determined analytically for a general case. The expressions for lateral earth pressure and its point of application depends upon 'ϕ' δ, α_v and α_h. For vertical smooth walls and horizontal ground surface, the dynamic increment due to horizontal acceleration only lies at two thirds the height above the base of the wall as specified in Indian Standard Code (IS: 1893-1966).

It is also interesting to note that this method helps us to determine earth pressure distribution theoretically depending on type of the wall also, e.g. (i) sliding (ii) Tilting of the wall about base.

INTRODUCTION

Several methods are available to determine lateral earth pressure behind retaining wall due to earthquake loading. These include either modification of existing static formula for dynamic case[1,2,3] (Mononobe-Okabe) or results of experiments using vibration tables[4,5,6,7,8]. Distribution of both Static and dynamic lateral earth pressure is assumed to be hydrostatic in all analytical solutions.

Terzaghi[9,10] has shown that in static case and for a rough wall, the equilibrium conditions are not satisfied if earth pressure distribution is assumed to be hydrostatic. Also experimental results obtained by various research workers such as Matsuo[4], Okamoto[5] show that the lateral earth-pressure distribution is non-hydrostatic, under dynamic case. The assumptions made in Coulomb's theory have been critically evaluated and a solution for general case of wall with inclined surcharge has been developed.

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DISCUSSION ON COULOMB'S THEORY OF LATERAL EARTHPRESSURE

Coulomb's theory of earthpressure is based on three Assumptions:-

i) The surface of rupture is plane.

ii) The shearing resistance along this plane is fully mobilised.

iii) Lateral earthpressure increase like hydrostatic pressure in simple proportion to depth.

As stated by Terzaghi no serious objection can be raised against the first two assumptions as applied to cohesionless soil (dry sand) because for practical purpose both the first two assumptions are justified.

Let us consider the equilibrium of the failure wedge[9,10]. The forces acting on the failure wedge and their point of application are shown in Fig.1.

W = Weight of wedge acting vertically downwards at BC/3 from AB.

P_a = Active earthpressure inclined at 'δ' to the horizontal acting at AB/3 from A.

R = Reaction of failure plane inclined at 'ϕ' to the normal of the failure plane acting at AC/3 from A.

Where 'δ' and 'ϕ' are angle of wall friction and angle of internal friction respectively.

It can be easily seen that R and W meet at a point 'D' and Pa does not pass through this point. Hence in computing the earth-pressure by coulomb's theory all the conditions of statics are not satisfied. In otherwords, if a set of forces acting on a body are in equilibrium the following condition must be satisfied.

1. The Algebraic sum of all the vertical forces should be zero.

2. The Algebraic sum of all the horizontal forces should be zero.

3. The Algebraic sum of all the moments about any point should be zero.

In the Coulomb's theory of earthpressure, the total active earthpressure is obtained by utilizing the first two equilibrium conditions $\Sigma V = 0$ and $\Sigma H = 0$ and the point of application of lateral earthpressure is assumed to lie at 1/3 distance from bottom. However if this assumption is to be correct, it should satisfy the third equilibrium condition. By taking moment about point 'D' it can be seen that $\Sigma M = 0$ is not satisfied, because there is net moment equal to $P_a H/3 \sin\delta \cot\psi$. This means the assumption that the point of application lies at 1/3 height from bottom is not justified. The following analysis is suggested to overcome this discrepency.

THEORETICAL INVESTIGATION

ASSUMPTIONS

(i) The material behind retaining wall is cohesionless (dry sand)

(ii) The rupture surface is plane.

(iii) The shear resistance along this plane is fully mobilised.

(iv) The vertical pressure on plane parallel to the ground surface is constant.

(v) The principle of superposition holds good.

The analysis will be made for the general case of a retaining wall with inclined surcharge and considering the earthquake forces which give results for static case by putting $\alpha_v = \alpha_h = 0$.

ACTIVE CASE

Fig. 2a shows a retaining wall having slope 'ψ' and vertical height 'H'. The top surface of the backfill rises at slope 'β' .

Let xx and yy be the horizontal and vertical reference axis respectively. The origion of this axis lies at the right bottom edge of the retaining wall. Rotate these axes of reference through an angle 'β' . Let these new axes of reference be x 'x' and y'y'. The following forces are acting on the failure wedge:-

W_1 = The weight of failure wedge ADC $\dfrac{\gamma H^2 \sin^2(\psi_1+\beta)\cot(\psi-\beta)}{2\sin^2\psi_1}$
and acts vertically downwards at 2 xAD/3 and DC/3 from axis of reference x'x' and y'y' respectively.

W_2 = The weight of failure wedge ADB $\dfrac{\gamma H^2 \sin^2(\psi_1+\beta)\cot(\psi_1+\beta)}{2\sin^2\psi_1}$
acting vertically downwards at 2xAD/3 and DB/3 from axis of reference x'x' and y'y' respectively.

P_a = The active earth pressure acting on the retaining wall inclined at an angle $(\psi_1 + \beta - \delta)$ to the reference axis y'y' at distance 'y'_a from axis x'x'.

R = The reaction acting on the assumed failure plane AC acting at an angle $(\psi - \beta - \phi)$ to the axis y'y' at distance 'y'_r from the axis x'x'.

$W_1 \alpha_v$ = Vertical inertia force $\dfrac{\gamma H^2 \sin^2(\psi_1+\beta)\cot(\psi-\beta)}{2\sin^2\psi_1}\alpha_v$ of failure wedge ADC acting vertically at D/C3 and 2xAD/3 from axis of reference.

$W_2 \alpha_v$ = Vertical inertia force $\dfrac{\gamma H^2 \sin^2(\psi_1+\beta)\cot(\psi_1+\beta)}{2\sin^2\psi_1}\alpha_v$ of failure wedge ADB acting vertically at DB/3 and 2xAD/3 from axes of reference.

$W_1 \alpha_h$ = Horizontal inertia force $\frac{\gamma H^2 \sin^2(\psi_1+\beta) \cot(\psi-\beta)}{2 \sin^2(\psi_1)} \alpha_h$ of failure wedge ADC acting horizontally left or right at DC/3 and 2xAD/3 from axes of reference.

$W_2 \alpha_h$ = Horizontal inertia force $\frac{\gamma H^2 \sin^2(\psi_1+\beta) \cot(\psi_1+\beta)}{2 \sin^2 \psi_1} \alpha_h$ of failure wedge ADC acting horizontally left or right at DC/3 and 2xAD/3 from axes of reference.

Where

γ = bulk density of the backfill material.

δ = Angle of wall friction.

ϕ = Angle of internal friction of backfill material.

β = Slope of backfill material.

ψ = Slope of assumed failure plane.

ψ_1 = Slope of retaining wall with horizontal.

α_v = Vertical seismic coefficient.

α_h = Horizontal seismic coefficient.

Conditions of equilibrium give us only 3-equations, while there are 4-unknowns. However, the problem can be solved by making an assumption regarding distrubition of vertical pressures on any surface parallel to the surcharge.

The problem may further be split into two for simplification purposes as follows:-

i) Considering only the component of body forces acting parallel to reference axis x'x' (See Fig. 2b)

ii) Considering only the component of body forces acting perpendicular to the reference axis x'x' (See Fig. 2c).

(i) Considering only the Component of Body Forces Acting Parallel to Reference Axis x'x'.

Fig 2b shows the forces acting and their point of application. These forces are

i) P'_a = Active earth pressure due to body forces which are parallel to x'x' acting at a distance y'_a from reference axis.

ii) R'_x = Reaction on assumed failure plane due to body forces which are parallel to x'x' acting y'_r from reference axis x'x'.

iii) W'_{1x} , W'_{2x} = Components of all the body forces acting parallel to x'x'.

Equating the component of all the forces acting parallel to y'y'

$$P'_a \cos(\psi_1+\beta-\delta) = R'_x \cos(\psi-\beta-\phi)$$

$$R'_x = P'_a \frac{\cos(\psi_1+\beta-\delta)}{\cos(\psi-\beta-\phi)} \qquad \ldots \qquad (1)$$

Equating the component of forces acting parallel to x'x'

$$P'_a \sin(\psi_1+\beta-\delta) + R'_x \sin(\psi-\beta-\phi) = W'_{1x} + W'_{2x}$$

$$P'_a \sin(\psi_1+\beta-\delta) + R'_x \sin(\psi-\beta-\phi) = \frac{\gamma H^2 \sin^2(\psi_1+\beta)}{2\sin^2\psi_1}\left[\cot(\psi_1+\beta)+\cot(\psi-\beta)\right]\left[(1\pm\alpha_v)\sin\beta+\alpha_h\cos\beta\right]$$

Substituting the value of R'_x from equation (1) in equation (2) and simplifying

$$P'_a = \frac{\gamma H^2 \sin^2(\psi_1+\beta)\left[\cot(\psi_1+\beta)+\cot(\psi-\beta)\right]\left[(1\pm\alpha_v)\sin\beta+\alpha_h\cos\beta\right]}{2\sin^2\psi_1\left[\tan(\psi_1+\beta-\delta)+\tan(\psi-\beta-\phi)\right]\cos(\psi_1+\beta-\delta)}$$

Taking moment about 'A' (Fig. 2b)

$$P'_a \sin(\psi_1+\beta-\delta)\, y'_a + P'_a \cos(\psi_1+\beta-\delta)\cot(\psi_1+\beta)\, y'_a + R'_x \cos(\psi-\beta-\phi)\cot(\psi-\beta)\, y'_r$$
$$+ R'_x \sin(\psi-\beta-\phi)\, y'_r = \frac{\gamma H^3 \sin^3(\psi_1+\beta)}{3\sin^3\psi_1}\left[\cot(\psi_1+\beta)+\cot(\psi-\beta)\right]\left[(1\pm\alpha_v)\sin\beta+\alpha_h\cos\beta\right]$$

Where y' = perpendicular distance from x'x' to the point of application of active earth pressure due to component of body forces parallel to x'x'.

Substituting the value of R'_x from equation (1) in equation (4)

$$P'_a \sin(\psi_1+\beta-\delta)\, y'_a + P'_a \cos(\psi_1+\beta-\delta)\cot(\psi_1+\beta)\, y'_a + P'_a \tan(\psi-\beta-\phi)\cos(\psi_1+\beta-\delta)\, y'_r$$
$$+ P'_a \cos(\psi_1+\beta-\delta)\cot(\psi-\beta)\, y'_r = \frac{\gamma H^3 \sin^3(\psi_1+\beta)}{\sin^3\psi_1}\left[\cot(\psi_1+\beta)+\cot(\psi-\beta)\right]\left[(1\pm\alpha_v)\sin\beta+\alpha_h\cos\beta\right]$$

Determination of y'_a and y'_r

Consider an element of thickness 'dy' parallel to x'x' at a distance y'_1 from top of ground surface.

$P'_a(y)$ = Pressure on plane AB at a distance y'_1 from top of ground surface.

$r'_x(y)$ = Pressure on plane AC at a distance y_1 from top of ground surface.

Since the element is to be in equilibrium -

$$\Sigma V = 0 = \frac{p'_a(y)\cos(\psi_1+\beta-\delta)\,dy}{\sin(\psi_1+\beta)} - \frac{r'_x(y)\cos(\psi-\beta-\phi)\,dy}{\sin(\psi-\beta)}$$

$$p'_a(y) = r'_x(y)\ \frac{\cos(\psi - \beta - \phi)\sin(\psi_1 + \beta)}{\cos(\psi_1 + \beta - \delta)\sin(\psi - \beta)}$$

the point of application of active earthpressure P'_a from top of ground surface is

$$Y - y'_a = \frac{\int_0^Y p'_a(y)\,dy\ y}{\int_0^Y p'_a(y)\,dy}$$

the point of application of reaction on failure plane from top of ground surface is

$$Y - y'_r = \frac{\int_0^Y r'_x(y)\,y\ dy}{\int_0^Y r'_x(y)\ dy}$$

divide equation (7) by equation (8)

$$\frac{Y-y'_a}{Y-y'_r} = \frac{\int_0^Y p'_a(y)\,y\ dy}{\int_0^Y p_a(y)\ dy} \times \frac{\int_0^Y r'_x(y)\,dy\ y}{\int_0^Y r'_x(y)\ dy\ y}$$

Substituting the value of $P'_a(y)$ from equation (6) in equation (9) and simplifying

$$\frac{Y-y'_a}{Y-y'_r} = 1 \quad \text{or} \quad y'_r = y'_a \qquad \ldots (10)$$

Substituting the value of P'_a and Y'_r from equation (3) and (10) in equation (5) and simplifying.

$$y'_a = \frac{2H\sin(\psi_1+\beta)}{3\sin\psi_1}\left[\frac{\tan(\psi-\beta-\phi) + \tan(\psi_1+\beta-\delta)}{\tan(\psi-\beta-\phi) + \tan(\psi_1+\beta-\delta) + \cot(\psi-\beta) + \cot(\psi_1+\beta)}\right]$$

$$h'_a = \frac{\sin\psi_1}{\sin(\psi_1+\beta)}\,y'_a$$

$$= \frac{2H}{3}\left[\frac{\tan(\psi_1+\beta-\delta) + \tan(\psi-\beta-\phi)}{\tan(\psi_1+\beta-\delta) + \tan(\psi-\beta-\phi) + \cot(\psi_1+\beta) + \cot(\psi-\beta)}\right]$$

taking moment of the forces acting on the retaining wall about base point 'A'.

$$M'_a = \frac{\gamma H^3 \cos\delta \sin^2(\psi_1+\beta)}{3\sin^3\psi_1 \cos(\psi_1+\beta-\delta)} \left[\frac{\{\cot(\psi_1+\beta)+\cot(\psi-\beta)\}\{(1\pm\alpha_v)\sin\beta+\alpha_h\cos\beta\}}{\tan(\psi_1+\beta-\delta)+\tan(\psi-\beta-\phi)+\cot(\psi_1+\beta)+\cot(\psi-\beta)} \right]$$

(ii) Considering only the Component of Body Forces Acting Parallel to Reference Axis y'y'

Fig.2c shows the forces acting and their point of application.

Equating the component, of all the forces acting, parallel to x'x'

$$P''_a \sin(\psi_1+\beta-\delta) = R''_x \sin(\psi-\beta-\phi)$$

$$R''_x = P''_a \frac{\sin(\psi_1+\beta-\delta)}{\sin(\psi-\beta-\phi)} \qquad \ldots (13)$$

Equating the component, of all the forces, acting parallel to y'y'.

$$P''_a\cos(\psi_1+\beta-\delta)+R''_x\cos(\psi-\beta-\phi) = \frac{\gamma H^2 \sin^2(\psi_1+\beta)}{\sin^2\psi_1}\left[\cot(\psi_1+\beta)+\cot(\psi-\beta)\right]\left[(1\pm\alpha_v)\cos\beta+\alpha_h\sin\beta\right]$$

Substituting the value of R''_x from equation (13) in equation (14) and simplifying:

$$P''_a = \frac{\gamma H^2}{2} \frac{\sin^2(\psi_1+\beta)}{\sin^2\psi_1 \cdot \sin(\psi_1+\beta-\delta)} \left[\frac{\{\cot(\psi_1+\beta)+\cot(\psi-\beta)\}\{(1\pm\alpha_v)\cos\beta+\alpha_h\sin\beta\}}{\cot(\psi_1+\beta-\delta)+\cot(\psi-\beta-\phi)} \right]$$

Where P''_a = Active earthpressure due to body forces parallel to y'y' acting at a distance ya" from reference axis.

R''_x = Reaction on failure plane due to body forces parallel to y'y' acting at y''_r from reference axis x'x'.

Taking moment about 'A'

$$P''_a\sin(\psi_1+\beta-\delta)\,y''_a + P''_a\cos(\psi_1+\beta-\delta)\cot(\psi_1+\beta)\,y''_a - R''_x\cos(\psi-\beta-\phi)\cot(\psi-\beta)\,y''_r$$

$$-R''_x\sin(\psi-\beta-\phi)\,y''_r = \frac{\gamma H^3\sin^3(\psi_1+\beta)}{6\sin^3\psi_1}\left[\cot^2(\psi_1+\beta)-\cot^2(\psi-\beta)\right]\left[(1\pm\alpha_v)\cos\beta+\alpha_h\sin\beta\right]$$

Substituting the values of R''_x from equation (13) in equation (16) and simplifying,

$$P_a'' \sin(\psi_1+\beta-\delta)(y_a''-y_n'') + P_a'' \sin(\psi_1+\beta-\delta) \quad y_a'' \cot(\psi_1+\beta-\delta)\cot(\psi_1+\beta) - y_n'' \cot(\psi-\beta-\phi)\cot(\psi-\beta)$$

$$= \frac{\gamma H^3 \sin^3(\psi_1+\beta)}{6 \sin^3\psi_1}\left[(1 \pm \alpha_v)\cos\beta + \alpha_h \sin\beta\right]\left[\cot^2(\psi_1+\beta) - \cot^2(\psi-\beta)\right]$$

Determination of y''_a and y''_r

Let at a distance 'y' from top of ground surface pressure on plane AB (Fig 2c) be P'' (y) and on plane AC (Fig. 2c) be R'' (y) respectively where $P''_a(y)$ and $r''_x(y)$ may be different function of y

Then

$$Y - y''_a = \frac{\int_0^Y p''_a(y)\, y dy}{\int_0^Y p''_a(y)\, dy} \qquad \ldots (18)$$

$$Y - y''_r = \frac{\int_0^Y r''_x(y)\, y dy}{\int_0^Y r''_x(y)\, dy} \qquad \ldots (19)$$

divide equation (18) by equation (19)

$$\frac{Y - y''_a}{Y - y''_r} = \frac{\int_0^Y p''_a(y)\, y dy}{\int_0^Y p''_a(y)\, dy} \times \frac{\int_0^Y r''_x(y)\, dy}{\int_0^Y r''_x(y)\, y dy} \qquad \ldots (20)$$

Let us consider an element of dimension 'dx' and 'dy' lying on failure plane at a perpendicular distance y' from ground surface. Fig. 2c shows the stresses acting on this element p''(y), p''(x) and (x,y) are normal and shear stresses acting on this element. Using these stresses Mohr's diagram (Fig.2f) has been drawn.

$$ON = OM + MN = \frac{OP}{\cos\phi}\left[1 + \sin\phi \sin(2\psi - 2\beta - \phi)\right]$$

$$OP = \frac{ON \cos\phi}{1 + \sin\phi \sin(2\psi - 2\beta - \phi)}$$

But $OP = r''_x$ = The resultant force on failure plane.

and $ON = p''(y)$

$$r''_x(y) = p''(y) \frac{\cos\phi}{1 + \sin\phi \sin(2\psi - 2\beta - \phi)} = K_1 p''(y) \qquad \ldots (21)$$

Where $K_1 = \dfrac{\cos\phi}{1 + \sin\phi \sin(2\psi - 2\beta - \phi)}$

Similarly $p''_a(y) = K_a p''(y)$ ... (22)

Where K_a is constant and function of ψ_1, ϕ and δ only

$p''_a(y)$ = active earthpressure at a perpendicular distance y' from top of ground surface.

From equation (21) and (22)

$$r''_x(y) = \frac{K_1}{K_a} p''_a(y) \qquad \ldots (23)$$

Substituting the value of $r''_x(y)$ from equation (23) in equation (20) and simplifying

$$\frac{Y-y''_a}{Y-y''_r} = 1, \text{ or } y''_r = y''_a \qquad \ldots (24)$$

Substituting the value of y''_r from equation (24) in equation (17) and simplifying:-

$$y''_a = \frac{H \sin(\psi_1+\beta)}{3 \sin\psi_1} \left[\frac{\{\cot(\psi_1+\beta) - \cot(\psi-\beta)\}\{\cot(\psi_1+\beta-\delta) + \cot(\psi-\beta-\phi)\}}{\cot(\psi_1+\beta-\delta)\cot(\psi_1+\beta) - \cot(\psi-\beta-\phi)\cot(\psi-\beta)} \right]$$

$$h''_a = \frac{\sin\psi_1}{\sin(\psi_1+\beta)} y''_a$$

$$h''_a = \frac{H}{3} \left[\frac{\{\cot(\psi_1+\beta) - \cot(\psi-\beta)\}\{\cot(\psi_1+\beta-\delta) + \cot(\psi-\beta-\phi)\}}{\cot(\psi_1+\beta-\delta)\cot(\psi+\beta) - \cot(\psi-\beta-\phi)\cot(\psi-\beta)} \right]$$

Considering the moment of forces acting on the retaining wall about 'A'

$$M''_a = P''_a h''_a \frac{\cos\delta}{\sin\psi_1}$$

$$= \frac{\gamma H^3 \cos\delta \sin^2(\psi_1+\beta)}{6 \sin^3\psi_1 \sin(\psi_1+\beta-\delta)} \left[\frac{\{\cot^2(\psi_1+\beta) - \cot^2(\psi-\beta)\}\{(1 \pm \alpha_v)\cos\beta + \alpha_h \sin\beta\}}{\cot(\psi_1+\beta-\delta)\cot(\psi_1+\beta) + \cot(\psi-\beta-\phi)\cot(\psi-\phi)} \right]$$

Determination of Centre of Total Earthpressure

By pronciple of superposition

$$M_a = M'_a + M''_a \qquad \ldots (27)$$

$$P_a = P'_a + P''_a \qquad \ldots (28)$$

Substituting the values of M_a' and M_a'' from equation (12) and (26) in equation (27), P_a' and P_a'' from equation (3) and (15) in equation (28) and simplifying

$$M_a = \frac{\gamma H^3 \cos\delta \sin^2(\psi_1+\beta)\,[\cot(\psi_1+\beta)+\cot(\psi-\beta)]}{6\sin^3\psi_1 \sin(\psi_1+\beta-\delta)} \left[\frac{2\tan(\psi_1+\beta-\delta)\,\{(1\pm\alpha_v)\sin\beta+\alpha_h\cos\beta}{\tan(\psi_1+\beta-\delta)+\tan(\psi-\beta-\phi)+\cot(\psi_1+\beta)+\cot(\psi-\beta)} + \frac{\{\cot(\psi_1+\beta)-\cot(\psi-\beta)\}\,\{(1\pm\alpha_v)\cos\beta+\alpha_h\sin\beta}{\cot(\psi_1+\beta-\delta)\cot(\psi_1+\beta)-\cot(\psi-\beta-\phi)\cot(\psi-\beta)} \right]$$

$$P_a = \frac{\gamma H^2 \sin^2(\psi_1+\beta)\,[\cot(\psi_1+\beta)+\cot(\psi-\beta)]}{2\sin^2\psi_1 \sin(\psi_1+\beta-\delta)} \left[\frac{\{(1\pm\alpha_v)\sin\beta+\alpha_h\cos\beta\}\tan(\psi_1+\beta-\delta)}{\tan(\psi_1+\beta-\delta)+\tan(\psi-\beta-\phi)} + \frac{(1\pm\alpha_v)\cos\beta+\alpha_h\sin\beta}{\cot(\psi_1+\beta-\delta)+\cot(\psi-\beta-\phi)} \right]$$

$$h_a = \frac{H}{3}\left[\frac{\dfrac{2\tan(\psi_1+\beta-\delta)\,\{(1\pm\alpha_v)\sin\beta+\alpha_h\cos\beta}{\tan(\psi_1+\beta-\delta)+\tan(\psi-\beta-\phi)+\cot(\psi_1+\beta)+\cot(\psi-\beta)} + \dfrac{\{\cot(\psi_1+\beta)-\cot(\psi-\beta)\}\,\{(1\pm\alpha_v)\cos\beta+\alpha_h\sin\beta}{\cot(\psi_1+\beta-\delta)\cot(\psi_1+\beta)+\cot(\psi-\beta-\phi)\cot(\psi-\beta)}}{\dfrac{\tan(\psi_1+\beta-\delta)\{(1\pm\alpha_v)\sin\beta+\alpha_h\cos\beta}{\tan(\psi_1+\beta-\delta)+\tan(\psi-\beta-\phi)} + \dfrac{(1\pm\alpha_v)\cos\beta+\alpha_h\sin\beta}{\cot(\psi_1+\beta-\delta)+\cot(\psi-\beta-\phi)}} \right]$$

The value of 'ψ' which gives maximum moment is to be taken as the critical failure plane, since overturning about the base can take place when moment is maximum. If there is no overturning and only sliding takesplace the value of 'ψ' which gives maximum active earthpressure is to be taken as the critical failure plane. Thus we can differenciate between tilting wall and sliding wall.

Expressions can be obtained for passive case proceeding in a similar manner.

NON-DIMENSIONAL COEFFICIENTS

Expressions for moments (M_a) and pressures (P_a) and distance of point of application of pressure (h_a) above the base depends upon the geometry of the figure also. In order to reduce these values to non-dimensional form the following parameters are defined.

$$C_{ma} = \frac{M_a}{\gamma H^3/6}, \quad C_{pa} = \frac{P_a}{\gamma H^2/2} \quad \text{and} \quad C_{ha} = \frac{h_a}{H/3}$$

TYPICAL COMPUTATIONS

TILTING WALL

C_{ma} was evaluated on IBM 1620 for $\psi_1 = 90^\circ$, $\phi = 30$ and $\delta = 7.5^\circ$

by assuming different values of 'ψ''. The maximum value of C_{ma} thus determined has been plotted in Figure 3(a) versus 'α_h'' for different values of 'α_ϑ''. C_{pa} and C_{ha} were also evaluated for that value of ψ which gives C_{ma} maximum and their variation with 'α_h'' for different values of α_ϑ also been shown in Figure 3b and 3c.

SLIDING WALL

Sliding wall is one which fails by sliding and not by overturning. For this type of failure the pressure is to be maximum. Since the maximum pressure and maximum moment cannot attain for the same value of 'ψ''. C_{pa} was also evaluated on IBM 1620 for $\psi_1 = 90^\circ$, $\phi = 30^\circ$ and $\delta = 7.5^\circ$ by assuming different values of 'ψ''. The maximum values of C_{pa} thus determined are also plotted in Figure 3(b) versus 'α_h'' for different values of 'α_ϑ''. C_{ma} and C_{ha} were also evaluated for that value of 'ψ'' which gives C_{pa} maximum and their variation with α_h for different values of α_ϑ also been shown in figure 3a and 3c respectively.

PRESSURE DISTRIBUTION

Let at a vertical distance 'h' from top of retaining wall the activ earthpressure p(h) be Kh^n where K and n are constant

ie $$p(h) = Kh^n \qquad \ldots \qquad (35)$$

From equation (35) the total active earthpressure is given by

$$P_a = \int_0^H Kh^n \, dh = \frac{K\,H^{n+1}}{n+1} \qquad \ldots \qquad (36)$$

Similarly the moment of active earthpressure about top of retaining wall.

$$M_t = \int_0^H Kh^{n+1} dh = \frac{KH^{n+2}}{n+2} \qquad \ldots \qquad (37)$$

Where M_t = Moment about top of retaining wall

Dividing equation (37) by equation (36) and simplifying

$$n = \frac{2M_t - P_a H}{P_a H - M_t} \qquad \ldots \qquad (38)$$

Substituting the value of 'n' in equation (36) we can obtain the value of K, With known values of 'k' and 'n' we can calculate the pressure distribution using equation (35).

EXAMPLE

Given $H = 10$ m, $\gamma = 1.76$ T/m^2, $\psi_1 = 90^\circ$, $\phi = 30^\circ$,
$\delta = 7.5^\circ$, $\beta = 0.0$, $\alpha_v = 0.0$

Sliding Wall

From Figure 3 for $\alpha_h = 0.1$, $C_{pa} = 0.378$ and $C_{ha} = 1.203$

$$P_a = C_{pa} \frac{\gamma H^2}{2} = 33.2 \text{ t/m}$$

Moment about top $= M_t = P_a H \left(1 - \frac{C_{ha}}{3}\right) \cos\delta = 192.0$ Tm/m

Substituting the values of M_t, P_a and H in equation (38), we get

$$n = 0.373$$

Substituting the value of 'n' in equation (36), we get

$$K = 1.93$$

Substituting the values of K and n in equation (35) we obtain the following values of p(h) (Column 3, Table-1) at different heights from the top of wall.

In the same table values of p(h) for $\alpha_h = 0$, is listed in column 2 of the same table.

Tilting Wall

Similarly the value of pressure of different height were calculated for $\alpha_h = 0$, and 0.1 and are listed in column 4 and 5.

The pressure distribution for both sliding wall and tilting wall were shown in Figure 4.

COMPARISON WITH MONONONE OKABE AND INDIAN STANDARD SPECIFICATIONS[11]

In Table 2, values of total earth pressure computed by Mononobe Okabe and new method have been shown in coulumns 2,4 and 5. for $\alpha_h = 0$ and 0.1. Also the height of point of application of the total earth pressure by the two methods are shown in columns 6,8 and 9. The difference in magnitude of the two pressures computed by different methods is negligible, however, the location of the point of application is fairly different in the two cases. If the earth pressure acts at a higher elevation from the base it constitutes more severe conditions of loading on the wall.

According to the provisions of I.S. Specifications for Earthquake Resistant Design of Structure[11], the static pressure P stat is assumed to act at one-third the height and the dynamic increase ($\Delta P_{dyn} = P_{dyn} - P_{stat}$) acts at two-third the height of the retaining wall. In order to compare how the results based on the proposed method compared with the provisions of Indian Standard, the point of application of resultant pressure was computed according to these provisions. The corresponding values of h_a for $\alpha_h = 0$ and 0.1g are shown in column 7 of the same table. It will be seen that the total dynamic pressure acts at approximately the same height as obtained from the provisions of the code for all practical purposes in this particular case.

CONCLUSIONS

From this study the following conclusions can be drawn:-

(i) The pressure distribution behind retaining wall is not hydrostatic and it is function of (a) Angle of internal friction 'ϕ' (b) Angle of wall friction 'δ' (c) Vertical acceleration 'α_v'' and (e) Horizontal acceleration 'α_h'' .

(ii) Depending on angle of wall friction 'δ' , the angle of internal friction affects pressure distribution. However, if angle of wall friction is zero it has no effect.

(iii) Vertical acceleration alone does not have any effect on pressure distribution. However even if 'α_h'' is acting its effect is much less compared to that of horizontal acceleration.

(iv) For a vertical and smooth wall, the pressure distribution is hydrostatic for static case.

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TABLE-1 PRESSURE AT VARIOUS DEPTHS

Depth 'h' from top of retaining wall in metre	Earth Pressure in t/m^2			
	Sliding wall		Tilting wall	
	$\alpha_h = 0$	$\alpha_h = 0.1$	$\alpha_h = 0$	$\alpha_h = 0.1$
1	2	3	4	5
0.1	0.236	0.820	0.248	0.834
0.5	0.662	1.493	0.682	1.507
1.0	1.033	1.933	1.054	1.946
2.0	1.611	2.503	1.629	2.511
4.0	2.513	3.241	2.518	3.241
6.0	3.260	3.770	3.248	3.762
8.0	3.921	4.197	3.891	4.182
10.0	4.524	4.561	4.476	4.54

TABLE -2

COMPARISON WITH MONONOBE-OKABE AND I.S. CODE[11]

	P_a				h_a			
	M-o' Method	I.S. Code	New Method		M-o' Method	I.S. Code	New Method	
			Tilting wall	Sliding wall			Tilting wall	Sliding wall
1	2	3	4	5	6	7	8	9
0	27.55	27.55	27.55	27.55	3.33	3.33	3.58	3.56
0.1	33.25	33.25	33.20	33.25	3.33	3.92	4.02	4.00

M - o' = Mononobe-Okabe.

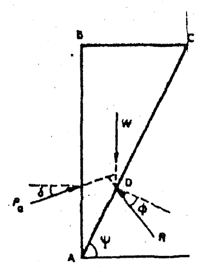
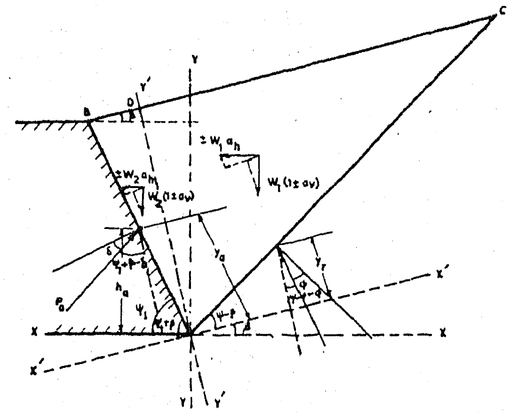
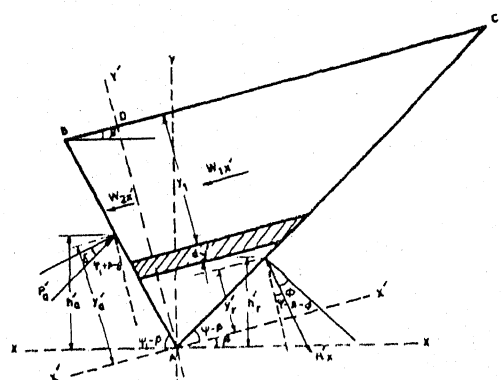


FIG. 1 _ FORCES ACTING ON THE FAILURE WEDGE ACCORDING TO COULOMB'S THEORY

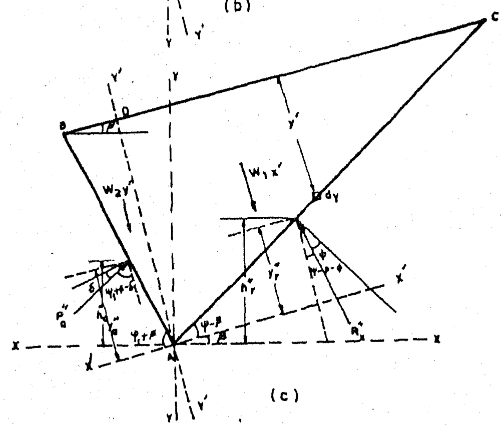


a _ RETAINING WALL WITH THE FORCES ACTING ON THE FAILURE WEDGE

FIG 2



(b)

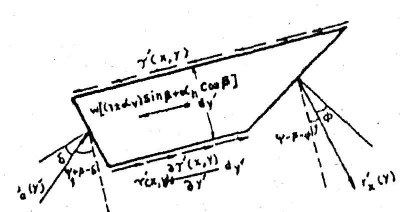


(c)

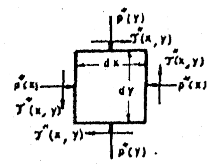
b_ FORCES ACTING ON THE FAILURE WEDGE WHEN VERTICAL BODY FORCES ARE ACTING

c_ FORCES ACTING ON THE FAILURE WEDGE WHEN HORIZONTAL BODY FORCES ARE ACTING

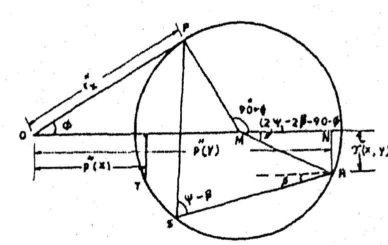
FIG. 2



(d)



(e)



(f)

d_ FORCES ACTING ON AN ELEMENT THICKNESS dy WHEN HORIZONTAL BODY FORCES ARE ACTING

e _ STRESSES ACTING ON AN ELEMENT dx,dy AT DISTANCE y′ FROM TOP OF GROUND SURFACE

f_ MOHR'S DIAGRAM FOR THE STRESSES ACTING ON THE ELEMENT

FIG. 2

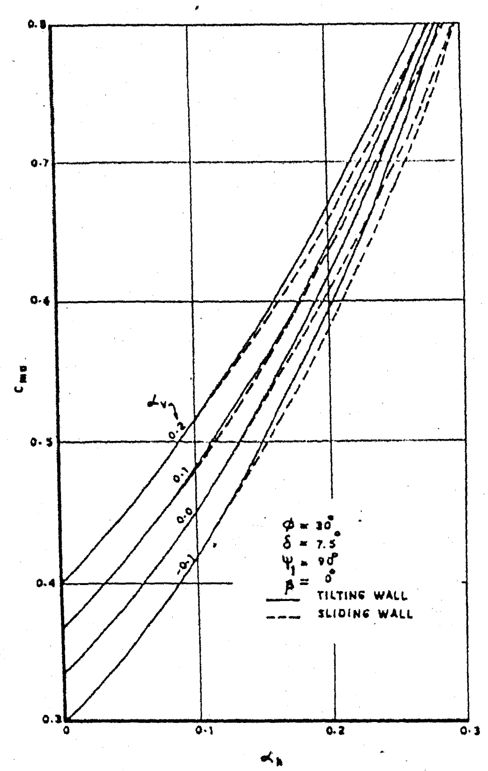


FIG.3(a) _ C_{ma} VERSUS α_h FOR BOTH TILTING WALL AND SLIDING WALL

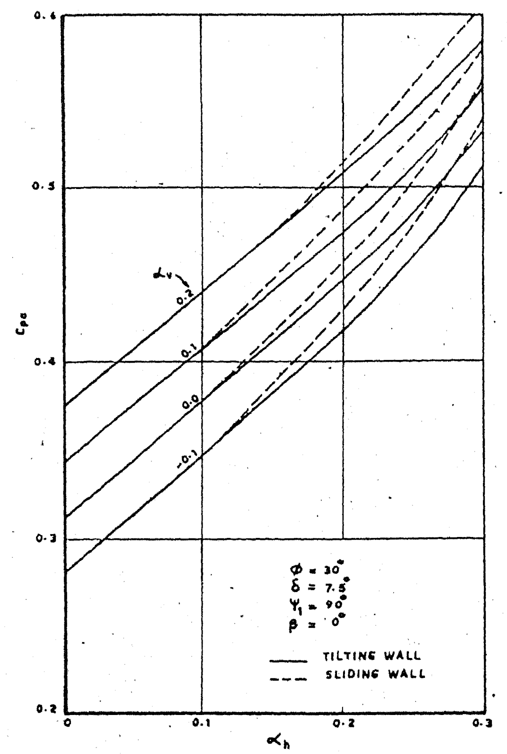


FIG.3(b)_ C_{pa} VERSUS α_h FOR BOTH TILTING WALL AND SLIDING WALL

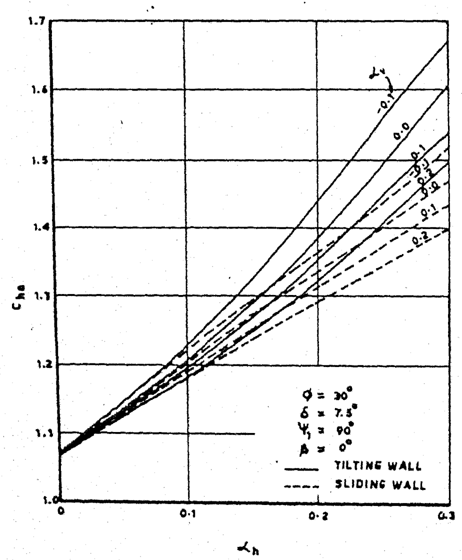


FIG.3(c)_ C_{ha} VERSUS α_h FOR BOTH TILTING WALL AND SLIDING WALL

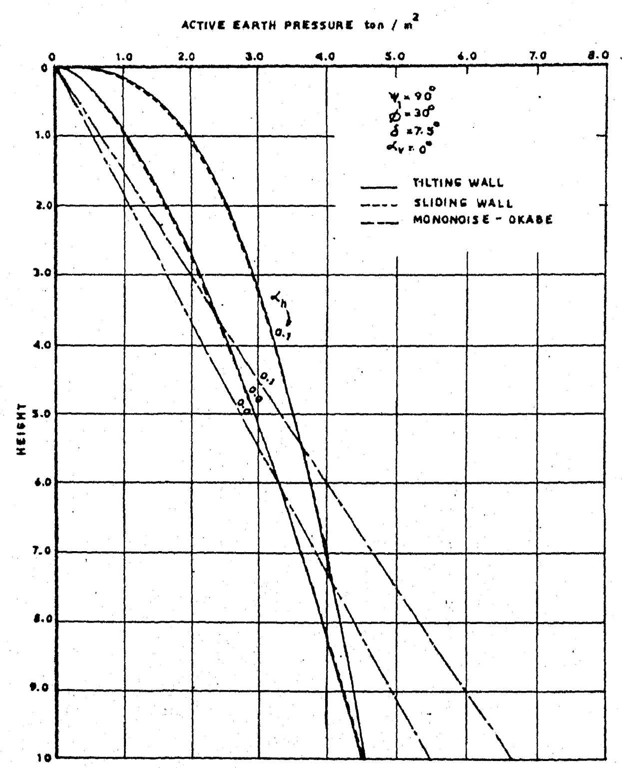


FIG. 4 _ PRESSURE DISTRIBUTION