HYDRODYNAMIC PRESSURES ON ARCH DAMS DURING EARTHQUAKES.

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SYNOPSIS:

The Hydrodynamic pressures generated on an arch Dam during horizontal Earthquake motions along the valley is presented. Pressures, due to steady-state harmonic motions of the Dam and the valley walls, at frequencies both higher and lower than the fundamental frequency of the reservoir-system have been obtained by using an electric analogy technique. This technique is simple and general, and can be used for any arbitrary configuration of the Dam and the valley walls; for a given configuration, dynamic pressures can be obtained for any type of motion, e.g, vertical, transverse to the valley, or even rotational motion, simply by processing what has been termed here as the " influence matrix ". Subsequent use of these results to evaluate pressure responses due to arbitrary ground motions has been discussed.

NOTATION:

a_x	, a	=	Linear accelerations.
C	, c	=	Non-dimensional coefficients, acoustic velocity in water.
G(xyz)	, g	=	Influence matrix, (function) gravity acceleration.
H		=	Height of Dam.
I(xy)	, I	=	Current intensity, unit matrix.
i	, i	=	Current density, $(-1)^{\frac{1}{2}}$.
L		=	Free surface at z=0.
m		=	Mass density of water.
n		=	(cT/H).
P	, p	=	Hydrodynamic pressures.
r		=	Specific electrical resistance.
T	, t	=	Period of oscillation, time.
V(xyz)		=	Electric potential function.
xyz		=	Rectangular cartesian coordinates.
z		=	(ω^2/c^2).
$\alpha\beta\gamma$		=	Coordinates.
ω		=	Circular frequency of oscillation.

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INTRODUCTION:

The behaviour of an arch Dam during Earthquakes is a complicated one, since, the motions of the Dam and the valley walls generate Hydrodynamic pressures, in excess of static pressures; these pressures, in turn, modify considerably the vibration characteristics of the Dam. Satisfactory evaluation of the structural and Hydrodynamic aspects of the problem and the interaction between them in particular, is a difficult proposition.

To begin with, the Dam is an elastic structure and its elasticity affects the dynamic pressures. If, for the time being, we assume that the Dam moves as a rigid body, then the dynamic pressures on it can be obtained either theoretically, when the geometry permits it, or experimentally. Conventional approach (1,2) to the problem, is then to consider the nett effect of the dynamic pressures as an equivalent increase in the mass of the Dam; its dynamic characteristics are then found to evaluate its response to a given motion.

This simplified approach has, however, one fundamental drawback, in that, when the frequency of motion of the Dam is higher than the fundamental frequency of the reservoir-system the pressure response has a component which is out-of-phase with the acceleration acting on the Dam (3). Under these circumstances, it becomes meaningless to consider the Dam as moving with an 'attached mass'. Thus, this approach holds, only if the dominant frequencies of motion are lower than the fundamental frequency of the reservoir-system; unfortunately, Fourier spectra (4) of past Earthquakes indicate significant amplitudes over a wide range of frequencies, and this invalidates the use of such a technique.

Although, as we have seen, the coupling of the structural and Hydrodynamic aspects is difficult , a seperate study of them gives one considerable insight into the problem ; the 'uncoupled' Hydrodynamic aspect has been studied in this paper.

Even this uncoupled problem presents formidable difficulties, owing to the relatively complicated geometry of an arch Dam, as far as its theoretical solution is concerned. Notwithstanding these difficulties, Kotsubo (5) derived an analytical solution for an arch Dam with simplified boundary conditions; he assumed the Dam to be a cylindrical one, in a rectangular valley and with radial valley walls. Solution to this problem was obtained for uniform accelerations, parallel and perpendicular to the valley; damped, out-of-phase components of pressure were found by neglecting all the waves returning to the Dam.

However, analytical solution of these problems is difficult and very rare indeed. Further, no general experimental technique seems to be available for obtaining dynamic pressures on arbitrary configurations of the arch Dam and the valley walls.

In this paper, the subject of study chosen is an arch Dam of cylindrical shape, located in a rectangular valley, (Fig. 1). Although, the shape chosen here is a relatively simple one, the technique used is general, and can be used for any arbitrary configuration, however complicated.

EQUATIONS OF MOTION:

The geometry of the arch Dam under consideration is shown in Fig. 1.

If the motion of the Dam and the valley is such that the resulting fluid velocities are small , then the terms representing convective acceleration can be neglected ; if, in addition, viscosity is neglected, then the simplified equations of motion can be written as (6),

$$\nabla^2 p = (1/c^2)\frac{\partial^2 p}{\partial t^2} \qquad ....1$$

where, ∇^2 = Laplacian operator in three dimensions, $p(xyz,t)$ = Hydrodynamic pressures, in excess of static pressures, and c= acoustic velocity in water. Solution of equation 1 with appropriate initial and boundary conditions will give the dynamic pressures.

Whereas, solution to Eq. 1 may be attempted for any form of acceleration, a(t), of the Dam and the valley, we shall investigate the case when $a_x(t)$ is periodic; in particular, let the steady-state acceleration of the Dam and the valley be represented by,

$$a_x = \mathrm{Re}(ae^{i\omega t}) \qquad ....2$$

where, ω = angular frequency of motion and $i=(-1)^{\frac{1}{2}}$; then, the steady-state dynamic pressure response will be given by,

$$p = \mathrm{Re}(Pe^{i\omega t}) \qquad ....3$$

Substituting for p into Eq. 1 , we obtain,

$$\nabla^2 P + (\omega^2/c^2)P = 0 \qquad ....4$$

Eq. 4 must now be solved with relevant boundary conditions to give the dynamic pressure amplitudes, P(xyz).

No condition can be imposed on the tangential component of fluid velocity (or acceleration) at the Dam surface , S , since the fluid is frictionless. However, since no seperation is permitted, we can show (6) that we must have,

$$\frac{\partial P}{\partial n} + ma = 0 \quad \text{on S} \qquad \text{..........5}$$

in which, n is the direction along the normal to S, and m = mass density of water. Further, we assume that P vanishes at the free surface, z = 0, and at large but finite distances from the Dam. That P=0 at the free surface, holds only when the exciting frequencies are relatively high; for low frequencies of motion, a linearised condition, such as,

$$\frac{\partial P}{\partial z} - (\omega^2/g)P = 0 \quad \text{on } z=0 \qquad \text{..........6}$$

should give better results. The technique for incorporating Eq. 6 into the present analysis will be discussed in conclusion.

We now build up the complete solution to Eq. 4, by considering its particular solution, and the solution of its associated homogeneous equation. Thus, expressing the complete solution in terms of an integral equation of the second kind, we can write, bearing in mind that the Dam moves as a rigid body,

$$P(xyz) = z\int_R G(xyz,\alpha\beta\gamma)P(\alpha\beta\gamma)dv + ma\int_S G(xyz,\alpha\beta\gamma)ds \quad \text{.....7}$$

where, $z = (\omega^2/c^2)$.

The first integral is carried out throughout the volume R, while the second on the entire surface, S. The kernel , $G(xyz,\alpha\beta\gamma)$ in the above equation is recognised as the "influence function" (of dynamic pressures) within the domain R+S , since, by definition, it represents the effect at (xyz) when unit cause is active at $(\alpha\beta\gamma)$. $G(xyz,\alpha\beta\gamma)$ is a symmetric function, owing to the reciprocity of cause and effects in a linear system; it is harmonic within R+S, and satisfies relevant boundary conditions; its eigenvalues and eigenvectors respectively represent frequencies of dynamic pressure oscillations and their amplitudes. If z is not an eigenvalue of $G(xyz\,\alpha\beta\gamma)$, then Eq. 7 converges to a solution; however, if z happens to be an eigenvalue of $G(xyz,\alpha\beta\gamma)$, then the solution diverges, giving infinite pressure amplitudes; (strictly speaking, a solution still exists, if the general solution is orthogonal to the common eigenfunction).

We now approximate the continuous domain (R+S), by a finite number of points; let R contain m points, while the surface of the Dam, S, is approximated by n points . We can thus write Eq. 7 in the matrix notation, as

$$\{P\} = z\,[G][v]\{P\} + ma\,[G]\{S\} \qquad \text{..........8}$$

in which, [] and { } denote square matrices and vectors respectively. [G] is now defined as the "influence matrix" of dynamic pressures, while, [v] is a diagonal weighing matrix of elemental volumes within R. The vector {S} represents the surface elements on the Dam, projected along the direction of motion, x . We finally write Eq. 8 as,

$$\{P\} = [I - zGv]^{-1}\{P'\} = [K]\{P'\} \qquad \text{.........9}$$

in which, I = unit matrix of order (m+n) and {P'} is the general solution, representing incompressible dynamic pressures on the Dam, so that,

$$\{P'\} = ma\,[G]\{S\} \qquad \text{.........10}$$

The compressible pressures, {P} , on the Dam, due to a given excitation frequency can now be obtained from Eqs. 9 and 10.

PROBLEM SOLUTION:

The electric potential function, V(xyz), in an isotropic conducting medium satisfies the Poisson's Equation,

$$\nabla^2 V + ri = 0 \qquad \text{.........11}$$

and its associated homogeneous equation, $\nabla^2 V = 0$; r in the above equation is the specific electrical resistance of the medium, and i(xyz) is the current density distribution. Further, from Ohm's law, we have,

$$\frac{\partial V}{\partial n} + rI = 0 \qquad \text{.........12}$$

in which, I(xyz)= current density across a surface, to which the normal is n. From the similarity of these equations , to those governing the dynamic pressures, the influence matrix , [G] can be obtained, by analogy, in terms of the influence matrix of voltages, [V] .

The experimental set-up for obtaining [V] was very simple. The Dam and the valley walls were moulded out of perspex; the region R and the Dam surface S were provided with finite electrodes (7); there were 24 electrodes within R, while S was provided with 28 electrodes. It was found later, that the sub-matrix of [V], which couples points in R to those on S, decayed exponentially away from the Dam. It would have been sufficient therefore, to have only a few points within R in the immediate vicinity of the Dam. The free surface was represented by a brass plate, with respect to which all potentials (voltages) were measured. The electrolyte used was ordinary tap water, whose specific electrical resistance was accurately measured by a specially made apparatus.

Unit currents were now fed into the electrodes, and the voltages measured at all the electrodes, in turn, by using a precision voltmeter; thus [V], and consequently [G] was obtained. The dynamic pressures were then obtained from Eqs.9 and 10.

RESULTS:

The dynamic pressures obtained for different sections of the Dam have been plotted in Fig. 2. The dotted lines in these drawings are the incompressible pressures, on a straight gravity Dam with a vertical upstream face, obtained by Westergaard (1). Also shown in the crown section (Fig.2a) are the results obtained by Kotsubo (5) for an arch Dam with radial valley walls.

The first resonance occured at $n(=cT/H) = 4.08$; this compares well with the theoretical value of 4.0 (5). However the pressures obtained here are larger than those in ref. 5, presumably since, although we have $R/H=1.09$, which is larger than Kotsubo's 0.5, the angle at which the Dam meets the valley walls, is, in our case much smaller than in ref. 5, where it is 90°. In our case it is only 23°.

Some other resonant frequencies occured at n=2.22, 1.60 and 1.31 etc. Change in the shape of pressure distribution curves, for values of n less than 4.0 is worth noting.

CONCLUSIONS:

We make the following observations in conclusion:

(1) The matrix [K], in Eq. 9 obviously represents the magnification of Hydrodynamic pressures, due to the compressibility of water; as expected, magnification becomes unity when $z=0$, i.e, $\omega = 0$.

(2) It is interesting to note, that, the general solution $\{P'\}$ also represents compressible dynamic pressures, corresponding to $\omega = 0$.

(3) When z happens to be an eigenvalue of $G(xyz,\alpha\beta\gamma)$, resonance occurs, at least theoretically. However, in a real situation, the system will have some amount of damping , and consequently, resonant amplitudes will be attenuated, depending on the actual amount of damping present in the system.

(4) For relatively low frequencies of motion, one should expect the formation of surface waves, which will invalidate the assumption, P = 0 at the free surface. However, we can obtain improved results by substituting the linearised condition at the free surface, expressed by Eq. 6. Thus, Eq. 7 now reads,

$$P(xyz) = z\int_R G(xyz,\alpha\beta\gamma)P(\alpha\beta\gamma)dv + \int_{S+L} G(xyz,\alpha\beta\gamma)\dot{f}(\alpha\beta\gamma)ds \quad ..13$$

in which,

$$f(xyz) = -(\omega^2/g)P(xy) \quad \text{at } z=0$$
$$= ma \quad \text{on } S$$

Again, we see that iteration of the above equation will give the Hydrodynamic pressures, provided of course that the "influence matrix" for the domain (R+S+L) has already been obtained.

We also note, that, generally for low frequencies of motion, the effect of compressibility is negligible, and the wave effect at the free surface becomes relatively more important. For high frequencies of motion at small amplitudes, the reverse is true. At the present time, experiments are being conducted to evaluate the effect of surface waves on the dynamic pressures on arch Dams, and the results will be reported in subsequent papers.

(5) Having obtained the periodic response, these can be used to investigate the pressures due to an arbitrary motion provided, the frequency spectrum is obtained in the usual manner.

(6) The most important aspect of the present technique, is that, once the influence matrix has been obtained, dynamic pressures can be obtained (a) for any configuration of the Dam and valley walls, however complicated, and (b) for a given configuration, dynamic pressures due to horizontal , vertical or even rotational motion can be obtained, simply by adjusting the general solution accordingly.

BIBLIOGRAPHY:

1. Westergaard, H.M, "Water pressure on Dams during Earthquakes", Trans. ASCE, Vol. 98, pp. 418-471, 1933

2. Tremmel, E, "Estimation of the influence of forced vibration on the stresses imposed on arch Dams", Trans, 7 th. congress on large Dams, C 14, 1961.

3. Chopra, A.K, "Hydrodynamic pressures on Dams during Earthquakes", Proc. ASCE, EM-6, Dec. 1967, pp. 205-223.

4. Jenschke, V.A, Clough, R.W, and Penzien, J, "Analysis of Earth motion accelerograms", Report No. SESM 64-1, Institute of Engineering research, University of California, Berkeley, Jan. 1964.

5. Kotsubo, S, "Water pressure on Dams during Earthquakes ", Proc. 2nd. World conference on Earthquake Engineering, Vol.2, 1960.

6. Zienkiewicz, O.C, " Hydrodynamic pressures due to Earthquakes", Water Power, London,Sept. 1964, pp. 382-388.

7. Zienkiewicz, O.C and Nath, B, " Earthquake Hydrodynamic pressures on arch Dams- an electric analogue solution" , Proc. Institution of Civil Engineers, London, Vol. 25, June 1963, pp. 165-175.

FIG.1 Geometry of the Dam.

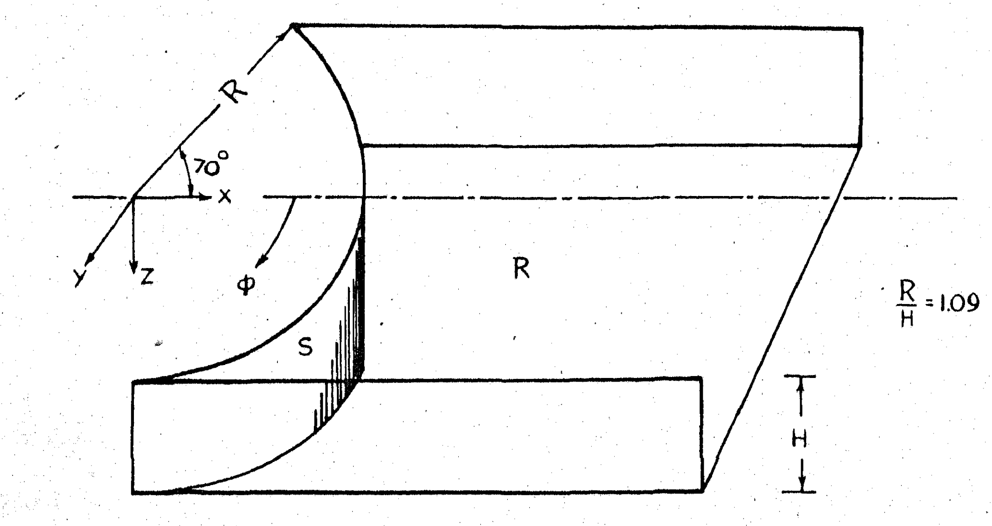

FIG.2 Dynamic pressures on the Dam.

(a)

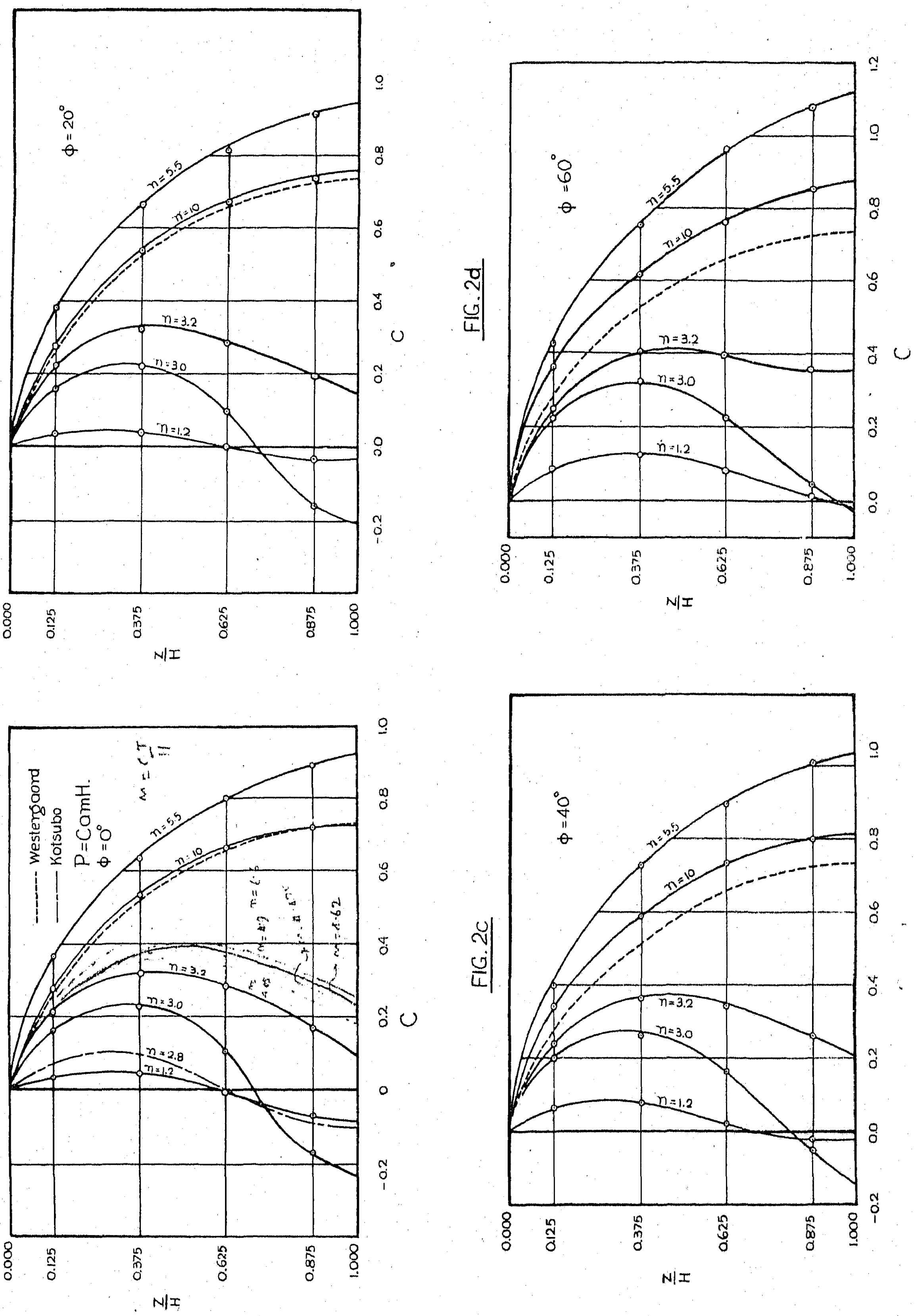

FIG. 2b.

FIG. 2c

FIG. 2d