THE EFFECT OF SEISMIC ACTION ON THE DYNAMIC BEHAVIOUR OF ELEVATED WATER TANKS

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ABSTRACT

A study is presented on the effect of the interaction between the structure and the mass of water upon the dynamic response of elevated tanks.

The motion of the water mass is described by the general equations of hydrodynamics, the conditions of uniqueness being explicitely expressed.

The equation of energy is establisted under the assumption of an irrotational, non-permanent potential motion, and the perturbind and gravitational fields are characterized by the scalar functions of the velocity potentials.

From the analysis of these relations it follows that the oscillations of the liquid are equivalent to the oscillations of a system with one degree of feedom. The general relations of equivalence corresponding to a reservoir of arbitrary form, are presented.

By taking into account the elastic characteristics of the structure and the geometrical configuration of the reservoir containing the liquid, the dynamic behaviour of the aggregate can be investigated by means of an oscillating system with two degrees of dynamic feedom.

In conclusion, the results supplied by the wethod described are compared with those published in the litterature and referring to particular shapes of the reservoir.

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INTRODUCTION

Water accumulations often found, water towers are built in various shapes that combine serviceable and architectural features. The size of these constructions as well as the various, geometrical shapes of the tab (cylinder, truncated cone, paraboloid, sphere, eliptical etc) call for a complex dynamical analysis.

Generally the dynamical response of the hydrotechnical structure is the result of the joint action of the fluid mass and of the elastic structure. Interesting theoretical and practical studies have been made concerning the phenomena which appear in the fluid mass and in the structure comes in contact with it. The theoretical researches made by Poisson (1828) and Reyleigh (1876) on the osscilation of fluid masses led to new studies of complex analysis by up to date welhods : Westergaard H.M. (1933), Hoskins L.M. (1934), Morris B.T.(1938) Ruge C.A. (1937-8), Jacobsen L (1934,1949,1951), Carder D.S. (1936), Werner W -Sundquist K.L. (194o), A.Arias (1948,1963), Ayre R.S.(1951), Housner G.W. (1957,1963), Moran D.F.-Cheney J.A. (196o), Sretenskii L.N. (1951), Graham E.W.-Rodriguez A.M. (1952), Krishna J- Shekaran A.R. (196o), Steinbrugge O.C.- Nath B (1963), S.Moran G (1963), Nikolaenko N.A. (1961,1963), Goldenblat I.I. (1961), Bratu C. (1965,1966, 1968), Ramiah-Rajate Mohana Gupta D.S. (1966) etc.

Since the investigations effected were theoretical and practical solutions of particular cases, the cylindrical and rectangular tanks were the most frequently treated cases. In the following a "fluid - elastic" analysis is presented for tanks of arbitrary shape.

EFFECT OF WATER AND EQUIVALENT MASS

The fluid mass stored in a tank is in motion under the joint action of the gravitation field and of the field of the perturbing force ; the motion of a fluid particle being described by the equations :

$$\frac{DV_x}{dt} = f_x - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \Delta (V_x) \qquad (x,y,z) \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \vec{V}) = 0 \tag{2}$$

These equations are valid for the non-stationary motion of a real compressible fluid in a simply connected domain and explicit conditions of uniqueness. If in the equations of motion Navier-Stokes (1) we perform the Ruark transformation and denote the non-dimensional magnitudes (index ",") and the constant dimensional magnitudes which are included in the conditions of uniqueness (index "$_0$") we shall get the equations in a non-dimensional form :

$$\frac{1}{S_0}\frac{\partial V'_x}{\partial t'} + V'_x\frac{\partial V'_x}{\partial x'} + V'_y\frac{\partial V'_x}{\partial y'} + V'_z\frac{\partial V'_x}{\partial z'} = \frac{1}{Fr_0} f'_x - \tag{3}$$

$$-Eu_0\,\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \frac{1}{Re_0}\,\nu'\Delta'(V'_x) \qquad (x,y\,z)$$

It should be noted that the equations include the non-dimensional criteria of the phenomenon, namely :

- Strouhal's criterion $S_0 = \dfrac{L_0}{V_0\,t_0}$
- Froude's criterion $Fr_0 = \dfrac{V_0^2}{L_0\,f_0}$
- Euler's criterion $Eu_0 = \dfrac{\Delta p_0}{\rho_0\,V_0^2}$
- Reynolds criterion $Re_0 = \dfrac{V_0\,L_0}{\nu_0}$

The dynamic and kinematic characteristics by the conditions of uniqueness are introduced in the equations of motion by these criteria. The model of perfect fluid where the contribution of viscosity stresses is neglected can be adapted provided that. The term $\frac{1}{Re}\nu'\Delta'$ is very small with respect to the others ; this is mainly the case when :

- Re is large, i.e. when the motion occurs, in a sufficiently large space (tanks, lakes,etc).
- S_0 is small, i.e. when the motion has a high degree of non-permanence and takes place during small time intervals in the form of schocks.

In the case of an earthapnake acting on a tank of the type usually met with in technical applications, these conditions are fulfilled since the tank dimensions are sufficiently large and the external forces acting on the tank have a high frequency.

Both the experiments effected by Ruge, Jacobse, Keulegan and the theoretical relation established by Nicolaenko for cylindrical and rectangular tanks show that the system may be assumed to be linear, the dissipasion of energy during the short interval of the oscillations being a negligeable phenomenon.

Under these conditions the equations of motion and of continuity may be written for a perfect incompressible liquid in the non-stationary potential motion :

$$\frac{p}{\rho} + \frac{\partial\varphi}{\partial t} + \frac{1}{2}V^2 + U = C(t) \tag{4}$$

$$\Delta\varphi(x,y,z,t) = 0 \tag{5}$$

$$\vec{V} = grad\ \varphi \tag{6}$$

Considering the kinetic energy of the oscillatory motion of a particle as being small with respect to the other energies and considering that the phenomenon takes place in sinch a way that C (t) may be introduced in the φ (x,y,z,t) equation (4) becomes /7,17/ :

$$\frac{p}{\rho} + \frac{\partial \varphi}{\partial t} + g z = 0 \tag{7}$$

In follows that in order to describe the motion of the liquid in the tank, the absolute potential function of the velocities φ (x,y,z,t), most be determined. This function most satisfy Laplace's equation, being an harmonic function in the respective domain with explicite conditions of uniqueness.

Two fields of force, the gravitational field and the field of the perturbing forces, act on particles of fluid which make ap the mass (fig.1). The scalar potential function of velocities may be obtained by superposing the two fields.

$$\varphi_{(x,y,z;t)} = \Phi_1(x,y,z)\cdot \dot{f}_1(t) + \Phi_2(x,y,z)\cdot \dot{f}_2(t) \tag{8}$$

where

$$\varphi_1(x,y,z;t) = \Phi_1(x,y,z)\cdot \dot{f}_1(t) \tag{9}$$

is the velocity potential resulting from effect the gravitational field, white

$$\varphi_2(x,y,z;t) = \Phi_2(x,y,z)\cdot \dot{f}_2(t) \tag{10}$$

is the velocity potential resulting from the effect of the perturbing field.

It should be noted that by introducing (8) in (5) it follows that Φ_1 and Φ_2 are also harmonic functions which are dependent only on the shape of the tank.

$$\Delta \Phi_1(x,y,z) = 0 \qquad\qquad \Delta \Phi_2(x,y,z) = 0 \tag{11}$$

To find the expressions of these functions means the solving of the Laplace equation (11) under explicite conditions of uniqueness. The boundary conditions may be expressed as shown in fig.1 :

- the motion takes place with a free surface which under atmospheric pressure

$$\left.\frac{\partial \varphi}{\partial t}\right|_{z=\zeta} + g\zeta = 0 \tag{12}$$

- the field of perturbing forces determines the motion of the particles in contact with the wall surface (Neumann's condition).

$$\left.\frac{\partial \varphi_2}{\partial n}\right|_{(C)} = V \cos(\bar{V}, \bar{n}) \tag{13}$$

- the gravitational field does not cause velocities normal to the wall as the liquid is in permanent contact with the latter (the occurence of cavitation is very unlikely), i.e. a condition of the Dirichlet type.

$$\left.\frac{\partial \varphi_1}{\partial n}\right|_{(C)} = 0 \qquad \therefore \qquad \left.\varphi_1\right|_{(C)} = ct \tag{14}$$

The initial condition may be expressed by considering the mass of liquid at rest

$$\left.\varphi\right|_{t=0} = 0 \qquad \left.\dot{\varphi}\right|_{t=0} = 0 \tag{15}$$

The equation of energy (7) which is valid over the whole domain under investigation, may take two particular forms :

$$p = -\rho \frac{\partial \varphi}{\partial t} \tag{16}$$

which represents the dynamic pressures due to oscillations and

$$\left.\frac{\partial \varphi_1}{\partial t}\right|_{z\sim 0} + g\zeta_1 + \left.\frac{\partial \varphi_2}{\partial t}\right|_{z\sim 0} + g\zeta_2 = 0 \tag{17}$$

which represents the equation of energy of the free surface.

Introducing in (17) the expressions (9) and (1o) and taking into account that / 6,7,8/ :

$$\zeta = \int_0^t \left.\frac{\partial \phi}{\partial z}\right|_{z\sim 0} \cdot \dot{f}(t)\, dt = \left.\frac{\partial \phi}{\partial z}\right|_{z\sim 0} \cdot f(t) = \phi^{*}_{z\sim 0} \cdot f(t) \tag{18}$$

it follows that

$$\ddot{f}_1(t) + g \left.\frac{\phi_1^{*}}{\phi_1}\right|_{z\sim 0} \cdot f_1(t) = -\left[\left.\frac{\phi_2}{\phi_1}\right|_{z\sim 0} \cdot \ddot{f}_2(t) + \left.\frac{\phi_2^{*}}{\phi_1}\right|_{z\sim 0} \cdot f_2(t) \right] \tag{19}$$

which describes the forced linear oscillations of a system with one degree of freedom. The frequency of the free oscilations is given by the general relation /7/.

(2o)

The conclusion is that the mass of liquid contained in the tank will effect oscillations with the frequency Ω (20) similar to those of a system with one degree of freedom. Since the function ϕ is harmonic the possibility of its series expansion leads to different values Ω^i corresponding to the mode "i" of oscillation of the mass of liquid.

By particularizing the function ϕ in various domains (cylindrical, rectangular) we obtain the results pointed out by Jacobsen /14,15/, Ries /9/, Lamb /18/, Bratu /6,7/, Kocin /17/ and even Housner /12,13/ who operated on simplified physical model.

In the case of a water tower with a tank of a arbitrary shape (fig.2) we consider the natural harmonic oscillations of the type

$$\{x\} = \{e^{i\omega t}\} \tag{21}$$

where

$$\{x\} = \begin{Bmatrix} \delta \\ \theta \end{Bmatrix} \qquad \{\omega\} = \begin{Bmatrix} \omega_\delta \\ \omega_\theta \end{Bmatrix} \tag{22}$$

or

$$\ddot{f}_1(t) + \Omega^2 f_1(t) = e^{i\omega t}\left[\frac{\phi_2}{\phi_1}\bigg|_{z\sim 0}\omega^2 - g\,\frac{\phi_2^*}{\phi_1}\bigg|_{z\sim 0}\right] \tag{23}$$

which leads to the solution

$$f_1(t) \sim \frac{\dfrac{\phi_2}{\phi_1}\bigg|_{z\sim 0}\omega^2 - g\,\dfrac{\phi_2^*}{\phi_1}\bigg|_{z\sim 0}}{\Omega^2 - \omega^2}\; e^{i\omega t} \tag{24}$$

δ, θ are the displacement by the flexural and torsional vibrations.

Negleting the natural oscillations with respect to the others and introducing (21) and (24) in (8) and then in (16) we obtain an expression for the hydrodynamic pressures

$$p = -a\rho\left[\phi_2 + \phi_1\;\frac{\dfrac{\phi_2}{\phi_1}\bigg|_{z\sim 0}\omega^2 - g\,\dfrac{\phi_2^*}{\phi_1}\bigg|_{z\sim 0}}{\Omega^2 - \omega^2}\right] \tag{25}$$

The hydrodynamic action on the walls of the reservoir is the resultant of the pressures

$$\vec{P} = -a\rho\int_{(C)}\left[\Phi_2+\Phi_1\,\frac{\frac{\Phi_2}{\Phi_1}\Big|_{z\sim 0}\omega^2 - g\,\frac{\Phi_2^*}{\Phi_1}\Big|_{z\sim 0}}{\Omega^2-\omega^2}\right]\vec{n}\,ds \qquad (26)$$

where

$$M_e = \rho\int_{(C)}\left[\Phi_2+\Phi_1\,\frac{\frac{\Phi_2}{\Phi_1}\Big|_{z\sim 0}\omega^2 - g\,\frac{\Phi_2^*}{\Phi_1}\Big|_{z\sim 0}}{\Omega^2-\omega^2}\right]ds \qquad (27)$$

is the mass of the system equivalent to the action of the liquid. This emphasizes the fact that to every mode "i" there corresponds an equivalent mass M_e^i

In the particular case where the lateral walls are vertical we note that $\Phi_2^* = 0$ and $\Omega^2 \ll \omega^2$, thus leading to simpler form :

$$M_e = \rho\int_{(C)}\left[\Phi_2 - \Phi_1\,\frac{\Phi_2}{\Phi_1}\Big|_{z\sim 0}\right]ds \qquad (28)$$

By writing explicitly Φ_1 and Φ_2 in the cylindrical and rectangular domain we obtain the results found by Jacobsen, Housner, Graham-Rodriguez, Ries, Arias, Bratu.

Returning to the functions Φ_1 , Φ_2 and taking into account that they must satisfy the Laplace equation (11), it should be noted that for truncated cone, spherical or parabolic domains, and in general for domains the contour (C) of which is a surface of revolution, the series expansion (Lagrange, Mathieu, Bessel etc) may supply general solutions in which to every term of the series there corresponds an oscillation mode of the liquid mass. For domains of arbitrary shape numerical methods can be applied (finite-differences, finite-elements) or experimental ones (electro-hydrodynamic analogy).

For instance, in the case of a tank of the shape shown in fig.3. we divide the liquid domain into a number of "finite elements" ; then the Laplace equation may be represented by the linear system

$$[A]\left\{\varphi^{i}\right\}=\begin{Bmatrix}0\\ \vdots\\ 0\\ c_1\\ \vdots\\ c_i\end{Bmatrix}\begin{Bmatrix}0\\ \vdots\\ 0\\ v\\ \vdots\\ v\end{Bmatrix} \tag{29}$$

or

$$\left\{\varphi^{i}\right\}=[A]^{-1}\begin{Bmatrix}0\\ \vdots\\ 0\\ c_1\\ \vdots\\ c_i\end{Bmatrix}\begin{Bmatrix}0\\ \vdots\\ 0\\ v\\ \vdots\\ v\end{Bmatrix} \tag{30}$$

whence, the pressures are expressed by

$$\left\{p^{i}\right\}=-\rho[A]^{-1}\begin{Bmatrix}0\\ \vdots\\ 0\\ c_1\\ \vdots\\ c_i\end{Bmatrix}\begin{Bmatrix}0\\ \vdots\\ 0\\ a\\ \vdots\\ a\end{Bmatrix}=\left\{m_e^{i}\right\}\left\{a\right\} \tag{31}$$

It follows therefore, that both the pressures and the hydrodynamic forces resulting from the oscillations of the liquid mass contained in the tank, can be expressed by moans of equivalent oscillators with a single degree of freedom (m_e), in the same way in which the oscillations of elastic structure have been treated by Balan St., Ifrim M and Pacoste C /2,3,4/. It is obvious that a computer program can be worked out for the solution of the system of equations (29-31).

The fact that the effect of the liquid in contact with vibrating surfaces can be replaced by a system of masses $\{m_e\}$ has led to intensting investigations dealing with containers on shipp or aircraft, with the design of grates, weirs, piping operating unde turbulent flow conditions, earthquake engineering etc. The dynamic effect of the free or forced oscillations of a mass of liquid stored in a water tower can be too, replaced by a system of oscillators with one degree of freedom.

The latters location may be determined by the relation (fig.4).

$$\vec{r}_P = \frac{\int_{(c)} p \cdot r \, \vec{n} \, ds}{P} \tag{32}$$

where p and F have the general expressions (25) and (26) respectively.

The expression of the free surfade follows from

$$\zeta(x,y;t) = \frac{1}{\gamma} a \left[\Phi_2\Big|_{z\sim 0} + \Phi_1\Big|_{z\sim 0} \frac{\frac{\Phi_2}{\Phi_1}\Big|_{z\sim 0} \varpi^2 - g \frac{\Phi_2^*}{\Phi_1}\Big|_{z\sim 0}}{\Omega^2 - \varpi^2} \right] \tag{33}$$

The solution of the Laplace equation and the expressions or the values of the potential functions Φ_1, Φ_2 for various particular cases, have been the object of many investigations. The general relations (25)- (33) become valid for the particular case considered, e.g. for the rectangular tank we obtain /6,9,1o/

$$\varphi_1 = -\left[\sum_p{}' \frac{4d}{\pi^2 p^2} \cos \frac{p\pi}{d} x - \sum_q{}' \frac{4b}{\pi^2 q^2} \cos \frac{q\pi}{b} y \right] \dot{U}(t)$$

$$\varphi_2 = \left[\sum_p{}' \sum_q{}' \frac{ch\, k(z+h)}{ch\, kh} \cdot \cos \frac{p\pi}{d} x \cdot \cos \frac{q\pi}{b} y \right] : \dot{U}(t) \tag{34}$$

$$\Omega^2 = gk \cdot th \cdot kh \quad \therefore \quad K = \pi \sqrt{\frac{p^2}{d^2} - \frac{q^2}{b^2}} \quad \therefore \quad p.q = 1,3,5\ldots$$

$$M_e = M_0 \left[1 + \frac{1}{h_0} \frac{1}{\pi p \left(\frac{g\pi p}{d\varpi^2} - cth \cdot kh \right)} \right] = \mu M_0$$

There relations are valid when the tank is acted on along an arbitrary horizontal direction. Fig.5 shows the variation of the coefficient of equivalence μ as a function of the geometry of the reservoir ($a, h, h_0 = \frac{h}{a}$) and of the oscillation period of the support T. It should be noted that for the reservoirs usually met with in practice the latter's effect in very smal, the results being in agreement with those obtained by Housner /12/, Nikolaenko-Goldenblat /1o/, Graham-Rodriguez /11/ etc.

SEISMIC FORCES ACTING ON THE CONSTRUCTION

Under these conditions the physical model of the oscillations of a water tower may be of a type best suited - to the shape and importance of the construction.

For instance, for a water tower of a compact construction we may adopt a scheme like that shown in fig.6 (a,b,c) where for every oscillation mode of the liquid mass there is a corresponding dynamic equivalent (M_e^i, Ω^i).

The scheme is generally found in the litterature and the dynamic equivalent of the structure (M,ω) can be determined with sufficient accuracy /4,5,16/.

In the case of a structure of the type shown in fig.6 (d) the foundation of which is in a soil with elastic characteristics allowing distorsions over a certain zone, the physical model may be reprezented by "finite elements".

Irrespective of the physical model selected, the general equation which governs the forced oscillations of the model under linear operating conditions, may be expressed matricially as follows

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [k]\{x\} = -[M]\{\ddot{U}_0\} \tag{35}$$

The notations used in equation (35) are those generally found in the litterature.

For the determination of the dynamic characteristics of the elastic system which corresponds to the equivalent physical model of the water tower, the undamped free oscillations characterized by the equation

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\} \tag{36}$$

shall be analyzed first.

In order to obtain the eigenvalues and the eigenvectors which characterize the natural pulsations ω_i , and the ordinates of the vibration modes $\psi^{(i)}$, we shall admit for equation (36) solutions of the form

$$\{x\} = \{\psi\} \sin(\omega t - \nu) \tag{36 bis}$$

Under these conditions,equation (36) becomes

$$\left([K] - \omega^2[M]\right)\{\psi\} = \{0\} \tag{37}$$

By marking the determinant of the matrix $[K] - \omega^2 [M]$ vanisch we obtain the following characteristic equation whose n roots represented the real and positive eingenvalnes

$$\left| [K] - \omega^2 [M] \right| = 0 \tag{38}$$

By replacing successively the eigenvalues in equation (37) we obtain the column matrix of the eigenvectors corresponding to every oscillation mode

$$\left([K] - \omega_i^2 [M] \right) \left\{ \psi^{(i)} \right\} = 0 \qquad (i = 1,2,\dots n) \tag{39}$$

were

$$\left\{ \psi(i) \right\} = \begin{Bmatrix} \psi_1^{(i)} \\ \psi_2^{(i)} \\ \vdots \\ \psi_n^{(i)} \end{Bmatrix} \tag{40}$$

It is obvious that in the case of the models shown in fig.6 (b) and (c) the calculation is considerably simplified since these systems include only two degrees of freedom.

From the point of view of the oscillations caused by an earthquake and characterized by the column matrix of the ground accelerations

$$\ddot{U}_0(t) = \begin{Bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{Bmatrix} \ddot{U}_0 \tag{41}$$

it is much more conveniant to express the forced vibrations of the system with n degrees of freedom by means of n equations independent with respect to the normal modes /16/.

The seismic forces corresponding to every vibration mode may be expressed with the help of the seismic response spectra. Thus, the general expression of a seismic force acting on the mass m_k in the vibration mode i will be

$$F_k^{(i)} = m_k a_k^{(i)} S_a^{(i)} \tag{42}$$

In the above formula (42) the expression

$$a_k^{(i)} = \frac{\sum_{k=1}^{n} m_k \psi_k^{(i)}}{\sum_{k=1}^{n} m_k [\psi_k^{(i)}]^2} \psi_k^{(i)} \qquad (43)$$

has been noted by $a_k^{(i)}$, which could be called a "form factor" /5/ since the geometrical configuration of the vibration mode $\psi_k^{(i)}$ depends on it.

The seismic acceleration spectrum has been denoted by $S_a^{(i)}$, ant the acceleration corresponding to the natural period $T_i = 2\pi/\omega_i$ (fig.7), by $S_a^{(i)}$. The ground shear force corresponding to the made $\underline{i}$ will be

$$F^{(i)} = \sum_{i=1}^{n} F_k^{(i)}$$

Torsional vibrations may be treated in the sameway thus confirming the advantages of the "finite elements" method /23/. The passage from the state of external forces applied to the structure to the state of internal stresses and distorsions may be effected by known methods and using to this end a computer.

The effect of the deformability of the foundation, ground on the dynamic response of water towers may be introduced fairly easily by the method described in work /4/.

Finally we remark that for the schemes shown in fig.6 and which represent the combinet equivalents liquid-structure, we can find single dynamic equivalents represented by a one-masse system with one degree of freedom. Under such conditions the qualitative evolution of the dynamic response of water towers acted on by seismic forces, is considerably simplified. This method of treating the problem will be dealt with in a future paper.

CONCLUSIONS

The dynamic analysis of hydrotechnical structure of the water tower type involves a hydro-elasto-dynamical treatment. The elements of hydrodynamic action expressed as a generalized equivalant,are emphasized.

The selection of a physical model with the help of a "finite elements" is convenient since it lends itself to the use of computers operating on a programm aimed at finding the eigenvectors and eigenvalues . When the vibration modes and the natural frequencies are known,, the dynamic analysis of the structure by means of response spactra constitues a usual computation method.

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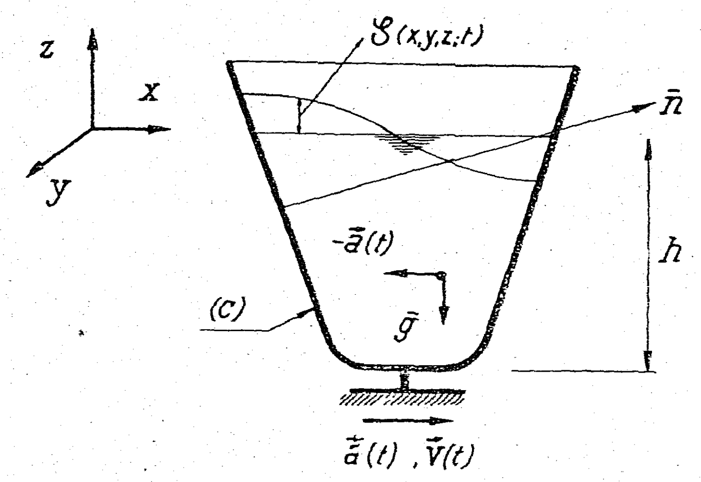

Fig. 1

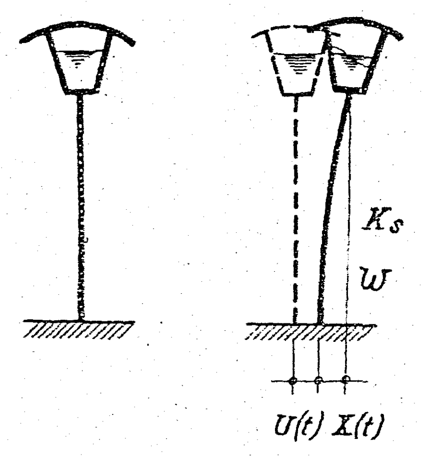

Fig. 2

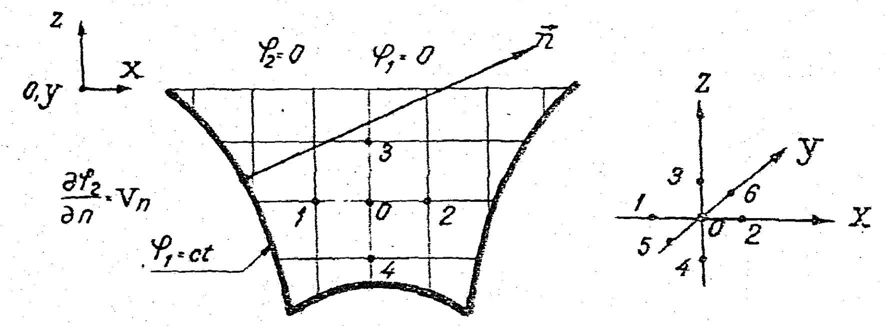

Fig. 3

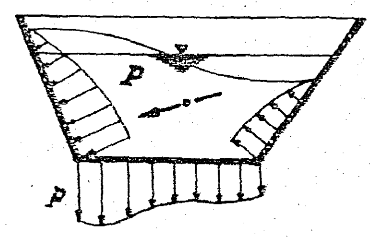

Fig. 4

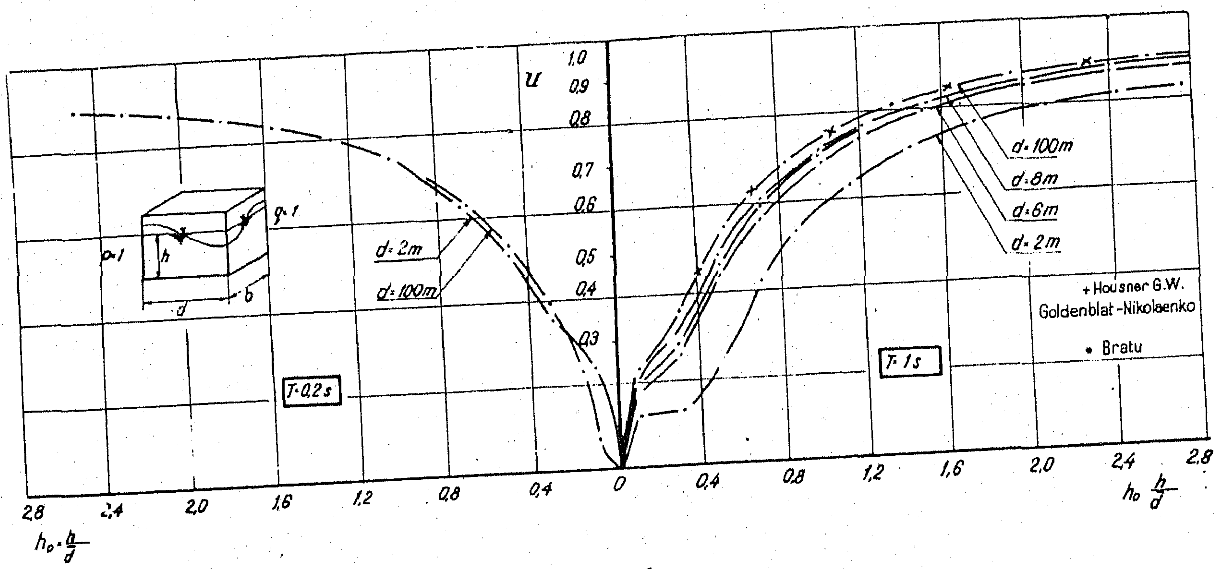

Fig.5

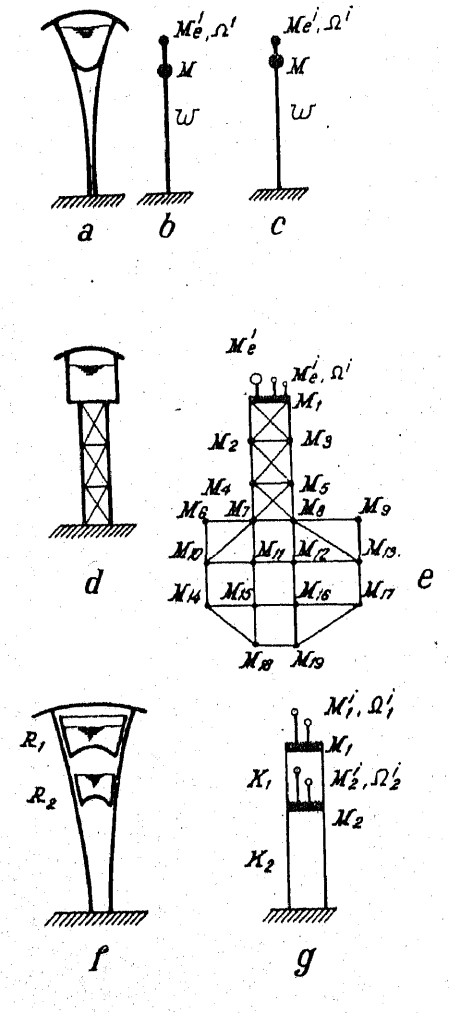

Fig.6

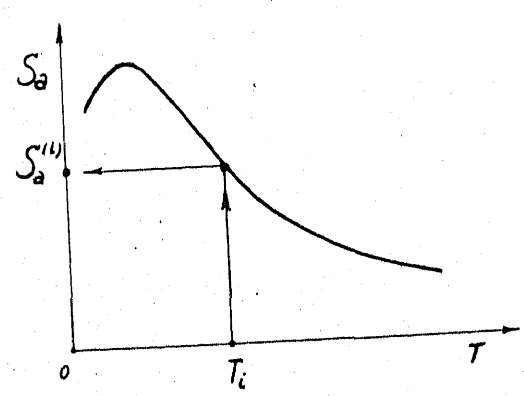

Fig.7