WATER-DAM SEISMIC INTERACTION

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Synopsis

Linear conservative water-dam interaction in case of given rock motion is dealt with. Formal solutions of seismic effects are derived first for dam and water each, using appropriate eigen-function expansions. A feed-back procedure leads to interaction equations, written for sine-function as well as for arbitrary deterministic disturbances. Non-stationary random disturbances are then dealt with. Computational techniques are discussed. The case of an infinitely long gravity dam is analyzed, in order to derive transfer functions for the interaction phenomenon. Conclusions are concerned mainly with accuracy of hypotheses and computational difficulties.

1. Introduction

Seismic effects on dams are dealt with in literature in most cases by means of analyzing either

-dynamic water pressures on a dam having a given motion, or

-dam vibrations in case of empty lake.

None of these approaches can give an accurate image on seismic effects in case of full lake, due to the fact of the important mutual influence between dynamic water pressures and dam vibrations. Experimental data obtained by several authors as by the writer, [7], show that motions of rock and dam points are greatly different, i.e. dam motions cannot be identified with rock motions in order to derive seismic water pressures on dams. These remarks led some specialists to the approach of water-dam interaction problems. The main ways to be mentioned up-to-date in this view, are the analysis of:

-free vibrations of submerged dam structures, [8], and of

-forced plane vibrations of gravity dams subjected to random seismic loading, [3].

This paper is concerned with the analysis of water-dam seismic interaction phenomena in a more general case than those dealt with in literature. The shapes of lake and dam are arbitrary and a system of equations is written (in case of deterministic or random disturbances) for the normal coordinates of motion parameters (related to the natural modes of the dam, corresponding to empty lake).

The main idealizing assumptions are following:

-geometric and rheologic linearity of the phenomenon;

-neglecting of dissipative properties of dam material and water;

-neglecting of interaction phenomena between rock and water on one hand, and between rock and dam on the other hand.

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An accurate approach of the problem should be in compliance with the scheme of fig. 1a. The latter of the idealizing assumptions adopted previously leads to an analysis according to the simplified scheme of fig. 1b. Computations on this basis are developed stepwise; dam vibrations are first analyzed in compliance with the scheme of fig. 2a, and water motion is then dealt with, according to fig. 2b. Relations according to the scheme of fig. 1b are derived by using for a feed-back procedure the results obtained by following fig. 2a and 2b. The initial deterministic approach leads to equations which may be used easily as a basis for a stochastic approach.

An illustrative example, referring to the water-dam interaction in case of an infinitely long gravity dam is then presented. Computations are based on the assumptions, but not directly on the interaction equations derived previously (since natural modes cannot be used in this case). Latter developments may be of interest, since they complete conclusions and results obtained by other authors.

Techniques adopted in this paper may be directly used for analyzing interaction problems under more general assumptions on shape and rheologic properties for both structure and filling up.

2. Dam Motion

The dam sine-function motion is further expressed by means of complex amplitudes, $\bar{u}_\omega(P)$:

$$\bar{u}(P,t) = \mathrm{Re}\,[\bar{u}_\omega(P)\ e^{i\omega t}] \quad P \in \Omega_d \quad (\Omega_d\text{: dam body domain}) \qquad (2.1)$$

The dam motion satisfies Lamé's equation,

$$-\omega^2 \rho_d\, \bar{u}_\omega - \mathrm{div}\, \bar{\bar{\sigma}}_\omega = \bar{X}_\omega = o \quad (\rho_d\text{: dam material density}) \qquad (2.2)$$

and the boundary conditions

$$\bar{n}_{df}\, \bar{\bar{\sigma}}_\omega = o \qquad P \in S_{df}{}^{(I)} \qquad (2.3a)$$

$$\bar{n}_{dl}\, \bar{\bar{\sigma}}_\omega = -\bar{n}_{dl}\, p_\omega = \bar{\sigma}_{n\omega} \ (p\text{: water pressure}) \qquad P \in S_{dl} \qquad (2.3b)$$

$$\bar{u}_\omega = \bar{u}_{g\omega} \quad (\bar{u}_g\text{: ground displacements}) \qquad P \in S_{dg} \qquad (2.3c)$$

The stress tensor $\bar{\bar{\sigma}}$ satisfies Hooke's law,

$$\bar{\bar{\sigma}} = \lambda\, \bar{\bar{I}}\ \mathrm{div}\ \bar{u} + 2\,\mu\ \mathrm{sym\ grad}\ \bar{u}^{\ (II)} \quad (\lambda,\mu\text{: Lamé's moduli}) \qquad (2.4a)$$

i.e. $\mathrm{div}\ \bar{\bar{\sigma}} = [(\lambda + \mu)\ \mathrm{grad\ div} + \mu\ \mathrm{div\ grad}]\ \bar{u}$ (2.4b)

Further computations require an eigen-mode expansion of the solution of equation (2.2) and boundary conditions (2.3). The eigenmodes are considered for the case of fixed dam-ground contact zone, and of free dam-lake boundary. A formal contradiction arises between the homogeneous boundary conditions for the eigen-modes, and the nonhomogeneous boundary conditions (2.3b) and (2.3c), and this fact makes a rigorous eigen-mode expansion of the solution $\bar{u}$ (P) impos-

(I) Subscripts of S denote various boundaries: g-ground, d-dam, l-lake, f-free (fig. 3a).

(II) sym denotes the operator which transforms a 2-nd order tensor into its symmetric part; $\bar{\bar{I}}$ denotes the unit 2-nd order tensor.

sible. An artifice further used leads to an expansion which is correct for internal points, but satisfies boundary conditions only for limits of sequences of internal points tending to boundary points.

The eigen-vectors of the dam, $\bar{e}_r(P)$, and the corresponding circular frequencies, ω_r, satisfy the equations

$$-\omega_r^2 \rho_d \bar{e}_r - \operatorname{div} \bar{\bar{\sigma}}_r = 0 \quad (\bar{\bar{\sigma}}_r\text{: stresses corresponding to } \bar{e}_r) \tag{2.5}$$

and the boundary conditions

$$\bar{n}_d \bar{\bar{\sigma}}_r = 0 \qquad P \in (S_{df} \cup S_{dl}) \tag{2.6a}$$

$$\bar{e}_r = 0 \qquad P \in S_{dg} \tag{2.6b}$$

The eigen-vectors are ortho-normal,

$$\int_{\Omega_d} \rho_d(P)\ \bar{e}_{r'}(P)\ \bar{e}_{r''}(P)\ d\Omega(P) = \delta_{r'r''} \tag{2.7}$$

and the eigen-values are expressed by means of the relation

$$\omega_r^2 = -\int_{\Omega_d} \bar{e}_r \operatorname{div} \bar{\bar{\sigma}}_r\ d\Omega(P) = \int_{\Omega_d} \bar{\bar{\sigma}}_r(P)\ \bar{\bar{\varepsilon}}_r(P)\ d\Omega(P) \tag{2.8}$$

A solution $\bar{u}_\omega(P)$, corresponding to an arbitrary free term $\bar{X}_\omega(P)$, but to homogeneous boundary conditions (2.3), could be expressed by means of an eigen-mode expansion,

$$\bar{u}_\omega(P) = \Sigma_r\ u_{r\omega}\ \bar{e}_r(P) \tag{2.9}$$

where the normal coordinates $u_{r\omega}$ are given by the relation

$$u_{r\omega} = \frac{1}{\omega_r^2 - \omega^2} \int_{\Omega_d} \bar{e}_r(P)\ \bar{X}_\omega(P)\ d\Omega(P) \tag{2.10}$$

derived by performing the scalar product of (2.2) and $\bar{e}_r(P)$, by integrating, and considering the relations (2.7), (2.8), (2.9).

To use expressions like (2.9) and (2.10) for the actual problem (2.2),(2.3)), an auxiliary problem, with homogeneous boundary conditions (2.3), and with a special kind of free term, $\bar{X}(P;\eta)$, is dealt with. Following properties are assumed in this view:

- the boundaries S_{dl} and S_{dg} are smooth;
- the free term $\bar{X}(P;\eta)$ is non-zero only within two layers L_l and L_g of η depth, adjacent respectively to S_{dl} and S_{dg} (fig. 3b);
- the resulting force obtained by integrating $\bar{X}(P;\eta)$ on the depth η of L_l is statically equivalent to the forces $\bar{\sigma}_{n\omega}(P)$, (2.3b);
- the strains produced within the layer L_g correspond to a displacement increase from zero at the boundary S_{dg}, to the values $\bar{u}_{g\omega}(P)$ at the homologous points on the internal layer boundary.

The integral of (2.10) may be split now into two terms:

$$\int_{\Omega_d} \bar{e}_r(P)\ \bar{X}_\omega(P)\ d\Omega(P) = \int_{L_l} \bar{e}_r(P)\ \bar{X}_\omega(P;\eta)\ d\Omega(P) + \int_{L_g} \bar{e}_r(P)\ \bar{X}_\omega(P;\eta)\ d\Omega(P)$$

The first term leads to the expression

$$\lim_{\eta \to 0} \int_{L_l} \bar{e}_r(P)\ \bar{X}_\omega(P;\eta)\ d\Omega(P) = \int_{S_{dl}} \bar{e}_r(P)\ \bar{\sigma}_{n\omega}(P)\ dS(P) \tag{2.11}$$

To obtain a simple expression for the second term, the solution corresponding to the free term $\bar{X}_\omega(P;\eta)$, $P \in L_g$, will be represented by means of displacements $\bar{u}_{g\omega}(P;\eta)$ and stresses $\bar{\bar{\sigma}}_{g\omega}(P;\eta)$. Due to (2.2),

$$\int_{L_g} \bar{e}_r(P)\ \bar{X}_\omega(P;\eta)\ d\Omega(P) = -\int_{L_g} \omega^2\ \rho_d(P)\ \bar{e}_r(P)\ \bar{u}_{g\omega}(P;\eta)\ d\Omega(P) - $$
$$ - \int_{L_g} \bar{e}_r(P)\ \mathrm{div}\ \bar{\bar{\sigma}}_{g\omega}(P;\eta)\ d\Omega(P)$$

The last term of the right member may be expressed by

$$-\int_{L_g} \bar{e}_r(P)\ \mathrm{div}\ \bar{\bar{\sigma}}_{g\omega}(P)\ d\Omega(P) = -\int_{L_g} \bar{u}_{g\omega}(P)\ \mathrm{div}\ \bar{\bar{\sigma}}_r(P)\ d\Omega(P) +$$
$$+ \int_{S(L_g)} \bar{n}_L(P)\ [\bar{\bar{\sigma}}_r(P)\ \bar{u}_{g\omega}(P;\eta) - \bar{e}_r(P)\ \bar{\bar{\sigma}}_{g\omega}(P;\eta)]\ dS(P)$$

but, since (2.5) is satisfied,

$$-\int_{L_g} \bar{u}_{g\omega}(P)\ \mathrm{div}\ \bar{\bar{\sigma}}_r(P)\ d\Omega(P) = \omega_r^2 \int_{L_g} \rho_d(P)\ \bar{e}_r(P)\ \bar{u}_{g\omega}(P;\eta)\ d\Omega(P)$$

The integral dealt with is thus expressed by

$$\int_{L_g} \bar{e}_r(P)\ \bar{X}_\omega(P;\eta)\ d\Omega(P) = (\omega_r^2 - \omega^2) \int_{L_g} \rho_d(P)\ \bar{e}_r(P)\ \bar{u}_{g\omega}(P;\eta)\ d\Omega -$$
$$- \int_{S(L_g)} \bar{n}_L(P)\ [\bar{e}_r(P)\ \bar{\bar{\sigma}}_{g\omega}(P;\ ) - \bar{\bar{\sigma}}_r(P)\ \bar{u}_{g\omega}(P;\eta)]\ dS(P)$$

When the layer depth tends to zero, the contribution of integrands containing $\bar{e}_r(P)$ vanishes, since $\bar{e}_r(P)$ are continuous functions, equal to zero on S_{dg}. The integration must be done on the internal layer boundary only, where $\bar{n}_L = -\bar{n}_{dg}$, and $\bar{u}_g\ (P;\eta) \neq 0$. Thus,

$$\lim_{\eta \to 0} \int_{L_g} \bar{e}_r(P)\ \bar{X}_\omega(P;\eta)\ d\Omega = -\int_{S_{dg}} \bar{n}_{dg}(P)\ \bar{\bar{\sigma}}_r(P)\ \bar{u}_{g\omega}(P)\ dS \quad (2.12)$$

The normal coordinates u_r of (2.9) become now

$$u_r = \frac{1}{\omega_r^2 - \omega^2}\ [\int_{S_{dl}} \bar{e}_r(P)\ \bar{\sigma}_{n\omega}(P)\ dS - \int_{S_{dg}} \bar{\sigma}_{nr}(P)\ \bar{u}_{g\omega}(P)\ dS] \quad (2.10')$$

As previously said, this solution is valid for internal points, but satisfies boundary conditions only if boundary points are considered as limits of sequences of internal points. The convergence of the solution for points located in the neighbourhood of boundary is slower than for the case of current continuous free terms $\bar{X}_\omega(P)$.

To avoid infinite predicted amplitudes $u_{r\omega}$ in case of any resonance, $\omega = \omega_r$, dissipative rheologic properties of dam material should be considered, which lead to complex values for ω_r^2.

3. Water Motion

The small sine-function vibrations of water, assumed to be a compressible non-viscous fluid, are analyzed by means of a velocity potential $\Phi(P,t)$, related to a complex amplitude $\Phi_\omega(P)$,

$$\Phi(P,t) = \mathrm{Re}\ [\Phi_\omega(P)\ e^{i\omega t}] \qquad P \in \Omega_1 \qquad (3.1)$$

Vibration velocity $\bar{v}(P,t)$ and dynamic pressure $p(P,t)$ are expressed in this case by means of the relations [2]

$$\bar{v}(P,t) = - \text{grad}\ \Phi(P,t) = \text{Re}\ [\bar{v}_\omega(P)\ e^{i\omega t}] \quad (3.2a)$$

$$\bar{v}_\omega(P) = - \text{grad}\ \Phi_\omega(P) \quad (3.2b)$$

$$p(P,t) = \rho_o\ \dot{\Phi}(P,t) = \text{Re}\ [p_\omega(P)\ e^{i\omega t}] \quad (3.2c)$$

$$p_\omega(P) = i\omega\rho_o\ \Phi_\omega(P) \qquad (\rho_o: \text{water density}) \quad (3.2d)$$

The velocity potential satisfies the wave equation,

$$- \nu^2\ \Phi_\omega - \text{div grad}\ \Phi_\omega = f_\omega = o \ (\nu = \frac{\omega}{c_o},\ c_o = \sqrt{k_o/\rho_o}), P \in \Omega_1 \quad (3.3)$$

(k_o: water bulk modulus, c_o: pressure wave velocity)
and the boundary conditions

$$\Phi_\omega(P) = o \qquad P \in S_{1f} \quad (3.4a)$$

$$\frac{\partial \Phi_\omega}{\partial n}(P) = - \bar{n}_{1g}(P)\ \bar{v}_{g\omega}(P) \qquad P \in S_{1g} \quad (3.4b)$$

$$\frac{\partial \Phi_\omega}{\partial n}(P) = - \bar{n}_{1d}(P)\ \bar{v}_{d\omega}(P) = \bar{n}_{d1}(P)\ \bar{v}_{d\omega}(P) =$$

$$= \bar{n}_{d1}(P)\ i\omega\ \Sigma_r\ \bar{e}_r(P)\ u_{r\omega} \qquad P \in S_{1d} \quad (3.4c)$$

To express the solution of the boundary problem ((3.3),(3.4)), the eigen-functions $\varphi_s(P)$ and eigen-values ν_s of an associated eigen-value problem will be used. The equation (3.3) leads to the equation

$$- \nu_s^2\ \varphi_s - \text{div grad}\ \varphi_s = 0 \quad (3.5)$$

with the boundary conditions

$$\varphi_s(P) = o \qquad P \in S_{1f} \quad (3.6a)$$

$$\frac{\partial \varphi_s}{\partial n}(P) = o \qquad P \in (S_{1g} \cup S_{1d}) \quad (3.6b)$$

The eigen-functions $\varphi_s(P)$ are ortho-normal,

$$\int_{\Omega_1} \varphi_{s'}(P)\ \varphi_{s''}(P)\ d\Omega(P) = \delta_{s's''} \quad (3.7)$$

and the eigen-values are expressed by means of the relation

$$\nu_s^2 = \frac{\omega_s^2}{c_o^2} = \frac{\rho_o}{k_o}\ \omega_s^2 = - \int_{\Omega_1} \varphi_s(P)\ \text{div grad}\ \varphi_s(P)\ d\Omega(P) = \int_{\Omega_1} [\text{grad}\ \varphi_s(P)]^2\ d\Omega(P) \quad (3.8)$$

An eigen-function expansion for the problem (3.3),(3.4),

$$\Phi_\omega(P) = \Sigma_s\ \Phi_{s\omega}\ \varphi_s(P) \qquad P \in \Omega_1 \quad (3.9)$$

gives for the normal coordinates $\Phi_{s\omega}$ (in case of non-zero free term $f_\omega(P)$ and of homogeneous boundary conditions (3.4)) the expression

$$\Phi_{s\omega} = \frac{1}{\nu_s^2 - \nu^2} \int_{\Omega_1} \varphi_s(P)\ f_\omega(P)\ d\Omega(P) \quad (3.1o)$$

The consideration of non-homogeneous boundary conditions by means of an approach, homologous to that which led to the expression (2.1o'), permits to derive a solution

$$\Phi_{s\omega} = \frac{1}{\nu_s^2 - \nu^2} \int_{(S_{1g} \cup S_{1d})} \varphi_s(P)\ \frac{\partial \Phi_\omega}{\partial n}(P)\ dS(P) =$$

$$= \frac{1}{\nu_s^2 - \nu^2} [- \int_{S_{1g}} \bar{n}_{1g}(P)\, \bar{v}_{g\omega}(P)\, \varphi_s(P)\, dS(P) + + i\omega \sum_r u_{r\omega} \int_{S_{1d}} \bar{n}_{dl}(P)\, \bar{e}_r(P)\, \varphi_s(P)\, dS(P)] \qquad (3.1o')$$

This solution satisfies the wave equation (3.3) at all internal points, but satisfies boundary conditions (3.4b) and (3.4c) only for limits of sequences of internal points tending to boundary points.

To avoid infinite predicted amplitudes $\phi_{s\omega}$ in case of any *resonance*, $\omega = \omega_s$, water viscosity should be considered. In this case, the equation of motion, the boundary conditions and the method of analysis would radically change.

4. Water-Dam Interaction

The response of a dam subjected to seismic disturbances acting at the boundaries $S_{1g} \cup S_{dg}$, may be described by means of relations to be derived from previous results, related to dam and water each.

A sequence of interaction coefficients,

$$I_{rs} = \int_{S_{dl}} \bar{n}_{dl}(P)\, \bar{e}_r(P)\, \varphi_s(P)\, dS(P) \qquad (4.1)$$

in which eigen-functions of dam and lake occur simultaneously, will be used throughout further relations. Sequences of free terms representing the seismic disturbance,

$$G_{r\omega} = - \int_{S_{dg}} \bar{\sigma}_{nr}(P)\, \bar{u}_{g\omega}(P)\, dS(P) \qquad (4.2a)$$

$$G_{s\omega} = - \int_{S_{1g}} \bar{n}_{1g}(P)\, \bar{v}_{g\omega}(P)\, \varphi_s(P)\, dS(P) \qquad (4.2b)$$

will be used beside them.

The relations (2.1o') and (3.1o') may be written in these terms,

$$(\omega_r^2 - \omega^2)\, u_{r\omega} = - i\omega \rho_o \sum_s I_{rs}\, \phi_{s\omega} + G_{r\omega} \qquad (4.3a)$$

$$(\nu_s^2 - \nu^2)\, \phi_{s\omega} = i\omega \sum_r u_{r\omega} I_{rs} + G_{s\omega} \qquad (4.3b)$$

The double system ((4.3a)-(4.3b)) represents the basic form of the interaction equations in case of a sine-function disturbance. This basic form may be replaced by systems of equations referring only to parameters related to either dam, or water, motion, respectively. Some new coefficients and free terms shall be adopted in this view:

$$D_{r\hat{r}\omega} = D_{\hat{r}r\omega} = k_o \sum_s \frac{I_{rs} I_{\hat{r}s}}{\omega_s^2 - \omega^2} \qquad (4.4a)$$

$$L_{s\hat{s}\omega} = L_{\hat{s}s\omega} = k_o \sum_r \frac{I_{rs} I_{r\hat{s}}}{\omega_r^2 - \omega^2} \qquad (4.4b)$$

$$W_{r\omega} = G_{r\omega} - i\omega k_o \sum_s \frac{I_{rs} G_{s\omega}}{\omega_s^2 - \omega^2} \qquad (4.4c)$$

$$\psi_{s\omega} = c_o^2 \, G_{s\omega} + i\omega c_o^2 \sum_r \frac{G_{r\omega} I_{rs}}{\omega_r^2 - \omega^2} \tag{4.4d}$$

The substitution of the right member of (4.3b) into (4.3a) leads to equations written for the dam motion,

$$(\omega_r^2 - \omega^2)\, u_{r\omega} - \omega^2 \sum_{\hat{r}} D_{r\hat{r}\omega} u_{\hat{r}\omega} = W_{r\omega} \tag{4.5}$$

The reciprocal substitution leads to equations for water motion,

$$(\omega_s^2 - \omega^2)\, \phi_{s\omega} - \omega^2 \sum_{\hat{s}} L_{s\hat{s}\omega} \phi_{\hat{s}\omega} = \psi_{s\omega} \tag{4.6}$$

Both systems are symmetric. The case of empty lake (or neglected water) corresponds to zero interaction coefficients I_{rs}, and $D_{r\hat{r}\omega}$, i.e. to the case of a dam vibrating according to the equations

$$(\omega_r^2 - \omega^2)\, u_{r\omega} = W_{r\omega} = G_{r\omega} \tag{4.5'}$$

valid in the case of seismic effects transmitted through ground only.

The interaction equations may be written for the case of arbitrary disturbances (acting after t = o) if the complex amplitudes dealt with previously are considered as images obtained by means of Fourier or Laplace integral transformations. Following functions must be introduced in this view:

$$\dot{D}_{r\hat{r}}(t) = k_o \sum_s \frac{I_{rs} I_{\hat{r}s}}{\omega_s} \sin \omega_s t \qquad (\text{for } t > o) \tag{4.7a}$$

$$\dot{L}_{s\hat{s}}(t) = k_o \sum_r \frac{I_{rs} I_{r\hat{s}}}{\omega_r} \sin \omega_r t \qquad (\text{for } t > o) \tag{4.7b}$$

and

$$G_r(t) = - \int_{S_{dg}} \bar{\sigma}_{nr}(P) \, \bar{u}_g(P,t) \, dS(P) \tag{4.8a}$$

$$G_s(t) = - \int_{S_{1g}} \bar{n}_{1g}(P) \, \bar{v}_g(P,t) \, \varphi_s(P) \, dS(P) \tag{4.8b}$$

$$\begin{aligned} W_r(t) &= G_r(t) - k_o \sum_s \frac{I_{rs}}{\omega_s} \int_o^t \sin \omega_s(t-q) \, \dot{G}_s(q) \, dq = \\ &= - \int_o^t [(t-q) \int_{S_{dg}} \bar{\sigma}_{nr}(P) \, \bar{w}_g(P,q) \, dS(P) - \\ &\quad - k_o \sum_s \frac{I_{rs}}{\omega_s} \sin \omega_s(t-q) \int_{S_{1g}} \bar{n}_{1g}(P) \, \bar{w}_g(P,q) \, \varphi_s(P) \, dS(P)] \, dq \end{aligned} \tag{4.8c}$$

$$\begin{aligned} \psi_s(t) &= c_o^2 \, G_s(t) + c_o^2 \sum_r \frac{I_{rs}}{\omega_r} \int_o^t \sin \omega_r(t-q) \, \dot{G}_r(q) \, dq = \\ &= - c_o^2 \, [\int_{S_{1g}} \bar{n}_{1g}(P) \, \bar{v}_g(P,t) \, \varphi_s(P) \, dS(P) + \\ &\quad + \sum_r \frac{I_{rs}}{\omega_r} \int_o^t \sin \omega_r(t-q) \int_{S_{dg}} \bar{\sigma}_{nr}(P) \, \bar{v}_g(P,q) \, dS(P) \, dq] \end{aligned} \tag{4.8d}$$

These functions occur in the integral equations of dam motion,

$$w_r(t) + \Sigma_{\hat{r}} \int_0^t d_{r\hat{r}}(t-q)\ w_{\hat{r}}(q)\ dq = W_r(t) \tag{4.9}$$

written for the normal acceleration coordinates,

$$w_r(t) = \ddot{u}_r(t) \tag{4.10a}$$

where the tensorial kernel $d_{r\hat{r}}(t)$ is given by the expression

$$d_{r\hat{r}}(t) = \omega_r\ \omega_{\hat{r}}\ \delta_{r\hat{r}} t + \dot{D}_{r\hat{r}}(t) \quad (\delta_{r\hat{r}}\text{: Kronecker's symbols}) \tag{4.10b}$$

A homologous system may be derived for water motion,

$$\psi_s(t) + \Sigma_{\hat{s}} \int_0^t l_{s\hat{s}}(t-q)\ \psi_{\hat{s}}(q)\ dq = \Psi_s(t) \tag{4.11}$$

if following symbols are introduced:

$$\psi_s(t) = \ddot{\Phi}_s(t) \tag{4.12a}$$

$$l_{s\hat{s}}(t) = \omega_s\ \omega_{\hat{s}}\ \delta_{s\hat{s}} t + \dot{L}_{s\hat{s}}(t) \tag{4.12b}$$

The basic form of equations, (4.3), corresponds the system

$$\ddot{u}_r + \omega_r^2\ u_r = -\rho_o\ \Sigma_s\ I_{rs}\ \dot{\Phi}_s + G_r(t) \tag{4.13a}$$

$$\dot{\Phi}_s + \omega_s^2\ \Phi_s = c_o^2\{\Sigma_r\ \dot{u}_r\ I_{rs} + G_s(t)\} \tag{4.13b}$$

In the case of empty lake (I_{rs}=o), the systems (4.9) and (4.13a) reduce both to a system corresponding to (4.5'),

$$w_r(t) + \omega_r^2 \int_0^t (t-q)\ w_r(q)\ dq = \ddot{u}_r + \omega_r^2\ u_r = G_r(t) \tag{4.14}$$

The systems (4.5),(4.6),(4.3),(4.5'), may be dealt with as images for the systems (4.9),(4.11),(4.13),(4.14), respectively.

5. Interaction in Case of a Random Disturbance

A realistic representation of seismic phenomena should consider the non-synchronous random variation of disturbances at different ground points. The most adequate mathematical model available at present for representing seismic motions, is that of non-stationary stochastic processes. A complete stochastic representation of seismic phenomena would require the distributions for any system of quantities (for instance, displacements at any system of points and time moments). Deriving of analytical solutions in this view, on the basis of Fokker-Planck equations, [1], is beyond actual possibilities. Aproximate solutions of the stochastic interaction problem could be obtained by means of a Monte-Carlo approach, [4], or of the analysis of lower order (practically 2-nd order) correlation tensors, [1].

The Monte-Carlo approach reduces practically to the analysis of previous equations in case of simulated random input. Computational difficulties arisen in this view are discussed in the next paragraph.

The correlation analysis should not be based on stationarity assumptions. Time intervals required for stress wave propagation along dams are comparable with the lower order natural periods of dam vibrations, while time intervals required for gravitational wav propagation along lakes are comparable with the whole earthquake du ation. These facts require the explicit consideration of non-synchronous non-stationary disturbances at various boundary points. The ground acceleration correlation tensor,

$$\bar{\mathcal{W}}_g(P,\hat{P},t,\hat{t}) = \overline{\bar{w}_g(P,t) * \bar{w}_g(\hat{P},\hat{t})} \quad (*\text{: dyadic product}) \tag{5.1}$$

will represent the main characteristic of seismic disturbances (zero mean values of any motion parameters can be reasonably assumed). This field of two systems of variables, P, t and $\hat{P}$, $\hat{t}$ respectively, yields the tensor of free terms of the correlational interaction equations,

$$\begin{aligned}
\mathcal{W}_{gr\hat{r}}(t,\hat{t}) &= \overline{W_r(t)\, W_{\hat{r}}(\hat{t})} = \int_0^t\int_0^{\hat{t}} [(t-q)(\hat{t}-\hat{q}) \iint_{P,\hat{P}\in S_{dg}} \bar{\sigma}_{nr}(P) * \bar{\sigma}_{n\hat{r}}(\hat{P}) \cdot \\
&\cdot \bar{\mathcal{W}}_g(P,\hat{P},q,\hat{q})\, dS(P)\, dS(\hat{P}) - k_o \Sigma_s \frac{I_{\hat{r}s}}{\omega_s} (t-q) \sin\omega_s(\hat{t}-\hat{q}) \cdot \\
&\cdot \int_{P\in S_{dg}} \int_{\hat{P}\in S_{1g}} \bar{\sigma}_{nr}(P) * \bar{n}_{1g}(\hat{P})\, \varphi_s(\hat{P})\, \bar{\mathcal{W}}_g(P,\hat{P},q,\hat{q})\, dS(P)\, dS(\hat{P}) - \\
&- k_o \Sigma_s \frac{I_{rs}}{\omega_s} (\hat{t}-\hat{q}) \sin\omega_s(t-q) \int_{\hat{P}\in S_{dg}} \int_{P\in S_{1g}} \bar{\sigma}_{n\hat{r}}(\hat{P}) * \bar{n}_{1g}(P) \cdot \\
&\cdot \bar{\mathcal{W}}_g(\hat{P},P,\hat{q},q)\, dS(\hat{P})\, dS(P) + k_o^2 \Sigma_{s\hat{s}} \frac{I_{rs}}{\omega_s} \frac{I_{\hat{r}\hat{s}}}{\omega_{\hat{s}}} \sin\omega_s(t-q) \cdot \\
&\cdot \sin\omega_{\hat{s}}(\hat{t}-\hat{q}) \iint_{P,\hat{P}\in S_{1g}} \bar{n}_{1g}(P) * \bar{n}_{1g}(\hat{P})\, \varphi_s(P)\, \varphi_{\hat{s}}(\hat{P}) \cdot \\
&\cdot \bar{\mathcal{W}}_g(P,\hat{P},q,\hat{q})\, dS(P)\, dS(\hat{P})]\, dq\, d\hat{q}
\end{aligned} \tag{5.2}$$

The dam response will be characterized in this view by the autocorrelation tensor of the normal acceleration coordinates,

$$\mathcal{W}_{r\hat{r}}(t,\hat{t}) = \overline{w_r(t)\, w_{\hat{r}}(\hat{t})} \tag{5.3}$$

which satisfies the double Volterra system derived from the interaction equation (4.9),

$$\begin{aligned}
\mathcal{W}_{r\hat{r}}(t,\hat{t}) &+ \Sigma_{\hat{r}'} \int_0^{\hat{t}} d_{\hat{r}\hat{r}'}(\hat{t}-\hat{q})\, \mathcal{W}_{r\hat{r}'}(t,\hat{q})\, d\hat{q} + \\
&+ \Sigma_{r'} \int_0^t d_{rr'}(t-q)\, \mathcal{W}_{r'\hat{r}}(q,\hat{t})\, dq + \\
&+ \Sigma_{r'\hat{r}'} \int_0^t\int_0^{\hat{t}} d_{rr'}(t-q)\, d_{\hat{r}\hat{r}'}(\hat{t}-\hat{q})\, \mathcal{W}_{r'\hat{r}'}(q,\hat{q})\, dq\, d\hat{q} = \mathcal{W}_{qr\hat{r}}(t,\hat{t})
\end{aligned} \tag{5.4}$$

6. Numerical Analysis of Interaction Problems

The analysis of either deterministic or stochastic interaction equations is to be carried out by means of computers only. Two kinds of data must be available for writing interaction equations:

- data on the natural modes, for dam and water respectively, and
- data on the disturbance characteristics.

The determination of a suitable number of natural modes, for dam and water both, is feasible, provided that a corresponding computer is available. The finite element method, [8], may be recommended for the analysis of both eigen-value problems.

The data on the characteristics of seismic disturbances should be derived on the basis of ground motion records and of theoretical developments on wave propagation phenomena. The actual miss of information in this field will oblige structural engineers to adopt some assumptions, having in view the geological features of the area.

The computational data once available, a step-by-step approach can be adopted for any of the equations (4.9) and (5.4).

The system (4.9), used in connection with the Monte-Carlo techniques, may be analyzed by means of a recursive finite sum method [5], made easy by the fact that both kernel and input-output quantities equal zero for zero argument. If the successive time moments $t_o=o$, $t_k=t_{k-1}+h$, $h=\text{const.}$, will be considered, this approach will be:

$$
\begin{aligned}
& w_r(t_1) = W_r(t_1) \\
& \dots\dots\dots\dots\dots \qquad\qquad (6.1) \\
& w_r(t_k) = W_r(t_k) - \Sigma_{\hat{r}} \Big[\sum_{l}^{1,k-1} d_{r\hat{r}}(t_{k-l})\, w_{\hat{r}}(t_l)\, h \Big]
\end{aligned}
$$

The solution of the system (5.4) can be obtained by means of similar techniques, generalized for a two-dimensional case:

$$
\begin{aligned}
& \mathcal{W}_{r\hat{r}}(t_1,t_1) = \mathcal{W}_{gr\hat{r}}(t_1,t_1) \\
& \dots\dots\dots\dots\dots\dots\dots\dots \\
& \mathcal{W}_{r\hat{r}}(t_k,t_{\hat{k}}) = \mathcal{W}_{gr\hat{r}}(t_k,t_{\hat{k}}) - \Sigma_{\hat{r}'} \sum_{\hat{l}}^{1,\hat{k}-1} d_{\hat{r}\hat{r}'}(t_{\hat{k}}-t_{\hat{l}})\, \mathcal{W}_{r\hat{r}'}(t_k,t_{\hat{l}})\, h - \\
& \qquad - \Sigma_{r'} \sum_{l}^{1,k-1} d_{rr'}(t_k-t_l)\, \mathcal{W}_{r'\hat{r}}(t_l,t_{\hat{k}})\, h - \qquad (6.2) \\
& - \Sigma_{r'\hat{r}'} \sum_{l}^{1,k-1} \sum_{\hat{l}}^{1,\hat{k}-1} d_{rr'}(t_k-t_l)\, d_{\hat{r}\hat{r}'}(t_{\hat{k}}-t_{\hat{l}})\, \mathcal{W}_{r'\hat{r}'}(t_l,t_{\hat{l}})\, h^2
\end{aligned}
$$

A step of computations will refer to the determination of the values $\mathcal{W}_{r\hat{r}}(t_k,t_{\hat{k}})$ for all positive values $k,\hat{k}$, satisfying the condition $k+\hat{k}=K=\text{const.}$, because this step is based on values of free terms, or of unknown quantities, already determined, for which $l+\hat{l}<K$.

7. Interaction in Case of a Gravity Dam

The case dealt with in this paragraph is that, of an infinitely long gravity dam. Developments presented herein are not based directly on the interaction equations derived previously, since solutions of eigen-value problems cannot be adopted in this case as a basis for analysis. The simple shape of the domain dealt with permits yet in this case the use of integral transformations with respect to time and space coordinates, and deriving of transfer functions for the phenomenon. Seismic disturbances are assumed in this view to be sine-functions of time and space variables.

The dam dealt with is a uniform section infinitely long gravity dam, with neglected beam stiffness, which can undergo displacements only as a result of ground displacements or compliance. The lake is represented by a half-infinite, constant depth, water layer (fig. 4a).

The seismic disturbance applied to the water consists of:
- vertical displacements of the lake bottom,

$$u_z(x,y,o,t) = u_{z\omega} \cos a(x-x_o) \cos by \cos \omega t \qquad (7.1a)$$

- horizontal displacements of the dam,

$$u_x(o,y,z,t) = u_{x\omega} \cos by \cos \omega t + \varphi_{y\omega} z \cos by \cos \omega t \qquad (7.1b)$$

The velocity potential will be assumed the expression

$$\Phi(x,y,z,t) = \text{Re}\, [\phi_{\omega y}(x,z)\, e^{i\omega t}] \cos by \qquad (7.2)$$

The function $\phi_{\omega y}(x,z)$ will satisfy the equation derived from (3.3),

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \nu^2 - b^2\right) \phi_{\omega y} = o \qquad (7.3)$$

and the boundary conditions derived from (3.4),

$$\phi_{\omega y}(x,H) = o \qquad (7.4a)$$

$$\frac{\partial \phi_{\omega y}}{\partial z}(x,o) = i\,\omega u_{z\omega} \cos a(x-x_o) \qquad (7.4b)$$

$$\frac{\partial \phi_{\omega y}}{\partial x}(o,z) = i\omega(u_{x\omega} + \varphi_{y\omega}\, z) \qquad (7.4c)$$

The solution of the problem ((7.3),(7.4)) is following:

$$\frac{1}{i\omega}\,\phi_{\omega y}(x,z) = -\frac{p_\omega(x,y,z)}{\omega^2 \rho_o \cos by} =$$

$$= \frac{4}{\pi} \sum_n^{1,\infty} \left[\frac{(-1)^{n-1}}{2n-1} X_n(x) \cos c_n z\right] u_{x\omega} +$$

$$+ \frac{4H}{\pi} \sum_n^{1,\infty} \left\{\frac{1}{2n-1}\left[(-1)^{n-1} - \frac{1}{c_n H}\right] X_n(x) \cos c_n z\right\} \varphi_{y\omega} +$$

$$+\left\{-\frac{1}{c\cos cH}\cos a(x-x_o)\sin c(H-z)+\right. \qquad (7.5)$$

$$\left.+\frac{2a}{H}\sin ax_o\sum_n^{1,\infty}\left[\frac{1}{c_n^2-c^2}X_n(x)\cos c_n z\right]\right\}u_{z\omega}$$

The solution (7.5) yields also the complex pressure amplitude, due to the relation (3.2d). Following symbols were adopted herein:

$$c_n = \frac{2n-1}{2}\frac{\pi}{H} \qquad (7.6a)$$

$$c^2 = \nu^2 - a^2 - b^2 \qquad (7.6b)$$

$$a_n^2 = \nu^2 - b^2 - c_n^2 \qquad (7.6c)$$

$$\hat{n} = \frac{1}{2} + \frac{H}{\pi}\sqrt{\nu^2 - b^2} \qquad (7.6d)$$

$$X_n(x) = \frac{1}{a_n}\sin a_n x - arb_n\, H\cos a_n x \quad (\text{for } n<\hat{n}) \qquad (7.6e)$$

$$X_n(x) = -\frac{1}{\sqrt{-a_n^2}}\exp(-\sqrt{-a_n^2}.x) \quad (\text{for } n>\hat{n}) \qquad (7.6f)$$

The case of integer $\hat{n}$ (i.e. the case when there exists an $a_{\hat{n}}=o$ according to (7.6a) and (7.6c)), appears as a resonant-like case, for which dissipative properties of water should be considered. The values of $\hat{n}$ and c must be real if rock wave velocities are greater than water pressure wave velocity.

The solution (7.5) is more general than solutions available in the literature, due to the occurrence of factors cos $a(x-x_o)$ and cos by, which permit an analysis of three-dimensional problems, with arbitrary characteristics of rock wave propagation. The solution contains arbitrary non-dimensional coefficients, arb_n, corresponding to free, stationary, pressure waves, not related to the seismic disturbance. Such waves could occur only in the case of missing viscosity. The coefficients arb_n could be determined in cases when data on dam displacements and on forces acting on the dam would be simultaneously available.

The complex amplitudes of resulting force, $F_\omega(y)$, and of resulting (overturning) moment, $M_\omega(y)$, per unit dam length (considered in the same sense as $u_{x\omega}$ and $\varphi_{y\omega}$ respectively) may be obtained by means of integration of the water pressure (7.5), along z:

$$-\frac{F_\omega(y)}{\omega^2\rho_o\cos by} = \int_o^H\frac{p_\omega(o,y,z)}{\omega^2\rho_o\cos by}dz = \frac{8H}{\pi^2}\left[\sum_n^{1,<\hat{n}}\frac{H}{(2n-1)^2}arb_n + \right.$$

$$\left.+\sum_n^{>\hat{n}}\frac{1}{(2n-1)^2}\frac{1}{\sqrt{-a_n^2}}\right]u_{x\omega} + \frac{8H^2}{\pi^2}\left[\sum_n^{1,<\hat{n}}\frac{H}{(2n-1)^2}\left(1-\frac{2(-1)^{n-1}}{(2n-1)}\right)arb_n + \right.$$

$$+\sum_{n}^{>\hat{n}} \frac{1}{(2n-1)^2} \left(1 - \frac{2(-1)^{n-1}}{(2n-1)}\right) \frac{1}{\sqrt{-a_n^2}}\Big] \varphi_{y\omega} + \left\{ \frac{1}{c^2} \left(\frac{1}{\cos cH} - 1 \right) , \right. \qquad (7.7)$$

$$\cos ax_o + \frac{a \sin ax_o}{\pi} \Big[\sum_{n}^{1,<\hat{n}} \frac{H}{c_n^2 - c^2} \frac{arb_n}{2n-1} + \sum_{n}^{>\hat{n}} \frac{1}{c_n^2 - c^2} \frac{1}{2n-1} \frac{1}{\sqrt{-a_n^2}} \Big] \Big\} u_{z\omega}$$

$$-\frac{M(y)}{\omega^2 \rho_o \cos by} = \int_o^H \frac{z\, p_\omega(o,y,z)}{\omega^2 \rho_o \cos by} dz = \frac{8\, H^2}{\pi^2} \Big[\sum_{n}^{1,<\hat{n}} \frac{H}{(2n-1)^2} \Big(1 -$$

$$- \frac{2(-1)^{n-1}}{(2n-1)} \Big) arb_n + \sum_{n}^{>\hat{n}} \frac{1}{(2n-1)^2} \left(1 - \frac{2(-1)^{n-1}}{(2n-1)}\right) \frac{1}{\sqrt{-a_n^2}} \Big] u_{x\omega} + \qquad (7.8)$$

$$+ \frac{8\, H^3}{\pi^2} \Big[\sum_{n}^{1,<\hat{n}} \frac{H}{(2n-1)^2} \left(1 - \frac{2(-1)^{n-1}}{(2n-1)}\right)^2 arb_n + \sum_{n}^{>\hat{n}} \frac{1}{(2n-1)^2} \Big(1 -$$

$$- \frac{2(-1)^{n-1}}{(2n-1)} \Big)^2 \frac{1}{\sqrt{-a_n^2}} \Big] \varphi_{y\omega} + \Big\{ \frac{1}{c^3} \frac{cH - \sin cH}{\cos cH} \cos ax_o +$$

$$+ \frac{4\, aH}{\pi} \sin ax_o \Big[\sum_{n}^{1,<\hat{n}} \frac{1}{c_n^2 - c^2} \frac{H}{2n-1} \left(1 - \frac{2(-1)^{n-1}}{(2n-1)}\right) arb_n +$$

$$+ \sum_{n}^{>\hat{n}} \frac{1}{c_n^2 - c^2} \frac{1}{2n-1} \left(1 - \frac{2(-1)^{n-1}}{(2n-1)}\right) \frac{1}{\sqrt{-a_n^2}} \Big] \Big\} u_{z\omega}$$

The water-dam interaction can be analyzed now by means of the equations of motion of the dam. They are written by neglecting the beam stiffness of the dam, and by using the expressions (7,7) and (7.8) for the dynamic effects of the water. The ground stiffness (defined for the case of dam mass centre located on the same vertical like the centre of the contact area) is characterized by a tensor [k], represented in fig. 4b. The equations of motion are in this case:

$$- \omega^2 (m\, u_{x\omega} + m\, h\, \varphi_{y\omega}) + k_{uu}(u_{x\omega} - u_{xg\omega}) + k_{u\varphi}(\varphi_{y\omega} - \varphi_{yg\omega}) - \frac{F_\omega(y)}{\cos by} = o \qquad (7.9)$$

$$- \omega^2 (m\, h\, u_{x\omega} + j\, \varphi_{y\omega}) + k_{\varphi u}(u_{x\omega} - u_{xg\omega}) + k_{\varphi\varphi}(\varphi_{y\omega} - \varphi_{yg\omega}) - \frac{M_\omega(y)}{\cos by} = o$$

where m represents the distributed mass of the dam, j - its mechanical moment of inertia with respect to the centre of the contact area, and h - the height of the mass centre above the contact area. The subscript g represents ground motion, not disturbed by the ground compliance, while the differences $u_{x\omega}-u_{xg\omega}$ and $\varphi_{y\omega}-\varphi_{yg\omega}$ represent the effect of ground compliance. The ground stiffness tensor [k] could be correctly derived as a function of ω and b, if the corresponding problem of half-space dynamics would be dealt with. After having solved the system (7.9), the total (base shear) force $F^S_\omega(y)$ and (overturning) moment $M^S_\omega(y)$ amplitudes, can be expressed as follows:

$$F^S_\omega(y) = F_\omega(y) + \omega^2(m\ u_{x\omega} + m\ h\ \varphi_{y\omega}) \qquad (7.10a)$$

$$M^S_\omega(y) = M_\omega(y) + \omega^2(m\ h\ u_{x\omega} + j\ \varphi_{y\omega}) \qquad (7.10b)$$

and, finally, as function of the quantities $u_{xg\omega}$, $\varphi_{yg\omega}$, and $u_{z\omega}$.

8. Conclusions

1. The equations of the seismic interaction between water and dam can be expressed in some relatively simple ways if the phenomenon is related to the natural vibration modes of dam and water, considered independently. There can be adopted either a form like (4.3), in which both kinds of coordinates occur, or forms like (4.5) and (4.6), in which only one kind of coordinates are used. The latter possibility is the more convenient, since it permits deriving of the Volterra integral systems (4,9) and (4.11), which can be easily handled.

2. The hypotheses adopted when writing the interaction equations (linearity, conservativity) are evidently idealizations. To improve this approach without cosiderably increasing difficulties, the dissipative nature of dam deformation, which appears to be the most important, could be considered, by means of substituting to the eigenvalues ω_r^2 complex values $\omega_r^2(1+i\gamma_r)$, where γ_r could depend on the disturbing circular frequencies ω, according to the rheologic laws of dam material.

3. The main computational difficulties arisen in investigating water-dam interaction phenomena, would be represented by the analysis of normal modes, and by the generation of free terms. The first kind of difficulty may be surmounted by means of a good computer, but the second one raises hard problems of dynamic seismology.

4. The normal modes of dam and water, used as a system of reference throughout the paper, could be replaced by means of any fundamental Ritz sequence. This way seems not to be advantageous, at least for theoretical analyses, since the more general expressions of parameters of interaction equations, occurring in latter case, would loose the simple sense of the expressions (4.1), (4.2), (4.4).

5. The seismic behaviour of simple structures can be investigated sometimes by means of analyzing their response to some simple types of disturbances. It is to be expected that quantitative results of sufficient reliability and accuracy concerning the interaction phenomenon could be obtained only on the basis of a stochastic approach, under non-stationarity assumptions. The necessity of this kind of analysis is due mainly to the features of seismic disturbances.

6. The main purpose of the use of stochastic approaches in problems of structural mechanics is that, of obtaining evaluations on the safety of structures dealt with, [4]. This paper was not concerned with safety evaluations, but only with the deterministic and stochastic aspects of the mechanical phenomenon of linear, conservative, dam-water interaction. Safety analysis should be based, beside the mechanical analysis, on a convenient representation of the seismic activity of the area, by means of stochastic techniques. Conceptions presented in [6] could be recommended in this view.

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References

1. V. V. BOLOTIN Statistical Methods in Engineering Mechanics (in Russian), 2-nd Ed., Moscow, 1965.

2. A. K. CHOPRA Hydrodynamic Pressures on Dams during Earthquakes. Rept. No. 66-2, Structural Engng. Lab. University of California.

3. " Reservoir-Dam Interaction during Earthquakes Bull. S. S. A., 4, Aug., 1967.

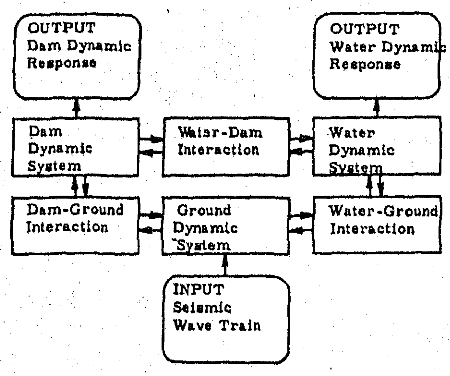
4. E. ROSENBLUETH Probabilistic Design to Resist Earthquakes Proc. A.S.C.E. em 5, 1964.

5. H. SANDI A Theoretical Investigation of the Interaction between Ground and Structure during Earthquakes. Proc. 2-nd W.C.E.E., Tokyo, 1960.

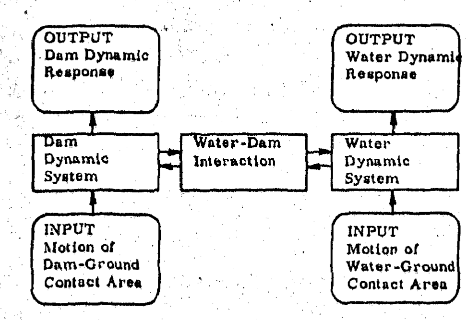
6. " Earthquake Simulation for the Estimate of Structural Safety. Proc. R.I.L.E.M. Symp. on the Effects of Repeated Loading of Materials and Structural Elements, Mexico City, 1966.

7. H. SANDI, D. DRAGOMIR, I. TOMA Seismic Vibrations of Arch Dams Proc. 9-th International Congress on Large Dams, Istanbul, 1967.

8. O. ZIENKIEWICZ The Finite Element Method in Structural and Continuum Mechanics Mc. Graw-Hill, London, 1967.

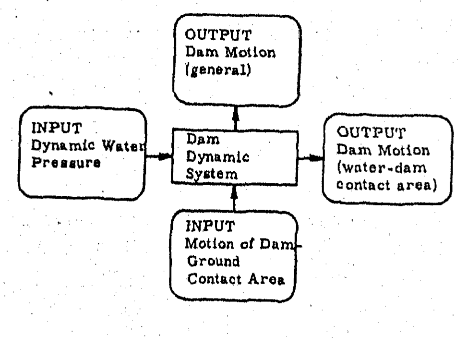


a. Actual Ground-Dam-Water Interaction

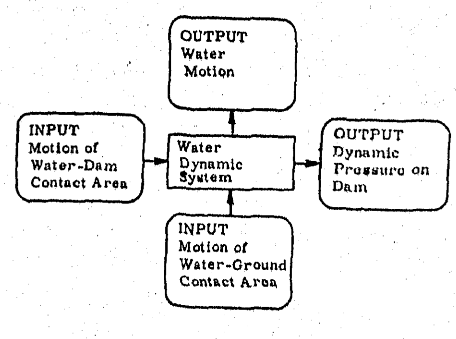


b. Idealized Dam-Water Interaction

Fig. 1 Idealization of the Interaction Problem

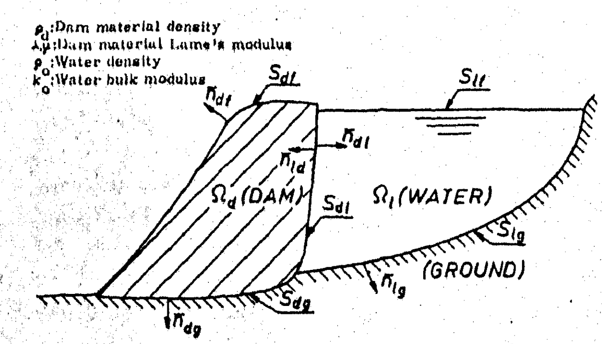


a. Dam Analysis

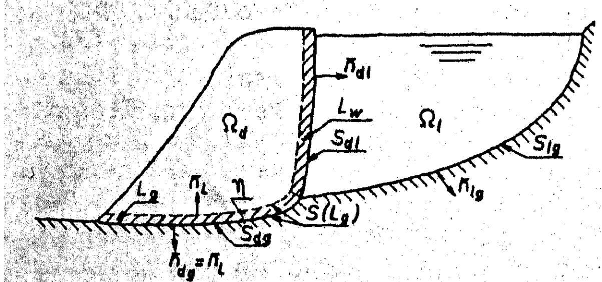


b. Water Analysis

Fig. 2 Independent Analysis of Basic Dynamic Systems

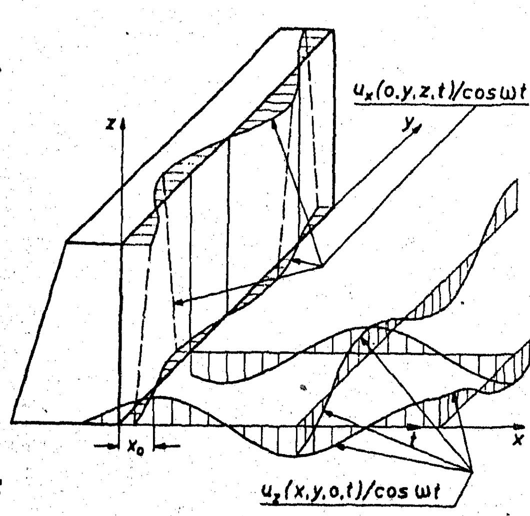


a. Domains and Boundaries Dealt with

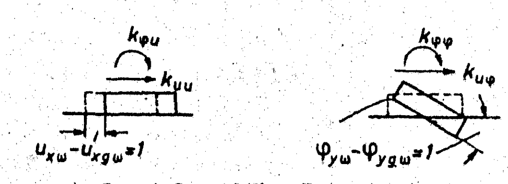


b. Layers Considered for Deriving the Expressions (2.11), (2.12), (2.10')

Fig. 3 Symbols for the Boundary-value Problems Dealt with



a. Horizontal and Vertical Disturbing Motions



b. Dynamic Ground Stiffness Factors k_{uu}, $k_{u\varphi}$, $k_{\varphi u}$, $k_{\varphi\varphi}$

Fig. 4 Symbols for the Gravity Dam Interaction Problem