"EARTHQUAKE RESPONSE ANALYSIS AND ASEISMIC DESIGN OF CYLINDRICAL TANKS".

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SYNOPSIS

A brief summary of the theory of ideal fluid dynamics of cylindrical tanks is given. The water-structure system theory is presented and a study of uncompling is developed. The theory is applied to several laboratory and full-scale tests'results and a very acceptable agreement is obtained.

The important parameters of real systems are determined and an earthquake-resistant design method is proposed for practical applications.

1.1.- Introduction.-

An equivalent mechanical system of elevated water tanks that permits the use of the earthquake response spectra theory is considered. This makes it necessary to know the behaviour of the water contained inside the tank, since its influence on the total response of the system is fundamental.

It is not the scope of this work to make a detailed analysis of the theoretical basis that allow to know this behaviour. References 1,2,3, contain detailed results of the theory mentioned above. First stops in theory, experimental studies, and laboratory tests are mentioned in references 4 to 8.

Several simplifying assumptions must be accepted to get a mathematical expression of the phenomenum. The liquid is a perfect fluid, its motion is irrotational and acyclic. The container has only one degree-of-freedom, because its rotation is ignored; its bottom is plane and horizontal. Displacements of the liquid and container are small.

This study is circunscribed to circular cylindrical tanks. The mechanical system is linear, which reduces the applicability of the theory only to constant-stiffness tanks.

2.2.- Free oscillations.-

The container shown in fig.1 is at rest and the liquid is vibrating freely.

The fluid has an infinite number of normal modes the displacement of the free surface, for a point $(r, \theta, 0)$ is given by:

$$\eta_i = - A_i p_i \cos\theta J_i(\lambda_i h/_a) \cosh(\lambda_i h/_a) \operatorname{sen}(p_i t - \xi_i) \qquad (1.1)$$

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where J_1 is the Bessel function of first kind and first order, λ_i are the zeros of J_i, A_i and A_i, ξ_i are parameters of each mode. p_i is called the frequency of the mode and is obtained from:

$$p_i^2 = \frac{g}{a} \lambda_i \ tgh (\lambda_i h / a) \qquad (i=1,2,\ldots,n) \qquad (1.2)$$

Table 1 gives the first five values of λ_i. It is noted that for large i, $\lambda_i \rightarrow \pi(i - \frac{1}{4})$.

Fig. 2 shows the shapes of the first three normal modes.

1.3.- <u>Reduced mass theory</u>.-

The tank has moved $\Delta_u(0)$ in the OX direction in Δ_t seconds. The $\bar{F}_0$ fluid force against the container walls is:

$$\bar{F}_0 = MR_0 \frac{dv(0)}{dt} \qquad (1.3)$$

in which M represents the total mass of liquid.

At the bottom of the tank there is an overturning moment due to liquid pressures against walls and bottom given by:

$$\bar{M}_0(0) = Mh(R_0 + 2S_0 - \tfrac{1}{2}) \frac{dv(0)}{dt} \qquad (1.4)$$

R_0 and S_0 are determined from:

$$R_0 = 1 - \sum_1 \frac{2\, tgh(\lambda_i h/a)}{\lambda_i(\lambda_i^2 - 1)\, h/a} \qquad S_0 = \sum \frac{2[1 - sech(\lambda_i h/a)]}{\lambda_i^2(\lambda_i^2 - 1)(h/a)^2} \qquad (1.5)$$

It is seen that $\bar{F}_0$ is only R_0 times the force that would be if the tank was full with a solid of the same specific weight. R_0 is called by Prof. Arturo Arias "reduced mass factor" (Ref.8) and describes in a cuantitative way the effects produced by the tank displacement on the liquid at the beginning of the motion.

The $R_0(h/a)$ function is plotted in fig.3. Its extreme values are $R_0(0)=0$ (empty tank) and $R_0(\infty)=1$ (tall and slender tank or roof confined tank)

The $\bar{M}_0(h/a)$ function is plotted in Fig.4 and it shows a minimum value at approximately $h/a=1,70$.-

h_0 is a parameter such that $\bar{F}_0 h_0$ gives the overturning moment $\bar{M}_0$, then:

$$h_0/h = (R_0 + 2S_0 - \tfrac{1}{2}) / R_0 \qquad (1.6)$$

Fig. 5 is a plot of: h_0/h curves.

1.4.- The hydrodynamical oscillators.-

The force exerted by the liquid against the container when the tank moves in a continuous way, u(t), in the interval (0,t) is given at the time t:

$$\bar{F} = M R_0 \ddot{u}(t) + \sum_i M R_i p_i I_s(i,r) \qquad (1.7)$$

where R_i is defined as follows:

$$R_i = \frac{2 \operatorname{tgh}(\lambda_i h/a)}{\lambda_i (\lambda_i^2 - 1) h/a} \qquad (i = 1,2,\ldots,n) \qquad (1.8)$$

and I_s a Duhamel type integral .

$\bar{F}$ can be considered as a summation of n+1 terms. The first, $\bar{F}_0 = MR_0 \ddot{u}(t)$ corresponds to the restitution force exerted upon an infinitely rigid oscillator with mass MR_0. The n remaining terms in (1.7) correspond to restitution forces of n elastic oscillators which have a natural frequency p_i and a mass $m_i = MR_i$.-

It is concluded that the effects of the liquid upon the container can be replaced, for dynamical purposes, by that of n+1 simple oscillators. Only one of those, whth a mass MR_0, is rigidly connected to the tank; the others are elastic and their stiffnesses are given by

$$k_i = m_i p_i^2 = M R_i p_i^2 \qquad (i = 1,2,\ldots,n) \qquad (1.9)$$

Each elastic oscillator (i=0) is related through p_i with a normal mode of vibration of the liquid and for this reason will be designated "hydrodynamical oscillators".

The $R_i(h/a)$ curves are plotted in Fig.3 for i= 1,2,3. From Table 2 is found that the summation of these terms is well representated by R_1+R_2.

The hydrodynamical and reduced mass factors satisfy:

$$R_0 + \sum_i R_i = 1 \qquad (1.10)$$

Which means,as it could be expected,that the summation of the masses of all the oscillators is equal to the mass of the liquid.

In ref.12, 13 expresions for the overturning moment $\bar{M}_i$ are given when the pressures upon bottom and walls are considered.

Fig. 5 presentes curves h_i/h for i=1 and 2 as a function of h/a.The parameter h_i is defined in such a way that $\bar{F}_i h_i$ is equal to the overturning moment corresponding to the i^{th} mode.

1.5.- The water-structure system.-

The equation of motion for an elevated tank is given by:

$$m\ddot{x}_0 + c(\dot{x}_0 - \dot{u}) + k(\bar{x}_0 - u) = -MR_0 \ddot{x}_0 - \sum_i m_i \ddot{x}_i \qquad (1.11)$$

where u(t) is the displacement of the foundation at time t, m is the mass of the container, c the viscous damping coefficient and k the constant stiffness of the tank supporting structure.

Using relative coordinates, relation (1.11) becomes:

$$m_o \ddot{x}_o + c_o \dot{x}_o + (k_o + \sum_i k_i)\, x_o - \sum_i k_i x_i = -m_o \ddot{u} \qquad (1.12)$$

where $m_0 = m + MR_0$ and k,c have been changed to k_0, c_0.

The equation for the i^{th} hydrodynamical oscillator is:

$$m_i \ddot{x}_i + k_i x_i - k_i x_o = -m_i \ddot{u} \qquad (i=1,2,\ldots,n) \qquad (1.13)$$

The water-structure system is represented by relations (1.12) and (1.13), which can be replaced by a matrix equation:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [\bar{K}]\{x\} = -\ddot{u}\{m\} \qquad (1.14)$$

The elements of the mass, damping and stiffness matrices are presented in Fig.7. The system is statically coupled and has n+1 degrees-of-freedom: one for the displacement of the container and n more for the modes of vibration of the water.

For $[C] \equiv 0$, normal modes and principal oscillators are defined. The frequency equation is:

$$k_o - m_o \omega^2 = \omega^2 \sum_i \frac{k_i m_i}{k_i - m_i \omega^2} \qquad (1.15)$$

When n=2, and only the first two modes of vibration of the water are considered, the frequency equation becomes: (1.16)

$$\omega^6 - [p_o^2 + (1+\eta_1) p_1^2 + (1+\eta_2) p_2^2]\,\omega^4 + [(p_1^2 + p_2^2)\, p_o^2 + (1+\eta_1+\eta_2)\, p_1^2 p_2^2]\,\omega^2 - p_o^2 p_1^2 p_2^2 = 0$$

where:

$$p_o^2 = k_o/m_o \qquad \eta_2 = m_2/m_o \qquad \eta_1 = m_1/m_o \qquad (1.17)$$

when n=1, and all the modes of vibration of the water superior to the first are disregarded, the frequency equation becomes:

$$\omega^4 - [p_o^2 + (1+\eta_1) p_1^2]\,\omega^2 + p_o^2 p_1^2 = 0 \qquad (1.18)$$

The general expression for the masses of the representative oscillators is:

$$M_r = \frac{K_o^2}{\omega_r^2}\, \frac{\psi_1(r)\cdot\psi_2(r)\cdots\psi_n(r)}{D(r,n)} \qquad (r=0,1,2,\ldots,n) \qquad (1.19)$$

where:

$$\psi_i(r) = k_i - m_i \omega_r^2 \qquad (r=1,2,\ldots,n) \qquad (1.20)$$

$$D(r,1) = k_o k_1 - m_o m_1 \omega_r^4 \qquad (1.21)$$

and

$$D(r,2) = k_o k_1 k_2 - [(k_2 m_1 + k_1 m_2)\, m_o + (k_o + k_1 + k_2)\, m_1 m_2]\,\omega_r^4 + 2\, m_o m_1 m_2\, \omega_r^6 \qquad (1.22)$$

The mechanic water-structure system is completely defined when the frequency equation (1.15) and the masses of the representative oscillator (1.19) are known.

It sufices, for practical purposes, as it will be shown later, to considerer only the fundamental mode of the water. This simplifies the expressions of the representative frequencies and masses notably.

2.- Comparison between theoretical and experimental results.-

2.1.- Works of Jacobsen.-

Many experiments using models of cilindrical containers on a vibrating table were done under the direction of Prof. Lydik S.Jacobsen at the University of Stanford[14].

The reaction and overturning moment of the fluid due to am impulsive displacement of the container were determined in those experiments. The experimental and theoretical results practically coincide, as it can be seen in Fig. 10 of Ref. 13.

The shape of the free surface in the plane of symmetry of the motion was also obtained. It was found that these shapes follow the Bessel function of first kind and first order, when the water is vibrating freely.

2.2.- Period Measurementes in elevated water tanks.-

The U.S. Coast and Geodetic Survey made an important study[15] of the dynamical characteristics of water tanks during the year 1934. Unfortunately, not enough data was published and only a quantitative comparison with the dynamic theory can be done.

Displacement curves due to several dynamic excitations were obtained: small vibrations, generated by wind or transit of vehicles ("spontaneous oscillations") and free oscillations generated using "pull-back" tests.

Carder (16) observed that the recordings were not purely sinusoidal but combinations of two and even three sinusoidal waves of different periods: a fundamental period, always present, varying between 1.0 and 1.7 secs. that seemed to depend on the elastic properties of the structure, volume of water and foundation conditions, a second period ranging between 0.20 and 0.45 secs. independent of the water volume and related to the torsional characteristics of the tank, and longer periods, of the order of 3 secs., that appeared in pull-back tests and can be attributed to motions of the water inside the container.

Applying the theory and neglecting torsional oscillations, the systems remains with two principal degrees-of-freedom: one in which the motion of the tank as a whole is preponderant ($M+MR_0$), and a second where the motion of the water is important (m_1).

2.3.- Period Measurements in Chile.-

A series of period measurements of different structures was performed in Chile, after the May 1960 earthquakes. Some of the structures were analyzed by W.K.Cloud[17]. It is of special interest the CAP water tank that Cloud used to apply Housner's simplified theory. He didn't obtain the period of the third mode, because Housner's theory only considers two degrees-of-freedom for the water-structure system. The characteristics of his oscillators are given only approximately, since no use of Bessel functions is made.

Using the results of the theory (relation 1.19) with the data of Cloud, three periods were obtained, and they differ from the experimentally measured in less than 4%.

Patricio Meller[18] studied the dynamical characteristics of almost all the important water tanks in Santiago. For steel tanks, theory and tanks the discrepance is larger, but still acceptable if consideration is made of the fact that the structure is not linear and the theory is not directly applicable.

2.4.- A.C.Ruge's experiments.

A.C. Ruge made many experiments with a model of an elevated steel water tank between 1934 and 1936 (Ref.20). Free and forced vibration test were done.

From free oscillation tests, Ruge concluded that the motion of the tank was composed by two compled vibrations: one corresponding to the tank and part of the liquid and the other only due to water displacements.

It is noted that a good qualitative agreement between theory and experience was obtained.

Ruge found periods of 0.39 and 0.24 secs. for the model that correspond to 2.7 and 1.6 secs. in the prototype. With Ruge's data the frequency equation (1.21) gives periods of. 0.2406 and 0.3927 secs.

By numerical integration of the equation of motion (1.14), fig.8 was plotted. It is seen that theoretical peaks agree very well with the experimental ones.

Fig. 9 shows Ruge's results from forced oscillations with cosinusoidal excitation. By numerical integration several theoretical points were computed and added to the figure. A good agreement is appreciated. Discrepances for certain points are discussed in ref. 12. It is noted that there appear extreme values for T=2.7 and 1.6 secs. which are the first two natural periods of the system as it should be expected.

3.- "EXACT" AND "APROXIMATE" RESPONSES.-

The "exact" theory is not practical because equations 1.12 and 1.13, must be solved numerically. A modal analysis, on the other Hand, is applicable but it requires the existence of normal modes. When damping is considered, normal modes will exist if the damping matrix can be expressed as a Caughey series.

Modal analysis was used for the particular case when [C] is a linear combination of [M] and [K] . This implies small amounts of viscous damping in the liquid. In this case it can be shown that [C] is proportional to [K] and the fraction of critical damping for each mode is given by:

$$\beta_r = \beta \omega_r : p \qquad (3.1)$$

where is the fraction of critical damping and p is the frequency of the empty tank.

The base shear ("exact" response) for the dynamic system, using notation of. fig. 6, is given by:

$$\bar{V}(t) = k_o x_o(t) \qquad (3.2)$$

Using normal damped modes, the base shear ("aproximate" response) is

$$\bar{V}(t) = \sum_r \{ K_r I_s(r, \beta_r) : \omega_r \} \qquad (3.3)$$

where: ω_r, β_r are the frequency and fraction of critical damping of the r normal mode, K_r is the stiffness of the r^{th} representative oscillator ($K_r = \bar{M}_r \omega_r^2$) and I_s (r, β_r) is a Duhamel damped integral.

For two real tanks and three earthquake accelograms the "exact" and "aproximate" responses were obtained by numerical integration using a third order Runge-Kutta method. The results are presented in Table 3. A comparison of the "exact" and "aproximate" responses shows that the differences are less than 4% and that the extreme values are obtained practically at the same instant.

It is known that the maximum response lies between the quadratic and the absolute methods of superposition for the modal responses(ref.24). Several methods have been proposed to obtain an estimation for the maximum response using modeal analysis. In this work the Arias-Husid superposition method has been used (ref.25.26)

In table 3, responses for the first three normal modes are given together with the quadratic, absolute and Arias-Husid superposition methods.

It is verified that the maximum response lies between the quadratic and absolute superpositions for the six cases considered. It is also seen that the Arias-Husid method has a very good agreement with the exact response.

4.- Evaluation of a practical method of dynamic analysis.-

The results of Table 3 show clearly that the second mode of the water can be disregarded (i=2). It is only necessary to work with two oscillattors: one rigid with mass $m+MR_0$ and another elastic with mass m_1.

A practical method of dynamic analysis has to follow the steps described below:

- Characteristics of the oscillators of the system.
  frequency : see eq. (1.2)
  mass : obtain R_0 and R_1 from fig. 3
  stiffness : see eq. (1.9)

- Characteristics of the water-structure systems.
  frequency : see eq. (1.18)
  mass : see eqts. (1.19 and 1.21)
  stiffness : $K_r = M_r \omega_r^2$

- Base shear. For each mode obtain the corresponding spectrum response: $\bar{V}_r$ máx.$=K_r S_D(r, \beta_r)$.
  where $S_D(r, \beta_r)$ is the displacement spectre selected for the site where the tank is located. Considering that acceleration, velocity and displacement spectre are aproximately relatod (ref.23), $\bar{V}_r$ máx can be expressed as a function of S_A or S_V.

  The base shear $\bar{V}$ máx is found as a linear combination of the quadratic and absolute methods of modal superposition.

- Overturning moment. The evaluation of the overturning moment is laborius and the complete analytical derivation can be found in ref. 12. It is suggested a simplified formule.

$$\bar{M} \text{ máx} = \bar{V} \text{ máx.} (l_i + \tfrac{1}{2} h) \quad (4.1)$$

where l_i the distance between foundation and tank bottom.

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TABLE # 1

i	1	2	3	4	5
λ_i	1.84119	5.33145	8.53633	11.70600	14.86359
$\lambda_i : \pi$	0.58607	1.69706	2.71720	3.72614	4.73123

TABLE # 2

h/a	R_o	R_1	R_2	R_3	R_4	R_5	$\frac{R_1+R_2}{1-R_o}$	$\frac{R_1}{1-R_o}$
0.00	0,0000	0.8371	0.0729	0.0278	0.0147	0.0091	0.9100	0.8371
0.10	0.0538	0.8277	0.0667	0.0226	0.0104	0.0055	0.9452	0.8747
0.20	0.1131	0.8012	0.0539	0.0153	0.0062	0.0030	0.9641	0.9033
0.35	0.2045	0.7377	0.0373	0.0093	0.0036	0.0018	0.9742	0.9273
0.50	0.2981	0.6603	0.0271	0.0065	0.0025	0.0013	0.9794	0.9408
0.75	0.4393	0.5342	0.0182	0.0044	0.0017	0.0008	0.9852	0.9527
1.00	0.5477	0.4324	0.0137	0.0033	0.0013	0.0006	0.9863	0.9560
1.25	0.6276	0.3565	0.0109	0.0026	0.0010	0.0005	0.9867	0.9574
1.50	0.6861	0.3007	0.0091	0.0022	0.0008	0.0004	0.9869	0.9579
2.00	0.7630	0.2271	0.0064	0.0016	0.0006	0.0003	0.9870	0.9582
2.50	0.8102	0.1819	0.0055	0.0013	0.0005	0.0002	0.9871	0.9583
3.00	0.8419	0.1516	0.0046	0.0011	0.0004	0.0002	0.9871	0.9583
3.50	0.8644	0.1299	0.0039	0.0009	0.0004	0.0002	0.9872	0.9583
4.00	0.8814	0.1137	0.0034	0.0008	0.0003	0.0001	0.9872	0.9584

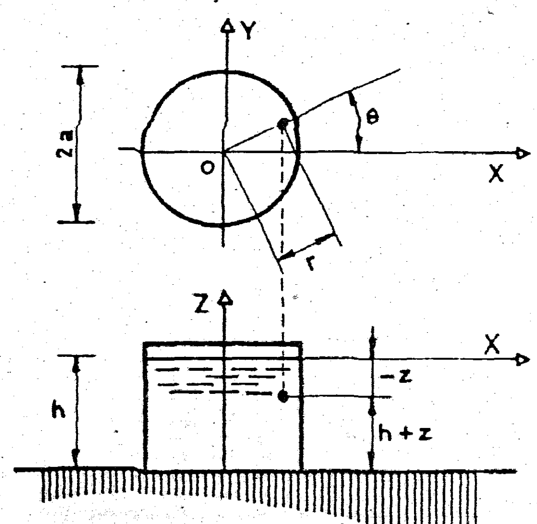

FIG. 1

T A B L E # 3.

BASE SHEARS (METRIC TONS)

TANK	EARTH QUAKE	"EXACT" RESP.	APROX. RESP.	DEV. %	NORMAL MODES			ABSOLUTE SUP.	CUADRATIC SUP.	ARIAS-HUSID SUP.	
					FIRST	SECOND	THIRD			2 GRADES	3 GRADES
RUGE	EC 40	70,927	69,920	-1.42	45,425	28,643	4,976	79,044	53,931	73,469	71,962
	Oly 49	43,374	41,672	-3.92	35,293	16,195	2,195	53,683	38,893	50,400	49,512
	Taft 52	55,627	54,589	-1.87	46,459	8,746	2,731	57,936	47,354	55,587	54,952
FACH	EC 40	72,620	71,569	-1.46	56,462	17,635	0,823	74,920	59,158	71,421	70,475
	Oly 49	29,163	28,887	-0.96	19,767	12,246	0,761	32,764	23,265	30,655	30,085
	Taft 52	12,474	12,451	-0.16	11.366	4,323	0,893	16,582	12,193	15,608	15,344

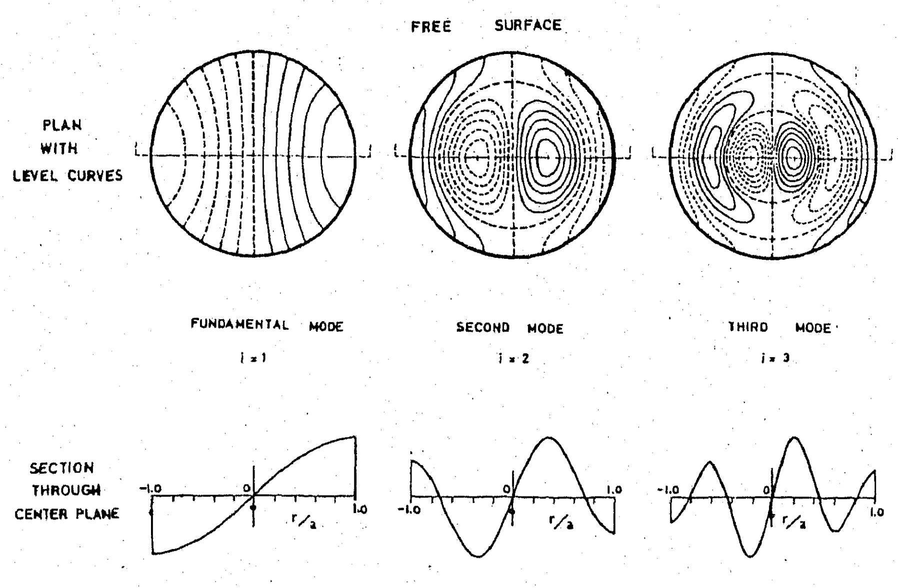

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FIG. 2 FREE OSCILLATIONS OF A PERFECT LIQUID

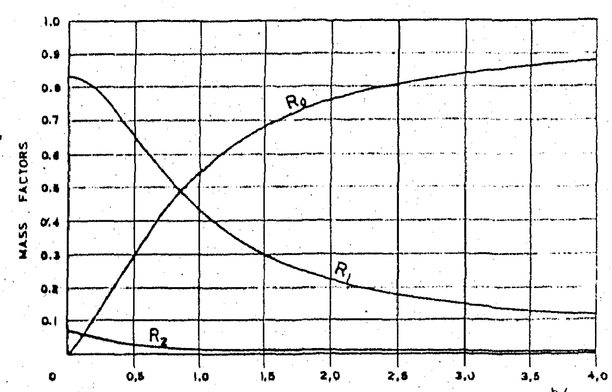

FIG. 3

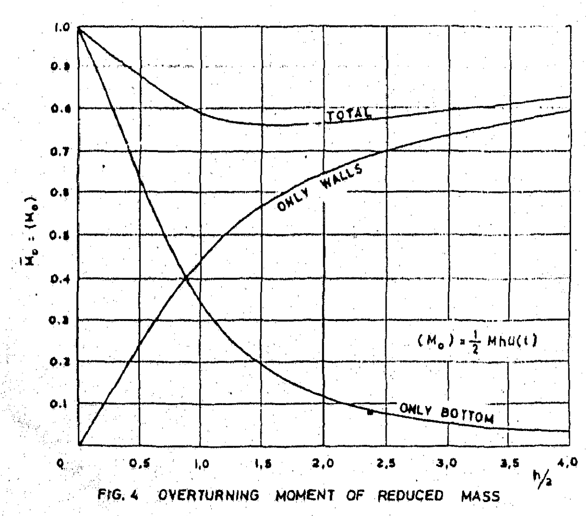

FIG. 4 OVERTURNING MOMENT OF REDUCED MASS

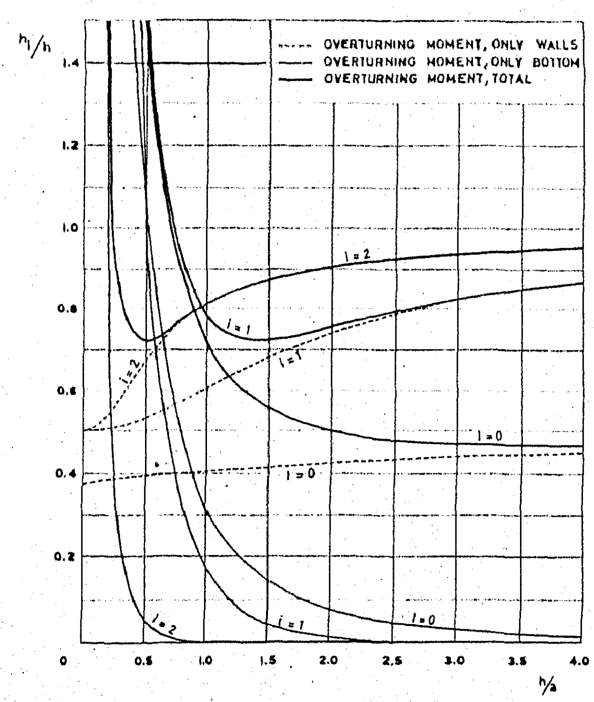

FIG. 5 RELATIVE HEIGHTS OF HIDRODINAMICAL OSCILLATORS

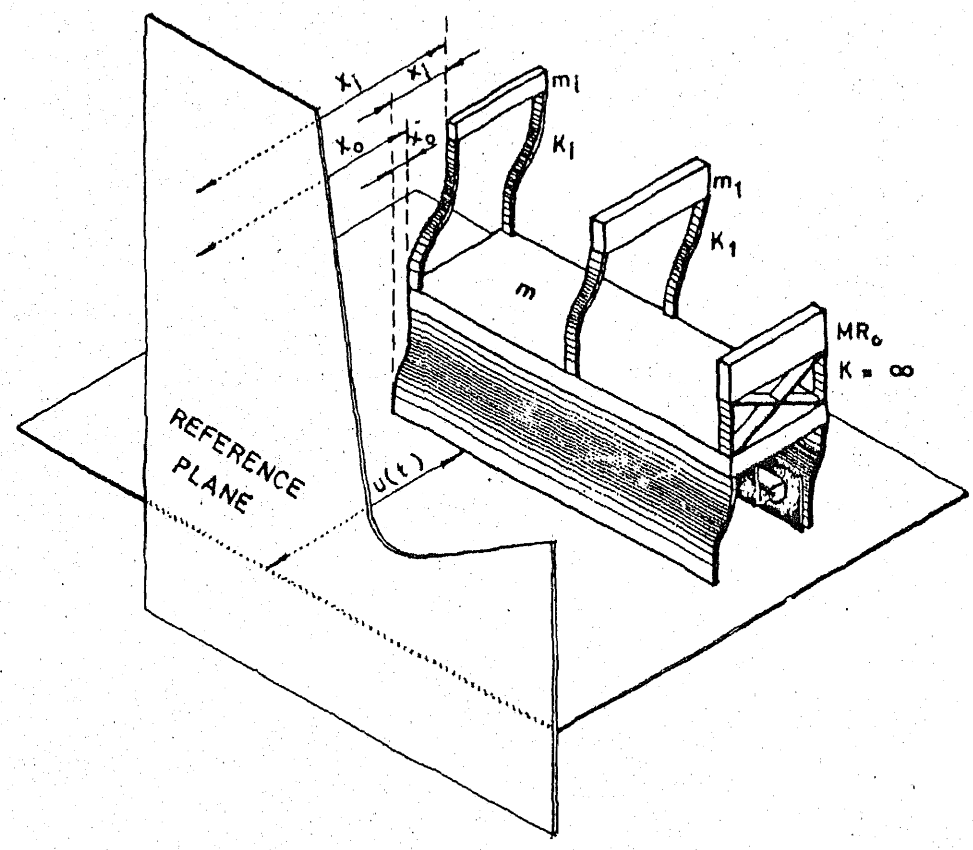

FIG. 6 DINAMIC REPRESENTATION OF WATER-STRUCTURE SYSTEM

$$[M] = \begin{vmatrix} m_0 & 0 & 0 & \dots & 0 \\ 0 & m_1 & 0 & \dots & 0 \\ 0 & 0 & m_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & & m_n \end{vmatrix} \qquad [C] = \begin{vmatrix} (c_0) & -c_1 & -c_2 & \dots & -c_n \\ -c_1 & c_1 & 0 & \dots & 0 \\ -c_2 & 0 & c_2 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -c_n & 0 & 0 & & c_n \end{vmatrix} \qquad [\bar{K}] = \begin{vmatrix} (k_0) & -k_1 & -k_2 & \dots & 0 \\ -k_1 & k_1 & 0 & \dots & 0 \\ -k_2 & 0 & k_2 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -k_n & 0 & 0 & \dots & k_n \end{vmatrix}$$

$$(c_0) = c_0' + \sum_i c_i \qquad (k_0) = k_0 + \sum_i k_i$$

$$c_i \to 0 \;\; si \;\; i \neq 0$$

FIG. 7

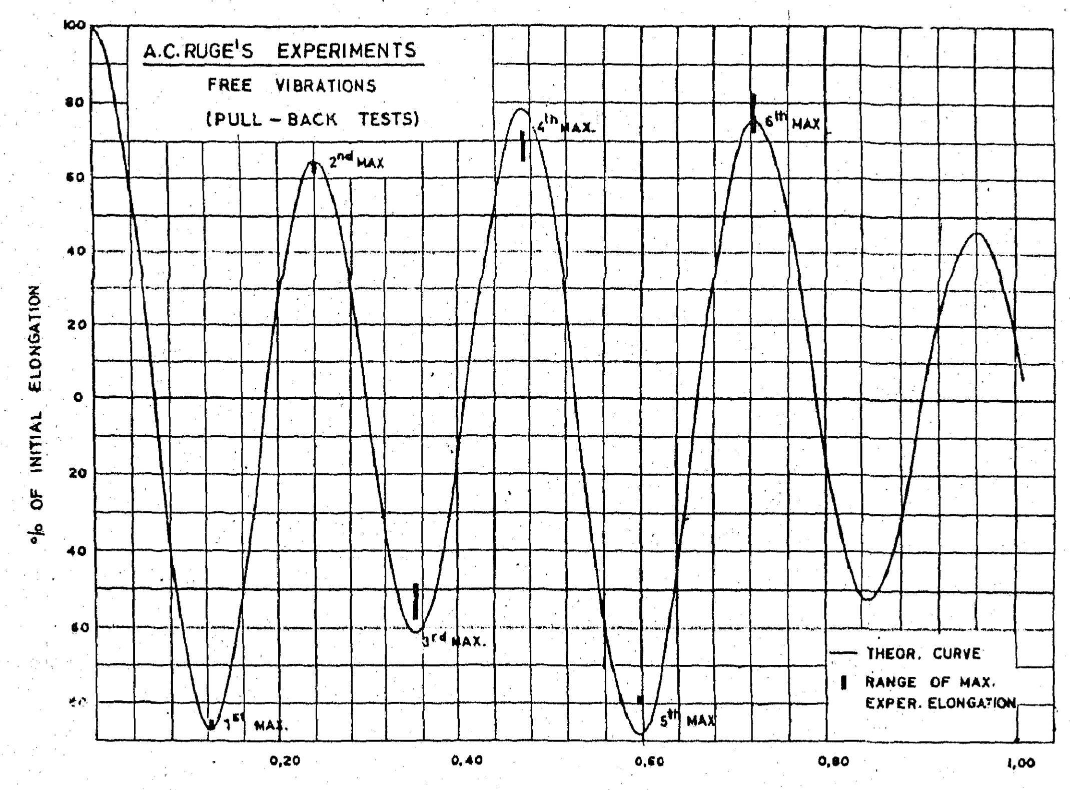

FIG. 8 DURATION OF FREE VIBRATIONS (SECS)

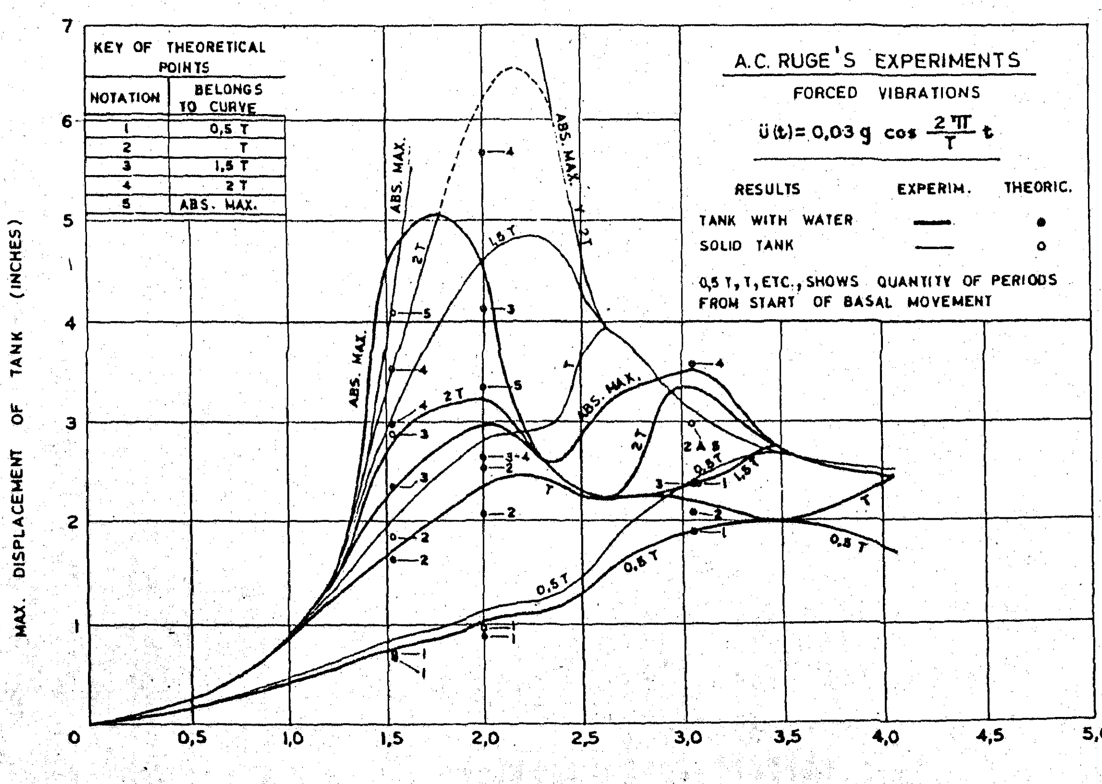

FIG. 9 PERIOD OF BASAL DISPLACEMENT (SECS.)