DYNAMICS OF EXTENDED-IN-PLAN STRUCTURES IN STRONG EARTHQUAKES

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Abstract

Oscillations of structures such as masts, supports of power transmission lines, large-span industrial buildings and higher extended-in-plan buildings in strong earthquakes are considered.

The equations of motion of the dynamic systems under consideration are expressed in the absolute system of coordinates. The horizontal component of the displacement (acceleration) of the foundation soil is assumed to be a stationary random function of time having the Gaussian distribution.

In the structures under consideration the distances between their supports are commensurable with the length of the running seismic wave. The spatial correlation is therefore taken into account between the displacements (accelerations) of the foundation soil at locations of structural supports.

Introduction

At present the design of structures and buildings for the seismic effect is based upon a design scheme in the form of a cantilever rod elastically restrained in the ground base, with masses concentrated in the centre of the zones of the structure or at the floor levels of buildings.

Such a scheme is quite reasonable for structures which dimensions in plan are significantly smaller than the lengths of seismic waves. The behaviour of extended-in-plan structures and buildings (such as bridges, support-wire system of power transmission lines, tall guyed masts, one-storey multi-spanned buildings and higher buildings which present a system of vertical diametric walls bound by

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floors rigid in the horizontal plane) with dimensions commesurable with lengths of seismic waves in earthquakes will differ from that of less extended structures.

On the one hand it is typical of such structures that when a seismic wave passes under the structure, the points of its ground base move with some lag of phases in time. On the other hand, such structures can produce both translational and translational-rotational movements depending on the angle between the direction of the displacement of soil particles and the direction of the seismic wave propagation. It is clear that these specific features are not reflected by the design scheme in the form of a cantilever rod.

G.W.Housner /8/ was the first to have paid attention to the allowance for extension in the studies of earthquake-resistance of buildings and structures. Vibrations of such structures under the assumption that displacements and acceleration of soil particles of the ground base are the given functions of time were further studied by Sinitsyn A.P. /7/, Korchinsky I.L. /5/ and Grossman A.B. /4/.

The present paper considers the problem of vibrations of extended-in-plan buildings and structures with the probabilistic approach.

I. <u>Statistical characteristics of displacement and acceleration of the ground base of the structure at two points located along the line of the propagation of the seismic wave.</u> If $a_1(t)$ is a stationary random process of the displacement of the ground base at some point 1, then at point 2 being at a distance x from point 1 along the line of the wave propagation the process $a_2(t)$ differs only by the time lag $\tau_1 = \frac{x}{c}$, where c-the velocity of the seismic wave propagation.

Owing to the stationary nature of processes, the correlation function $B_{a_2}(\tau,0)$ is equal to $B_{a_1}(\tau_1,0)$, and the function of cross-correlation of processes for points 1 and 2 differs from the correlation function by a time lag τ_1, too.

Applying the well-known expression for the cross-spectral density of processes at input and output of the dynamic system /1.6/, we find that the cross-spectral density of the processes at point 1 and 2 is

$$S_{a_1a_2}(\omega) = S_{a_1}(\omega)\,\Phi(i\omega) = S_{a_1}(\omega)\,e^{-i\omega\tau_1} \qquad (1.1)$$

where $S_{a_1}(\omega)$ is the spectral density at point 1,
$\Phi(i\omega)$ is the transfer function of the system.

Hence the function of cross-correlation

$$B_{a_1a_2}(\tau)=\overline{a_1(t)a_2(t+\tau)}=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{a_1}(\omega)e^{i\omega(\tau-\tau_1)}d\omega=$$

$$=\int_{-\infty}^{\infty}S_{a_1}(\omega)\cos\omega(\tau-\tau_1)d\omega \qquad (1.2)$$

Fig.1 presents normalized correlation functions and spectral densities $R_a(\tau)$ and $S_a^{H}(\omega)$ for four seismograms recorded in two regions of the USA at different time; curves of type $R_a(\tau)=e^{-\alpha|\tau|}(\cos\beta\tau+\frac{\alpha}{\beta}\sin\beta|\tau|)$. are plotted by a dashed line.

Spectral densities are computed by the formula:

$$S_a^{H}(\omega)=\frac{4\alpha m^2}{\omega^4+2a\omega^2+m^4} \qquad (1.3)$$

where α and β are parameters of the correlation function; $a=\alpha^2-\beta^2$; $m^2=\alpha^2+\beta^2$;

Substituting (1.3) into (1.2), we obtain (1)

$$B_{a_1a_2}(\tau)=\sigma_{a_1}^2\frac{4\alpha m^2}{2\pi}\int_{-\infty}^{\infty}\frac{\cos\omega(\tau-\tau_1)d\omega}{\omega^4+2a\omega^2+m^4}=$$

$$=\sigma_{a_1}^2e^{-\alpha|\tau-\tau_1|}\left[\cos\beta(\tau-\tau_1)+\frac{\alpha}{\beta}\sin\beta|\tau-\tau_1|\right] \qquad (1.4)$$

For $\tau=0$ expression (1.4) defines the variation of the mean square of the ground base displacement along the line of the wave propagation.

$$B_{a_1a_2}(0)=a_1(t)a_2(t-\tau_1)=B_{a_1}(0,\tau_1)=\sigma_{a_1}^2e^{-\alpha\tau_1}(\cos\beta\tau_1+\frac{\alpha}{\beta}\sin\beta\tau_1)=$$

$$=\sigma_{a_1}^2R_{a_1}(\tau_1)=\sigma_{a_2}^2 \qquad (1.5)$$

Similarly the variation of the mean square of acceleration of the ground base along the wave beam is determined:

$$B_{\ddot{a}_1\ddot{a}_2}(0)=B_{\ddot{a}_1}(0,\tau_1)=\sigma_{\ddot{a}_1}^2R_{\ddot{a}_1}(\tau_1)=\sigma_{\ddot{a}_2}^2 \qquad (1.6)$$

Graphs for $R_{a_1}(\tau_1)$ and $R_{\ddot{a}_1}(\tau_1)$ drawn by the parameters of the correlation functions of a few american seismograms and accelerograms for the wave velocity c=600 m/sec are given in Fig.2 and 3.

2. <u>Response of a one-tier extended-in-plan structure to the seismic effect in a stationary regime.</u> Let us assume that the floor of such a building in the horizontal plane is a rigid disk with mass M resting on a number of elastic supports.

The design scheme of the structure, regarded as a one-degree-of-freedom system. is presented in Fig.4.

In the absolute system of coordinates the equation for the translational movement of the structure has the form:

$$M\ddot{x}(t)+(u+iv)\sum_{k=0}^{n} c_k\left[x_k(t)-a_k(t)\right]=0 \tag{2.1}$$

Ignoring the imaginary part in the expression $(u+iv)\sum_{k=0}^{n} c_k a_k(t)$ and assuming here u=1, we obtain

$$\ddot{x}(t)+(u+iv)\omega_1^2 x(t)=\frac{1}{M}\sum_{k=0}^{n} c_k a_k(t) \tag{2.2}$$

Here M, $x(t)$ and C_k are respectively the mass of the system, its absolute displacement and the rigidity coefficient of the k-th strut.

$\omega_1^2=\frac{\sum c_k}{M}$ is the square of the circular frequency of free vibrations of the system;

$u=\frac{4-\gamma^2}{4+\gamma^2}$, $v=\frac{4\gamma}{4+\gamma^2}$; $\gamma=\frac{\delta}{\pi}$, δ is the logarithmic decrement of damping;

a(t) - stationary random process of the ground base displacement of the structure with an average value equal to zero.

The transfer function of the system is:

$$\Phi(i\omega)=\left[-\omega^2+(u+iv)\omega_1^2\right]^{-1}$$

The modulus square of the transfer function of the system:

$$|\Phi(i\omega)|^2=(\omega^4-2u\omega_1^2\omega^2+\omega_1^4)^{-1} \tag{2.3}$$

Let's calculate statistical characteristics of the exciting force F(t).

The average value $\overline{F(t)}=0$;

The mean square

$$\overline{F^2(t)}=\frac{\sigma_{a_0}^2}{M^2}\left[\sum_{k=0}^{n} c_k^2 R(\tau_k)+2\sum_{\substack{j,k=0\\ j\neq k}}^{n} c_j c_k R(\tau_j) R(\tau_k-\tau_j)\right] \tag{2.4}$$

The spectral density

$$S_F(\omega)=\frac{1}{M^2}\left[S_{a_0}(\omega)\sum_{k=0}^{n} c_k^2 R(\tau_k)+2\sum_{\substack{j,k=0\\ j\neq k}}^{n} c_j c_k S_{a_0}(\omega) R(\tau_j)\cos\omega(\tau_k-\tau_j)\right]; \tag{2.5}$$

It is assumed here that $\tau_\kappa - \tau_j = \frac{x_\kappa - x_j}{C}$; $\overline{a_\kappa^2(t)} = \sigma_{a_0}^2 R(\tau_\kappa)$;

$\overline{a_j(t)a_\kappa(t)} = \sigma_{a_j}^2 R(\tau_\kappa - \tau_j) = \sigma_{a_0}^2 R(\tau_j) R(\tau_\kappa - \tau_j)$; $S_{a_\kappa}(\omega) = S_{a_0}(\omega) R(\tau_\kappa)$;

$S_{a_j a_\kappa}(\omega) = S_{a_j}(\omega)\cos\omega(\tau_\kappa - \tau_j) = S_{a_0}(\omega) R(\tau_j)\cos\omega(\tau_\kappa - \tau_j)$;

$$S_{a_0}(\omega) = \sigma_{a_0}^2 \frac{4\alpha m^2}{\omega^4 + 2a\omega^2 + m^4}; \tag{2.6}$$

The mean square of the absolute displacement of the system *may be determined with the help of the known relation*

$$\overline{x^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_F(\omega)\,|\Phi(i\omega)|^2 d\omega \tag{2.7}$$

Substituting expressions (2.3) and (2.5) into (2.7) we arrive at

$$\overline{x^2(t)} = \frac{\sigma_{a_0}^2}{M^2\omega_1^4}\left[\xi^2 \sum_{\kappa=0}^{n} c_\kappa^2 R(\tau_\kappa) + 2\sum_{\substack{j,\kappa=0\\ j\neq\kappa}}^{n} c_\kappa c_j R(\tau_j) T_{\kappa j}\right] \tag{2.8}$$

where

$$\xi^2 = \frac{\omega_1^4 + 2\alpha\gamma\omega_1^3 + 4\alpha^2\omega_1^2 + \frac{2\alpha}{\gamma} m^2\omega_1}{\omega_1^4 + 2\alpha\gamma\omega_1^3 + 2a\omega_1^2 + 2\alpha\gamma m^2\omega_1 + m^4} \tag{2.9}$$

$$T_{\kappa j} = \kappa_1 e^{-\alpha(\tau_\kappa - \tau_j)}\cos\beta(\tau_\kappa - \tau_j + \psi_1) + \kappa_2 e^{-\frac{\gamma\omega_1}{2}(\tau_\kappa - \tau_j)}\cos\omega_1(\tau_\kappa - \tau_j + \psi_2) \tag{2.10}$$

Here

$\kappa_1 = \omega_1^4(m^2 A + B)\sqrt{1 + tg^2\varphi_1}$; $\kappa_2 = \frac{2\alpha}{\gamma} m^2\omega_1^3\left(c + \frac{D}{\omega_1^2}\right)\sqrt{1 + tg^2\varphi_2}$,

$tg\varphi_1 = \frac{\alpha(m^2A - B)}{\beta(m^2A + B)}$; $tg\varphi_2 = \frac{\gamma\left(c - \frac{D}{\omega_1^2}\right)}{2\left(c + \frac{D}{\omega_1^2}\right)}$; $\psi_1 = \frac{\varphi_1}{\beta}$; $\psi_2 = \frac{\varphi_2}{\omega_1}$;

$A = -C = \frac{2(\omega_1^2 + a)}{[4\omega_1^2(\omega_1^2 + a)(a\omega_1^2 + m^4) + (m^4 - \omega_1^4)^2]}$

$$B = \frac{2m^4(\omega_1^2 + a)A - 1}{m^4 - \omega_1^4}; \qquad D = \frac{1 - 2\omega_1^4(\omega_1^2 + a)A}{m^4 - \omega_1^4} \tag{2.11}$$

Graphs for ξ^2 in Fig.5 are plotted for the parameters of the correlation functions in Fig.1 and for the value $\gamma = 0,1$. For the graphs T_{kj} (Fig.6) parameters of seismograms 1 only were used.

The mean square of the absolute displacement being known, the mean square of the relative displacement of the k-th strut of the system is easily determined.

$$\overline{x_\kappa^2(t)} = \overline{[x(t) - a_\kappa(t)]^2} = \overline{x^2(t)} - \overline{2a_\kappa(t)x(t)} + \overline{a_\kappa^2(t)} \tag{2.12}$$

To calculate the covariance $\overline{a_k(t)x(t)}$ in (2.12) let's multiply the partial integral (2.2), determined by the equation

$$x(t)=\int_{-\infty}^{\infty}F(t-\tau)K(\tau)d\tau=\frac{1}{M}\sum_{K=0}^{n}c_k\int_{-\infty}^{\infty}a_k(t-\tau)K(\tau)d\tau, \qquad (2.13)$$

by $a_k(t)$ and average. We have

$$\overline{a_k(t)\,x(t)}=\frac{1}{M}\sum_{j=0}^{n}c_j\int_{-\infty}^{\infty}\overline{a_j(t-\tau)a_k(t)}\,K(\tau)d\tau \qquad (2.14)$$

Here $K(\tau)$ is an impulse transfer function of the system.

Replacing $K(\tau)$ in (2.14) by

$$K(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi(i\omega)e^{i\omega\tau}d\omega,$$

we get

$$\overline{a_k(t)x(t)}=\frac{1}{M}\sum_{j=0}^{n}c_j\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{a_ja_k}(\omega)\Phi(i\omega)d\omega=\frac{1}{M}\sum_{j=0}^{n}c_j J \qquad (2.15)$$

Applying the expression given in (2.6) for the cross-spectral density of processes a_j and a_K, we find

$$J=4\alpha m^2R(\tau_j)\sigma_{a_0}^2\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{(-\omega^2+\omega_1^2)\cos\omega(\tau_k-\tau_j)\,d\omega}{(\omega^4+2a\omega^2+m^4)(\omega^4-2u\omega_1^2\omega^2+\omega_1^4)} \qquad (2.16)$$

Hence

$$\overline{a_k(t)\,x(t)}=\frac{\sigma_{a_0}^2}{M\omega_1^2}\sum_{j=0}^{n}c_jR(\tau_j)S_{kj}, \qquad (2.17)$$

where

$$S_{kj}=k_3e^{-\alpha(\tau_k-\tau_j)}\cos\beta(\tau_k-\tau_j+\psi_3)+k_4e^{-\frac{\gamma\omega_1}{2}(\tau_k-\tau_j)}\cos\omega_1(\tau_k-\tau_j+\psi_4),$$

$$k_3=\omega_1^2(m^2A_1+B_1)\sqrt{1+tg^2\varphi_3};\quad k_4=\frac{2\alpha}{\gamma}m^2\omega_1^3\left(C_1+\frac{D_1}{\omega_1^2}\right)\sqrt{1+tg^2\varphi_4} \qquad (2.18)$$

The expressions for φ_3, ψ_3, φ_4 and ψ_4 are obtained from $\varphi_1,\psi_1,\varphi_2,\psi_2$ by means of substituting A_1, B_1, C_1 and D_1 for A, B, C and D.

$$A_1=-C_1=\frac{2\omega_1^2(a+\omega_1^2)-\omega_1^4+m^4}{[4(a+\omega_1^2)(m^4+a\omega_1^2)\omega_1^2+(\omega_1^4-m^4)^2]};$$

$$B_1=\frac{2m^4(a+\omega_1^2)A_1-\omega_1^2}{m^4-\omega_1^4};\qquad D_1=\frac{\omega_1^2-2\omega_1^4(a+\omega_1^2)A_1}{m^4-\omega_1^4};$$

Taking into account (2.17) for the mean square of the relative displacement of the k-th point of the system, we obtain the following expression

$$\overline{x_k^2(t)}=\frac{\sigma_{a_0}^2}{M^2\omega_1^4}\Big[\xi^2\sum_{k=0}^{n}c_k^2R(\tau_k)+2\sum_{\substack{j,k=0\\ j\neq k}}^{n}c_kc_jR(\tau_j)T_{kj}\Big]-$$
$$-2\frac{\sigma_{a_0}^2}{M\omega_1^2}\sum_{j=0}^{n}c_jR(\tau_j)S_{kj}+\sigma_{a_0}^2R(\tau_k); \qquad (2.19)$$

Formula (2.19) may be obtained directly from the solution of the equation for the translational movement of the structure in the system of coordinates connected with the ground base of the k-th strut.

Indeed, from equation (2.2), using the relationship $x(t)=x_k(t)+a_k(t)$, it follows

that

$$\ddot{x}_k(t)+\gamma\omega_1\dot{x}_k(t)+\omega_1^2x_k(t)=-\ddot{a}_k(t)-\omega_1^2a_k(t)+F(t)=F_1(t) \qquad (2.20)$$

The mean square of the exciting force

$$\overline{F_1^2(t)}=\overline{\ddot{a}_k^2(t)}+\omega_1^4\overline{a_k^2(t)}+\overline{F^2(t)}+2\omega_1^2\overline{a_k(t)\ddot{a}_k(t)}-2\omega_1^2\overline{a_k(t)F(t)}-$$
$$-2\overline{\ddot{a}_k(t)F(t)} \qquad (2.21)$$

The mean square of the seismic acceleration may be calculated by the spectral density of the ground base displacement.

$$\overline{\ddot{a}_k^2(t)}=\frac{R(\tau_k)}{2\pi}\int_{-\infty}^{\infty}\omega^4S_{a_0}(\omega)d\omega=R(\tau_k)\frac{4\alpha m^2\sigma_{a_0}^2}{2\pi}\int_{-\infty}^{\infty}\frac{\omega^4d\omega}{\omega^4+2a\omega^2+m^4} \qquad (2.22)$$

Integral (2.22) oscillates, therefore calculation $\ddot{a}_k(t)=\sigma_{\ddot{a}_k}^2$ has no sense in this case. It is, however, possible to transform $S_{\ddot{a}_k}(\omega)$ so, that the value $\sigma_{\ddot{a}_k}^2$ would retain its sense in similar cases as well.

$$\sigma_{\ddot{a}_k}^2=\sigma_{a_0}^2R(\tau_k)\frac{4\alpha m^2}{\pi}\int_0^\infty\left|\frac{-\omega^2}{\omega^2-2i\alpha\omega-m^2}+\left(\frac{\omega^2}{\omega^2-2i\alpha\omega-m^2}\right)_\infty\right|^2d\omega=$$
$$=\sigma_{a_0}^2R(\tau_k)\frac{4\alpha m^2}{\pi}\int_0^\infty\frac{(4\alpha^2\omega^2+m^4)d\omega}{\omega^4+2a\omega^2+m^4}=(4\alpha^2m^2+m^4)R(\tau_k)\sigma_{a_0}^2 \qquad (2.23)$$

Co-variations in equation (2.21) are determined as follows

$$\overline{a_k(t)\ddot{a}_k(t)}=\left[\frac{d^2}{d\tau^2}B_{a_k}(\tau)\right]_{\tau=0}=-B_{\dot{a}_k}(0)=-\sigma_{\dot{a}_k}^2=-m^2R(\tau_k)\sigma_{a_0}^2, \qquad (2.24)$$

since

$$\sigma_{\dot{a}_k}^2=\frac{1}{2\pi}\int_{-\infty}^{\infty}\omega^2S_{a_k}(\omega)d\omega=m^2R(\tau_k)\sigma_{a_0}^2$$

$B_{\dot a_k}(t)$ is the correlation function of the velocity $a_k(t)$

$$\overline{a_k(t)F(t)} = \sigma_{a_0}^2 \sum_{j=1}^{n} c_j R(\tau_j) R(\tau_k - \tau_j);$$
$$(2.25)$$
$$\overline{\ddot a_k(t)F(t)} = \sum_{j=0}^{n} c_j \left[\frac{d^2}{d\tau^2} B_{a_j a_k}(\tau)\right]_{\tau=0} = \sum_{j=0}^{n} c_j \left[\frac{d^2}{d\tau^2} \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{a_j a_k}(\omega) e^{i\omega\tau} d\omega\right]_{\tau=0} =$$
$$= -m^2 \sigma_{a_0}^2 \sum_{j=0}^{n} c_j R(\tau_j) R^*(\tau_k - \tau_j),$$

where

$$R^*(\tau_k - \tau_j) = e^{-\alpha(\tau_k - \tau_j)}\left[\cos\beta(\tau_k - \tau_j) - \frac{\alpha}{\beta}\sin\beta(\tau_k - \tau_j)\right] \quad (2.26)$$

Taking into account (2.24), (2.25) and (2.26) we obtain finally

$$\overline{F_1^2(t)} = (4\alpha^2 m^2 + m^4) R(\tau_k)\sigma_{a_0}^2 + \omega_1^4 R(\tau_k)\sigma_{a_0}^2 + \sigma_F^2 - 2m^2\omega_1^2 R(\tau_k)\sigma_{a_0}^2 -$$
$$- \frac{2\omega_1^2 \sigma_{a_0}^2}{M} \sum_{j=0}^{n} c_j R(\tau_j) R(\tau_k - \tau_j) + \frac{2m^2\sigma_{a_0}^2}{M}\sum_{j=0}^{n} c_j R(\tau_j) R^*(\tau_k - \tau_j); \quad (2.27)$$

the spectral density

$$S_{F_1}(\omega) = R(\tau_k)\omega^4 S_{a_0}(\omega) + R(\tau_k)\omega_1^4 S_{a_0}(\omega) + S_F(\omega) - \frac{2\omega_1^2}{M}\sum_{j=0}^{n} c_j S_{a_j a_k}(\omega) -$$
$$- \frac{2}{M}\sum_{j=0}^{n} c_j S_{a_j \ddot a_k}(\omega) + 2\omega_1^2 S_{a_k \ddot a_k}(\omega) = (\omega^4 - 2\omega_1^2\omega^2 + \omega_1^4) R(\tau_k) S_{a_0}(\omega) +$$
$$+ S_F(\omega) - \frac{2}{M}\sum_{j=0}^{n} c_j R(\tau_j)(\omega_1^2 - \omega^2) S_{a_0}(\omega)\cos\omega(\tau_k - \tau_j); \quad (2.28)$$

Substituting expressions (2.3), (2.5) and (2.28) into (2.7) we get (2.19).

The mean square of the inertia force acting on the mass M of the system

$$\overline{P_{\text{ин}}^2} = M^2 \overline{\ddot x^2(t)}$$
$$(2.29)$$

The expression for the mean square of the acceleration $\overline{\ddot x^2(t)}$ is obtained from (2.8) by substituting ξ_1^2 for ξ^2 and T_{kj}^* for T_{kj}

$$\xi_1^2 = \frac{m^2\omega_1^4\left(m^2 + \frac{2\alpha\omega_1}{\gamma}\right)}{(\omega_1^4 + 2\alpha\gamma\omega_1^3 + 2a\omega_1^2 + 2\alpha\gamma m^2\omega_1 + m^4)}$$
$$(2.30)$$

The parameter T^{*}_{kj} is determined by formulae (2.10) and (2.11) for magnitudes A_2, B_2, C_2 and D_2.

$$A_2 = -C_2 = -\frac{2\omega_1^2(m^4 + a\omega_1^2)}{4\omega_1^2(a+\omega_1^2)(m^4 + a\omega_1^2) + (\omega_1^4 - m^4)^2}$$

$$B_2 = \frac{m^4[1+2(a+\omega_1^2)A_2]}{m^4 - \omega_1^4}; \quad D_2 = -\frac{\omega_1^4[1+2(a+\omega_1^2)A_2]}{m^4 - \omega_1^4}; \tag{2.31}$$

The design value of the inertia force will be obtained by replacing in (2.29) the standard of the displacement by its design value a_{design}. In the first approximation this value may be determined by the design seismic acceleration and average values of parameters of correlation functions of soil displacements of the base recorded in the given seismic region.

$$a_{design} = \frac{gK_c}{\sqrt{4\alpha^2 m^2 + m^4}} \tag{2.32}$$

Here gK_c - design acceleration, K_c - seismic coefficient, g - gravity acceleration, α, m - parameters of the correlation function of the ground displacement.

3. Response of a multi-storeyed extended-in-plan building. The building is a system of vertical supports joint by floors rigid in the horizontal plane. The design scheme of the building is shown in Fig.7.

The equation for the translational movement of such a building as a system with a final number of degrees of freedom in the absolute system of coordinates has the form:

$$M_k \ddot{x}_k(t) + (u + iv)\sum_{\tau=1}^{s} c_{k\tau} x_\tau(t) = \sum_{j=0}^{n} c_{jk} a_j(t) = F_k(t)$$
$$(k = 1, 2 \ldots \ell \ldots \tau \ldots s) \tag{3.1}$$

Here $x_k(t)$ is the absolute displacement of the k-th floor, M_k is its mass, τ, ℓ are serial numbers of floors, $C_{k\tau}$- coefficient of rigidity of the building, $c_{jk} = \sum_{\tau=1}^{s} c_{jk\tau}$ is the sum of elements of the k-th line of the matrix of rigidity coefficients of the j-th strut. The latter is built by the matrix inversion of the coefficients of the influence $\| \delta_{k\tau} \|_1^s$. The value $\delta_{k\tau}$ is equal to the deflection at the point k of the strut j caused by a unit force at the point τ: $a_j(t)$ is a stationary random process of the ground displacement of the base at the j-th strut.

Dynamic displacements of floors of buildings with mass

M_k may be expressed through the modes of free vibrations of the building $\alpha_i(y)$ in the following way

$$x_k(t)=\sum_{i=1}^{N} q_i(t)\,\alpha_i(y_k), \tag{3.2}$$

where $q_i(t)$ are generalized coordinates; N is the number of modes of free vibrations of the system.

The lagrange equation for the i-th mode of vibrations of the building may be written as follows:

$$\ddot{q}_i(t)+(u+iv)\,\omega_1^2\,q_i(t)=\frac{Q_i(t)}{M_{i\,gen}}; \tag{3.3}$$

Here the generalized force is $Q_i(t)=\sum_{r=1}^{s}F_r(t)\,\alpha_i(y_r)$ the generalized mass is $M_{i\,gen}=\sum_{r=1}^{s}M_r\alpha_i^2(y_r)$, ω_i is the i-th circular frequency of free vibrations of the system; $\alpha_i(y_r)$ are coefficients of the distribution ofamplitudes of the displacement of the i-th mode at all points of the system where masses M_r are concentrated.

Covariance and cross-spectral density of exciting forces $F_r(t)$ and $F_\ell(t)$ have the form:

$$\overline{F_r(t)\,F_\ell(t)}=\sigma_{a_0}^2\sum_{j=0}^{n}\sum_{m=0}^{n}c_{jr}\,c_{m\ell}\,R(\tau_j)\,R(\tau_m-\tau_j)$$

$$S_{F_rF_\ell}(\omega)=\sum_{j=0}^{n}\sum_{m=0}^{n}c_{jr}\,c_{m\ell}\,S_{a_0}(\omega)\,R(\tau_j)\cos\omega(\tau_m-\tau_j) \tag{3.4}$$

The average values, mean square, spectral density and cross-spectral density of generalized forces are determined by the following formulae /2/.

$$\overline{Q_i(t)}=\sum_{r=1}^{s}\overline{F_r(t)}\,\alpha_i(y_r)=\sum_{r=1}^{s}\sum_{j=0}^{n}c_{jr}\,\overline{a_j(t)}=0$$

$$\overline{Q_i^2(t)}=\sum_{r=1}^{s}\sum_{\ell=1}^{s}F_r(t)F_\ell(t)\,\alpha_i(y_r)\,\alpha_i(y_\ell);$$

$$S_{Q_i}(\omega)=\sum_{r=1}^{s}\sum_{\ell=1}^{s}S_{F_rF_\ell}(\omega)\,\alpha_i(y_r)\,\alpha_i(y_\ell);$$

$$S_{Q_iQ_k}(\omega)=\sum_{r=1}^{s}\sum_{\ell=1}^{s}S_{F_rF_\ell}(\omega)\,\alpha_i(y_r)\,\alpha_k(y_\ell) \tag{3.5}$$

Statistical characteristics of generalized forces and transfer functions of the system being known, covariance and mean square of generalized coordinates are easily determined.

$$\overline{q_i(t)q_k(t)}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{S_{Q_iQ_k}(\omega)\left[\omega^4-u(\omega_i^2+\omega_k^2)\omega^2+iv(\omega_i^2-\omega_k^2)+\omega_i^2\omega_k^2\right]d\omega}{M_{i\,gen}M_{k\,gen}(\omega^4-2u\omega_i^2\omega^2+\omega_i^4)(\omega^4-2u\omega_k^2\omega^2+\omega_k^4)} \tag{3.6}$$

$$\overline{q_i^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{Q_i}(\omega)\,|\Phi_i(i\omega)|^2 d\omega \tag{3.7}$$

Here $|\Phi_i(i\omega)|^2 = [M^2_{i\,q_{i}n}(\omega^4 - 2u\omega_i^2\omega^2 + \omega_i^4)]^{-1}$ is the square of the modulus of the transfer function determined by equation (3.3). The mean square of displacements of the point K of the system may be written as

$$\overline{x_k^2(t)} = \sum_{i=1}^{N}\overline{q_i^2(t)}\,\alpha_i^2(y_k) + \sum_{r=1}^{N}\sum_{j=1}^{N}\overline{q_i(t)q_j(t)}\,\alpha_i(y_k)\alpha_j(y_k); \quad (i \neq j) \tag{3.8}$$

As is known /3/, in dynamic systems characterized by low damping and frequencies being rather far from one another the cross correlation between the generalized coordinates of the system may be neglected. In this case there remains only the first term in the right-hand part of (3.8).

The mean square of the relative displacement of the j-strut at the level of the k-th floor will be

$$\overline{x_{jk}^2(t)} = \overline{x_k^2(t)} - 2\overline{a_j(t)x_k(t)} + \overline{a_j^2(t)} \tag{3.9}$$

Covariance

$$\overline{a_j(t)x_k(t)} = \sum_{i=1}^{N}\overline{a_j(t)q_i(t)}\,\alpha_i(y_k) = \sum_{i=1}^{N}\alpha_i(y_k)\int_{-\infty}^{\infty}\overline{a_j(t)Q_i(t-\tau)}K(\tau)d\tau =$$

$$= \sum_{i=1}^{N}\sum_{r=1}^{s}\sum_{m=0}^{n}\alpha_i(y_k)\alpha_i(y_r)C_{mr}\int_{-\infty}^{\infty}\overline{a_j(t)a_m(t-\tau)}K(\tau)d\tau =$$

$$= \sum_{i=1}^{N}\sum_{r=1}^{s}\sum_{m=0}^{n}\alpha_i(y_k)\alpha_i(y_r)C_{mr}R(\tau_j)\sigma_{a_0}^2 S_{mj} \tag{3.10}$$

The parameter S_{mj} is calculated by formula (2.18).

Bibliography

1. Barstein M.F., Influence of An Irregular Wave on Lattice Engineering Structures. Stroitelnaya mekhanika i raschiot sooruzhenii, No.1, 1964.

2. Barstein M.F., Dynamic Design of Masts and Towers for Wind Action. Stroitelnaya Mekhanika i raschiot sooruzhenii,

No.4, 1967.

3. Bolotin V.V., Statistical Methods in Building Mechanics. Stroiisdat, Moscow, 1965.

4. Grossman A.B., Vibrations of Extended Structures under Seismic Effects. Sbornik nauchnykh trudov TashZNIIEP, issue 6, Tashkent, 1964.

5. Korchinsky I.L., Approximate Evaluation of Seismic Vibrations of Extended-in-Plan Structures. Study of Earthquake-Resistance of Building and Structures. Gosstroiisdat, Moscow, 1961.

6. Lanning D.H. and Battin R.G., Random Processes in Problems of Automatic Control. 11., 1958.

7. Sinitsyn A.P., Influence of a Running Seismic Wave on Massive Structures. Trudy Instituta Physiki Zemli "Voprosy Inzhenernoy Seismologii", Issue 5, 1961.

8. Housner G.W., Interaction of Building and Ground during an Earthquake. Bulletin of the Seismological Society of America, vol.47, No.3, 1967.

Statistical characteristics of the displacement of base estimated by american seismograms

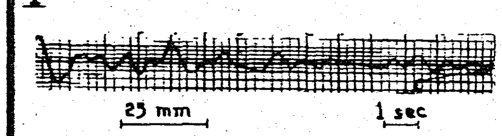
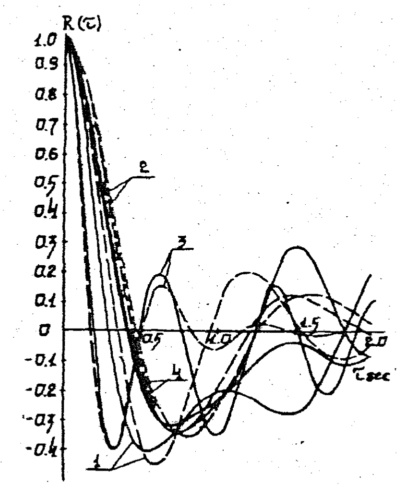
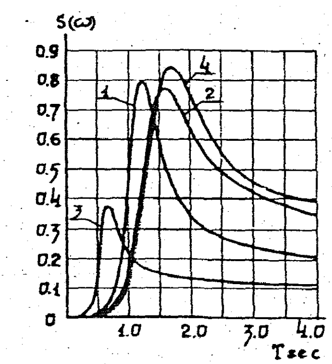
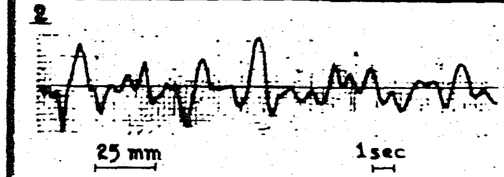
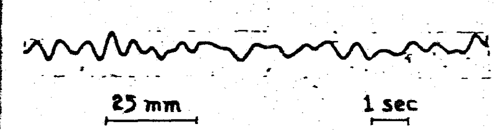
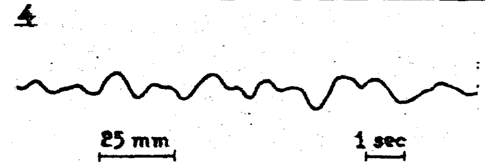
seismograms	region	data	duration in sec.	maximum value of displacement in cm.	standard deviation of displacement of base in cm	parameters of correlation function	normalized correlation functions	normalized spectral densities
1 	Hollister	19·I·1960	20·(15)	0,75	0,198	$\alpha=1,33;\ \beta=4,24$		
2 	Hollister	8·IV·1961	27·(21)	2,83	0,664	$\alpha=1,43;\ \beta=4,19$		
3 	Hollister	25·IV·1954	17·(12)	0,43	0,160	$\alpha=2,86;\ \beta=9,68$		
4 	El-Centro	12·XI·1954	28·(18)	0,52	0,235	$\alpha=1,31;\ \beta=3,925$		

Fig. 1

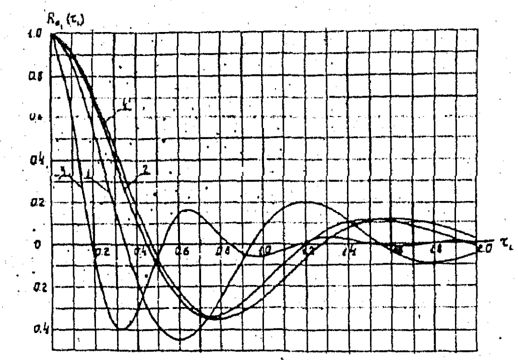


Fig.2 Parameter $R_{a_1}(\tau_1)$

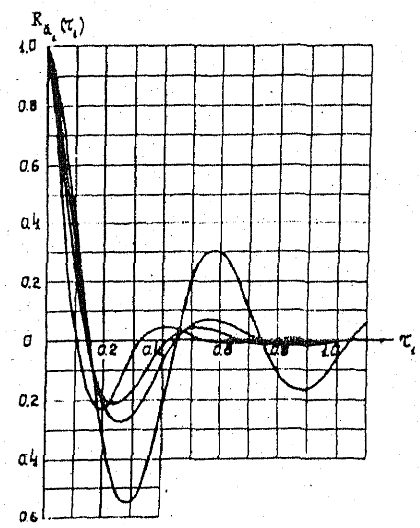


Fig.3 Parameter $R_{\ddot{a}_1}(\tau_1)$

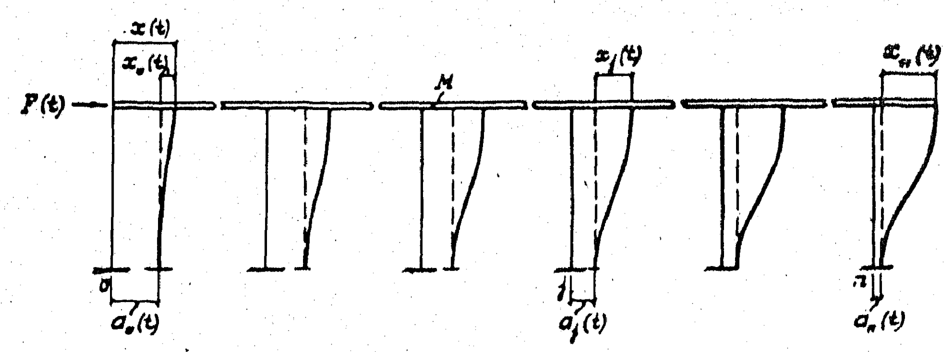


Fig. 4

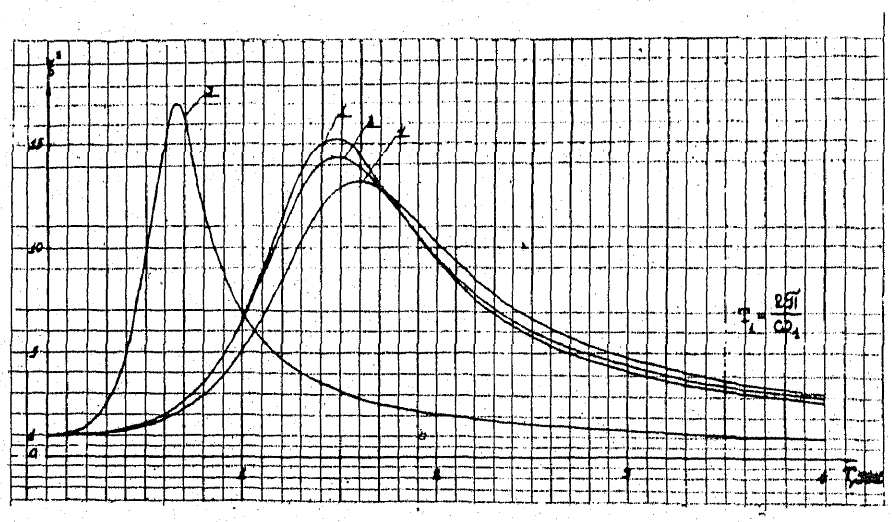


Fig.5 Graphs of ξ^2

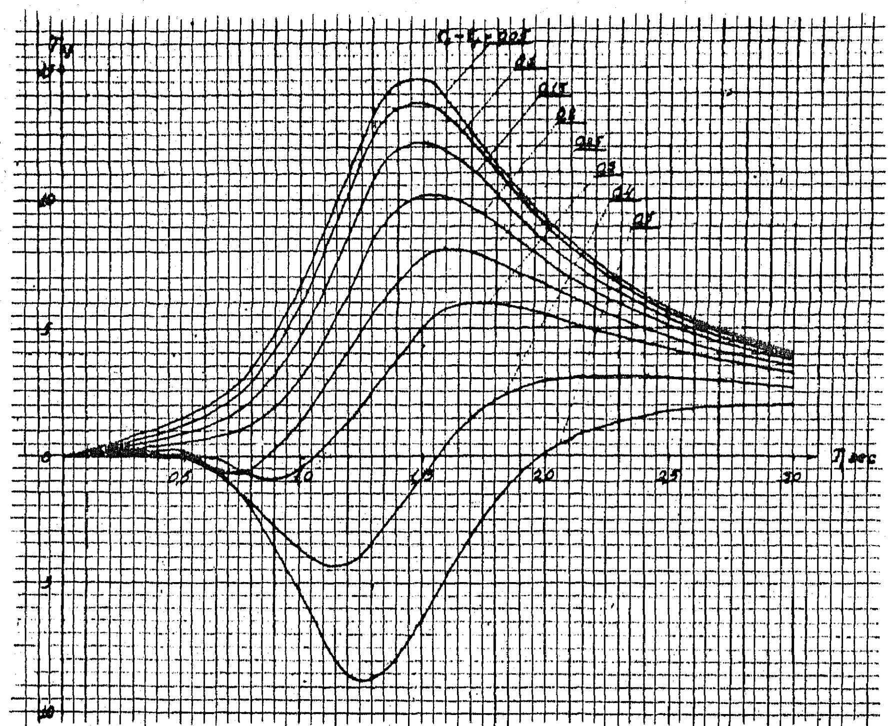


Fig. 6 Graphs of T_{kj} for parameters $\alpha = 1{,}33$; $\beta = 4{,}24$ calculated by seismogram 1

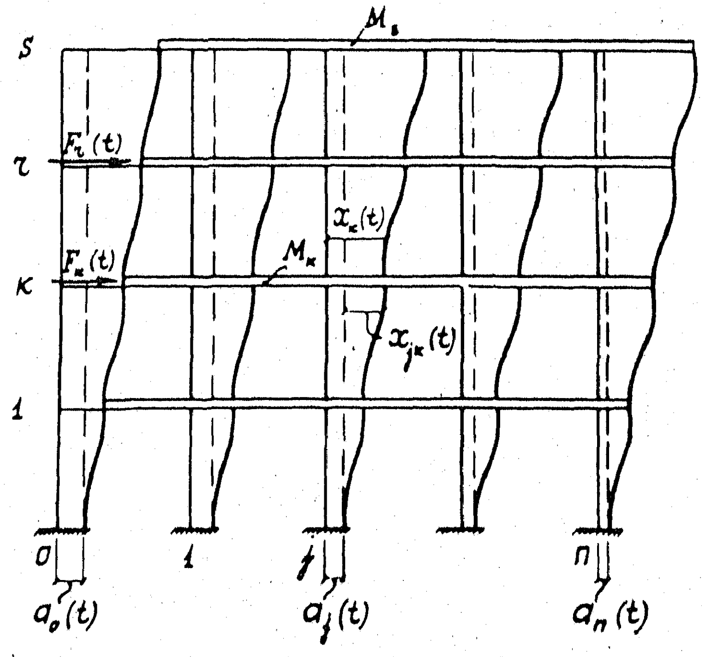


Fig. 7