STUDY ON THE EARTHQUAKE PROOF DESIGN OF ELEVATED WATER TANK

by: Y. Sonobe(I) and T. Nishikawa(II)

ABSTRACT

Elevated water tank has different vibratory characteristics compared with ordinary structures, because 'water' affects the vibration behavior. Several papers on this analysis have been published, but it is very complicated to analyze precisely its vibration behavior.

This report describes the possible method of replacing the elevated water tank by the appropriate equivalent vibratory system by experimental studies and of analyzing the dynamic response of the structures supporting the water tank and fluid in the tank.

This method may be applicable to the dynamic analysis of the same type of structures, such as petroleum tank, silo etc.

It was confirmed that measured responses of the supporting structures and the fluid by vibration tests of small models for the actually designed elevated water tanks were explained by the analysis by replacing these structures to the modified spring-mass system.

Two models were made: one is a cylindrical tank model, which has 60 cm diameter and the ratio of the model to the prototype of 1/10 - 1/20; another is a spherical model, of same size. Each tank was supported by a frame which has several kind of rigidity.

By free vibration tests, experimental equations of the fundamental periods of the water (Tw), corresponding to depth of the water and the most exciting periods of the frames (T_F), were obtained. After that, forced stationary vibration tests were carried out by the electro-magnetic vibration table. By these tests, response curves, periods of the elevated tank including higher order, damping coefficients, participation functions were obtained. Furthermore, forced vibration tests were carried out, using the input of pseudo El Centro NS 1940 record (time scale 1/2.5, max. acc. 80.2 gal), to the cylinderical tank. From its response, maximum displacement and acceleration of the frame and maximum response height of the water in the tank were found out.

Considering the test results, simplified two degree of freedom system, which consisted of the weight of frame plus dead water assumed to be fastened to the tank rigidly, and the weight of the free water assumed to be fastened to the tank by springs, was analyzed by analog computer 'SERAC.'

From this response analysis, the good agreement between the experiment and the analysis was confirmed. On this basis a possible method of the actual dynamic design of elevated tanks were proposed.

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SYNOPSIS

Vibration tests were carried out on 1/10 - 1/20 scale models of cylindrical and spherical elevated water tanks.

The test comprises free and forced vibration tests under sinusoidal and random seismic wave.

In addition, a response analysis was effected on an analog computer to compare the results with those obtained in the tests.

The present research enables the authors to clarify that vibration of elevated tanks may be expressed simply as vibration of two degrees of freedom system.

1. Introduction

Rigorous treatment of dynamic analysis of an elevated water tank is very complicated in comparison with other ordinary structures, because the fluid in the vessel takes a role as a part of vibrating body. Recently, however, an analytical method of responses due to arbitrary ground motion has been developed in the range of elasto-plasticity. Therefore, if the elevated water tank could be replaced by an appropriate vibratory system, it is able to predict the behaviors of the structure supporting the tank and fluid contained in the tank during earthquake.

The dynamic analysis of these structures is felt keenly necessary by the fact that great fire of petroleum company in 1964, Niigata earthquake, was caused mainly by the vibration of fluid in the tank.

Such a study was done in Prof. Muto's Laboratory at the University of Tokyo in 1957 by one of the authors. In this study the water in the tank was divided into two parts from the vibrational behavior, one was the dead water which was fastened to the tank rigidly and the other the free water fastened to the tank by spring. The ratio of water of each part was determined quantatively by experiments (ref. 3.2.(3)), and further it was proposed that the external force to be taken in the design of the elevated water tank was obtained by multiplying the sum of the equivalent weights of structure and dead water by the seismic coefficient. After that, the study concerning with the higher modes of vibration of fluid in the tank was done, but the purpose of this study was concentrated in the problem of the dead water.

Now in the current study, the above experiments have been more developed and, aiming at a dynamic design taking the coupling effect of structure and fluid into account, small type models were made to measure the vibration

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behavior of structure and fluid contained in the tank under the arbitrary ground motion besides the stationary vibration experiments on the free water and dead water.

From these experimental results, possibility was anticipated of replacing the structure simply with a two freedom vibratory system consisting of the weight of free water and of the dead load plus dead water. Confirming the above anticipation, the authors have attempted to make a guiding principle of the earthquake-proof design of structures of this kind.

2. Outline of experiment

2.1 Models

Two prototypes as shown in Fig. 1 were taken as the subject of study, that is, an actually designed cylindrical elevated water tank of 12 - 40 m in height and 250- 400 tons in volume and a spherical one of 12 - 20 m in height and 40 - 150 tons in volume. As a principle of model making, neglecting the viscosity of water and the weight ratio of each part being equal, the reduced scale for period may be taken as $\sqrt{S}$, where S is the reduced scale for length in the model, because the vibration period of water in the tank is proportional to the square-root of its diameter.

Since the vibration behavior had not been measured on the actual elevated water tank, the natural period was estimated following the previously-made studies and the result was applied in the design of models to be used.

The models were constructed in such a way that, as shown in Fig. 2, the water tank is supported on a steel frame of 4-pillar type. Three kinds of frames of different lengths are prepared so as to make the ratio of period of frame and water be in the range of that in the actual tank. The distance between the fixed points of frame as plate spring are 39 cm, 64.5 cm. 80 cm which are common for the both models. The tank is made of aluminium plates 1 mm thick and its diameter is 60 cm in the cylindrical tank and 50 cm in the spherical tank. The bottom shape of both models has a similar shape to the actual tank and the reduced scale of length corresponds to 1/17 - 1/20 in the cylindrical model and 1/10 - 1/14 in the spherical model.

2.2 Contents of experiment

Static loading tests: The experiment was conducted to calibrate the relation between the strain and deformation of the frame.

Free vibration tests: Free vibration of the frame was caused by releasing suddenly a slight forced deformation. Periods and damping coefficients of the frames were obtained. The vibration of water was caused by moving the tank by hand and when the fundamental vibration became stationary, the period and damping coefficient were obtained.

Stationary forced vibration tests: An electromagnetic vibration table was used in this forced vibration tests, keeping the total amplitude of the table at certain definite values, the frequency of the sinusoidal wave was varied continuously.

Forced vibration tests due to actual earthquake motion: El Centro NS 1940 Earthquake record in which the time axis is reduced to 1/2.5 was taken as an input, and dynamic strain-meter was used for the measurement of response as in the case of free and forced vibration tests mentioned above. Further, in order to examine the surface wave of water, uniformly spaced 18 floats were uniformly placed in the diameter line in the direction of vibration, and their vertical movements were photographed on a movie film. This experiment was carried out for the cylindrical type water tank model only.

3. Results of tests

3.1 Static loading tests

According to the test results, the relationship of the displacement and the strain of the frame are kept linear to the storey shearing force. But when the displacement becomes large, the rigidity is slightly lowered and a trivial hysteresis characteristic is observed. Table 1 shows the spring constant according to the secant modulus corresponding to the displacements of 4, 6 and 8 mm. The theoretical values in the table are the values calculated on the assumption that the pillars are fixed at the position of bolts. Table 2 shows experimental and theoretical values of period obtained on the models without water. From these data, it is considered that the statical and dynamic spring constants are nearly coincident.

3.2 Free vibration tests

(1) Periods and damping coefficients

Cylindrical model: Fig. 3 shows the relation between the period of the fundamental vibration of water T_W, the period of the most exciting vibration of the frame T_F and the depth of water H. H means the water depth from the position at the inter section of the bottom surface and the side surface.

In each case, the 1st period of water T_W is extremely large when the water depth is low, but it is decreased with the increase of the water depth, finally converging to a certain definite value ($0.11\sqrt{D}$ - $0.13\sqrt{D}$ sec D; diameter in cm). This characteristics may be also observable in the cases of other type of water tanks to be described in 3.2 (3). Also T_W increases as the rigidity of frame becomes higher when the water depth is kept constant.

Spherical model: Fig. 4 shows the relation between the period T_W, T_F and the water quantity in the case of spherical model. T_W approaches the value of $0.1\sqrt{d}$ as the water depth is higher. As for T_F, a similar relationship is observed as in the case of the cylindrical model.

Regarding to the damping in the case of cylindrical model, any damping characteristics is hardly found for the 1st vibration of water. The amount of damping of the most exciting vibration of frame can not obtained precisely, because the wave shape of vibration becomes irregular due to the effect of water, especially its tendency is enhanced when the rigidity of frame is lowered. It seems, however, that as a general tendency the lower the rigidity of frame the larger the damping, and there is no relation to the water quantity. The damping coefficient is estimated approximately at 1 - 2% for h = 39 cm, 2 - 3% for h = 64.5 cm, and 3 - 6% for h = 80 cm.

(2) Dead water and free water

From the most exciting period of frame, quantity of dead water W_D which is assumed having the fixing effect to the frame is calculated by the following formula.

$$W_D = \left[\left(\frac{T_F}{T_{FO}} \right)^2 - 1 \right] W_{FO} \qquad (1)$$

where

T_F : Period of main vibration of frame

T_{FO} : Period of frame without water

W_{FO}: Weight of frame

Fig. 5 shows the relation between the ratio of the dead water quantity W_D of the cylindrical model and the depth of water expressed by taking the tank diameter as unity, together with the experimental formula. The free water quantity tends to converge to a definite value as the water quantity is increased. Its value is estimated at H = 0.3D in case of h = 39 cm and 64.5 cm, and H = 0.33D in the case of h = 80 cm, taking the depth from the water surface. Fig. 6 shows the relation between W_D/W_W of the spherical model and the water quantity W_W, together with the experimental formula. The ratio of dead water is increased with the increase of water quantity. The values of W_D/W_W coincide well in case of h = 39 cm and 64.5 cm, but in case of 80 cm, the free water quantity is larger than the above two, which shows a similar tendency as in the case of cylindrical model.

(3) Period of free vibration, quantities of dead water and free water in the elevated water tanks of other types

Now the results of experiments carried out in Prof. Muto's Laboratory using the following 4 types of elevated water tanks are described.

Quadratic prism tank	200 (width) x 400 (depth)
Cylindrical tank with flat base	300 ø
Cylindrical tank with the half spherical base	300 ø
Truncated conical tank	slope of generator 3.3/1

The experimental methods were almost the same as the experiments described previously. Fig. 7 shows the relation between the free vibration period and the water depth. The most exciting period of frame T_F is increased with the water depth in all tanks, whereas the period of 1st mode vibration of water T_W converges to a certain definite value in each tank. Denoting the tank diameter by D, we obtain

$T_W = 0.11\sqrt{D}$ for prismatic tank

$T_W = 0.10\sqrt{D}$ for cylindrical tank

$T_W = 0.11\sqrt{D}$ for conical tank (T_W in sec, D in cm)

Fig. 8 shows the relation of the quantity of dead water and free water. The quantity of free water approaches a definite value as the quantity of water is larger. Denoting the free water quantity by the depth H_F;

we have

$H_F = 0.3$ D for prismatic and cylindrical tank

$H_F = 0.1$ H or $W_F = 0.27$ W (by weight) for conical tank

Each values of period was confirmed by the stationary vibration tests due to sinusoidal wave.

3.3 Forced vibration test due to sinusoidal ground motion

Cylindrical tank;

The experiments were carried out for the 3 different height of frame and 3 different depth of water at 10 cm, 20 cm, 25 cm, namely 9 cases in total. In each cases the response curves were obtained for various total amplitude of the vibration table of $2a_0 = 0.1$ mm, 0.2 mm and 0.4 mm (each being kept constant during one experiment).

In Fig. 9, 3 response curves are shown on the left side, and on the right side are shown the experimentally analyzed results of each response curve and the frequency of resonance vibration as well as the participation function βu. Here the participation function for the most exciting period of frame is taken as 1.0 and that for the fundamental vibration of water is shown only when it was found. The maximum response was unable to obtain precisely because of the lack of strength of model.

As a result of this experiment, the fundamental frequency and the higher frequencies of water and the frequency of the most exciting vibration of frame were obtained. When the height of frame is low and the rigidity of frame is high, the participation function of frame for the fundamental vibration of water is considerably so small that the effect of highervibration of the frame becomes negligible. On the other hand, when the frequency of vibration of the frame becomes small and the frequency of shaking table approaches to that of the fundamental period of water, the response of frame is fairly enhanced. Also the effect of the 2nd mode of vibration of water on the frame is increased, which indicates the fact that the coupling degree between water and frame is strengthened. The resonance period is found to agree well with that obtained by the corresponding experiment for free vibration.

The maximum response ratio $\bar{f}$ at the resonance point of fundamental vibration mode of the frame was larger than 30, so the damping coefficient may be assumed to be below 1.7%. As the maximum response ratio $\bar{f}$ near the frequency of fundamental vibration had been obtained. The damping coefficient was calculated by use of the participation function βu and the result is shown in Table 3, from which the damping coefficient of the fundamental vibration of water is presumed to be on the order of 0.5%.

Spherical model;

The experiments were conducted for the three different heights of the frame and the three different depth of water 1/4, 1/2 and 3/4 the diameter of the water tank, namely in 9 cases in total. The amplitude of the shaking table was equal to that of the case of the cylindrical model. The results of experiments shows a similar tendency as in the cylindrical model, and the effect of higher mode of water is observed.

3.4 Forced vibration test using earthquake motions

The experiments was made in the case of the cylindrical model. (h = 64.5 cm, H = 20 cm)

Referring to the shear force spectrum of El Centro NS 1940 earthquake, it is known that there occur peaks at about 0.5, 0.8, 2.5 sec. From the experiments for free vibration and stationary forced vibration, the fundamental period of water were estimated at 0.15 sec, 0.32 sec, and 0.41 sec for h = 39 cm, 64.5 cm, and 80 cm, respectively. On the other hand, if the El Centro earthquake record was reduced to 1/2.5 in time scale, the peaks are estimated at 0.2 sec, 0.32 sec and 1.0 sec. In consequence, there appear large responses to the 1st order vibration of water (about 1.0 sec) and the natural period of frame.

Then the pseudo El Centro earthquake reduced to 1/2.5 in time scale was used as the input. But as the vibration table used in this experiment is not equipped with a perfect feed back circuit, the record of accelerometer put on the table is considered as the ground motion.

Fig. 10 shows the acceleration record of El Centro and that of the table (max. acc. 80.2 gal), and Fig. 11 shows the response of frame. (max. acc. 239 gal) A vertical movement of water surface was obtained by plotting the movement of 18 floats photographed on a moving film as shown in Fig. 12.

4. Response analysis

The response in the cylindrical model (h = 64.5 cm, H = 20 cm) was analyzed in which the acceleration record of vibration table shown in Fig. 10 was used as an input. The response spectrum of one mass system of this ground motion is shown in Fig. 13. For the calculation of response the analog computer "SERAC" of University of Tokyo was used.

Set-up of vibration system

Adding the weights of the frame, the water tank and dead water altogether, and replacing this vibratory body to one mass vibratory system and then analyzing its response, it is possible to obtain the response acceleration and deformation of frame easily by a rough calculation.

Here, however, in order to examine the effect of free water, the free water was replaced to one mass. Thus response analysis was made on two mass vibratory system, as a whole as shown in Fig. 14 a). To obtain K_2, the free water was considered as a one-mass system connected to the frame by a spring, and from the formula

$$T_w = 2\pi\sqrt{\frac{m_2}{K_2}}$$

we have

$$K_2 = \frac{4\pi^2 m_2}{T_w^2}$$

The value of m_1, m_2, K_1, T_w were taken equal to those resulted from the experiment. The equation of vibration of this replaced two mass system are as follows:

$$m_2\ddot{Y}_2 + C_2(\dot{Y}_2-\dot{Y}_1) \qquad + K_2(Y_2-Y_1) = -m_2\ddot{y}_0$$
$$m_1\ddot{Y}_1 + C_1\dot{Y}_1 + C_2(\dot{Y}_1-\dot{Y}_2) + K_1Y_1 + K_2(Y_1-Y_2) = -m_1\ddot{y}_0$$

where Y_i = relative displacement of ith mass

$\ddot{y}_0$ = ground acceleration

Natural periods and modes of vibration forms

In the model used for calculation, h = 64.5 cm and H = 20 cm and the values of other factors are as follows.

m_2 = 0.042 $\mathrm{Kg\cdot sec^2/cm}$

m_1 = 0.044 $\mathrm{Kg\cdot sec^2/cm}$

K_2 = 2.02 $\mathrm{Kg/cm}$

K_1 = 17.0 $\mathrm{Kg/cm}$

Using these values, the periods and modes of this vibratory system are obtained as shown in Fig. 14 b), in which the periods agree well with the experimental values.

Calculation of response

Assuming for simplicity, the damping coefficient of water to be 0 and that of frame to be 0.05, the response was calculated.

The response acceleration of frame is compared with the experimental result as shown in Fig. 12. The analytical wave and experimental wave are nearly coincident in their maximum value, component of period and number of waves, which demonstrates the effectiveness of this analysis.

If, however, more accuracy is desired, it seems preferable to determine the periods from the model experiment. Then the value of m_1, m_2, K_2 may be obtained by assuming a coupling vibration with water and frame. Especially, in a case when the degree of coupling effect is high, that is, when the fundamental period of water is close to the most exciting period of frame, an excellent appropriate value of response could not be obtained unless such a mean is taken.

Speaking definitely, if the value of K_1, and $m = m_1 + m_2$ of the model are determined, and the 1st frequency n_1 and 2nd frequency n_2 are obtained from the coupled vibration experiment, we can pursue the following procedure;

From
$$\begin{vmatrix} K_1+K_2-m_1\cdot n^2 & -K_2 \\ -K_2 & K_2-m_2\cdot n^2 \end{vmatrix} = 0$$

we have
$$m_1(m-m_1)n^4-\left\{m_1K_2+(K_1+K_2)(m-m_1)\right\}n^2+K_1\cdot K_2=0$$

$$n_1^2+n_2^2 = \frac{m(K_1+K_2)-m_1K_1}{m_1(m-m_1)}$$

$$(n_1\cdot n_2)^2 = \frac{K_1\cdot K_2}{m_1(m-m_1)}$$

Solving these simultaneous equations, m_1, K_2 are obtained and then all unknown factors are determined.

The displacement of a mass equivalent to the free water is considered to represent the wave height at the side wall of tank. In the case of a cylindrical tank, if the ratio of wave height relative to the case of a cube tank is approximately calculated by assuming the volume of the moving water to be equal to that of the cylinder, we get a result that the ratic is 1.5. This fact has been clarified by experiment too, in other words, when the water quantity is sufficiently large, 1.5 times the relative displacement of the mass equivalent to the free water obtained from the analyzed response would give the wave height of the fundamental vibration of the water, the value being estimated at 3.2 cm.

5. Conclusion

Summarizing the results of this study, the following conclusions are given.

1) Natural periods

The fundamental period of water T_W obtained by the free vibration tests and the period of the most exciting vibration of the frame are coincide with those of the reasonance period obtained from the stationary forced vibration test. In the case of the cylindrical water tank, an experimental formula; $T_w = 0.13\sqrt{D} - 0.11\sqrt{D}$ sec is obtained, provided that the water depth is sufficiently large.

2) Dead water and free water

In each model, dead water and free water are examined. In the case of the cylindrical model, it has been clarified that the quantity of free water converges to a certain definite valve.

3) Response by stationary vibration tests

The characteristics of vibration behavior of model was obtained. Especially the existence of higher order vibration of water was confirmed.

4) Response by a pseudo earthquake

Replacing the elevated water tank with a two mass vibratory system, the response for random waves can be analyzed. If the water quantity in the cylindrical model is sufficiently large, 1.5 times the relative displacement of the mass equivalent to the free water obtained by analysis would correspond to the wave height of the fundamental vibration of water

If the earthquake-proof design of an elevated water tank is to be made by the statical seismic coefficient method, it seems more reasonable to ignore the free water than to take the sum of the dead load and the total weight of water as the seismic equivalent weight as pointed out by Dr. Muto 1947. However, the necessary conditions as mentioned below should be followed in neglecting the seismic force effect of free water.

a) The period of free water is large and the response acceleration against the ground acceleration is small.

b) The period of the frame is fairly small compared with the period of the free water.

c) The higher order vibration of water is negligible.

The present study has enable us to obtain the dead water and the free water as hitherto considered, to grasp the essential of the coupling effect, and to establish a method of the dynamical analysis of the coupling effect. Further study is expected to afford significant suggestion concerning the earthquake-proof design of structures of elevated water tanks and similar structures.

ACKNOWLEDGMENT

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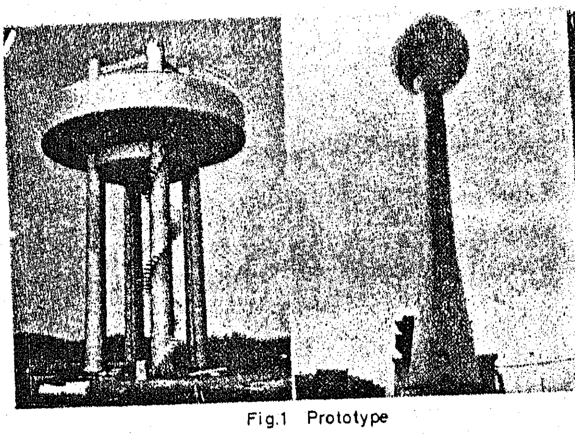


Fig.1 Prototype

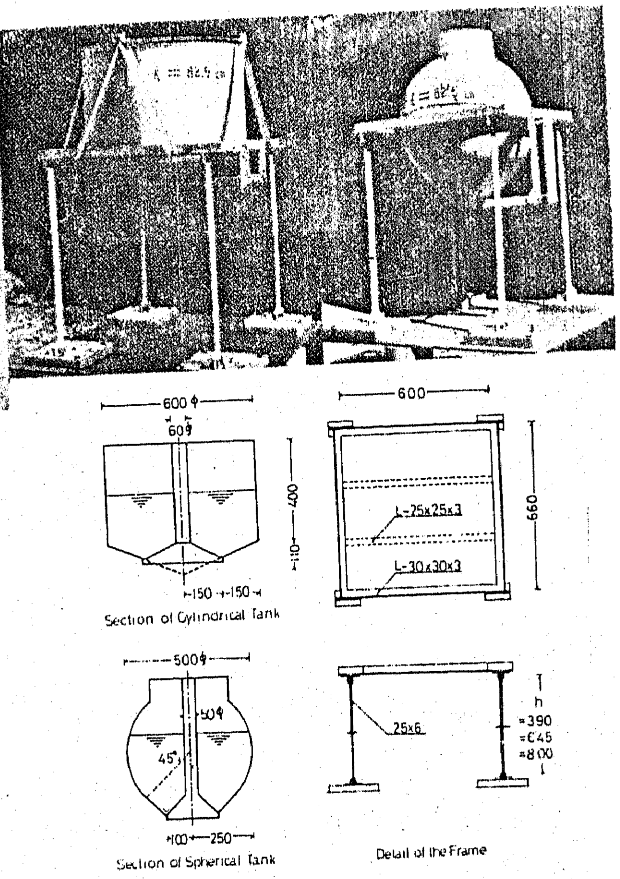


Fig.2 Model

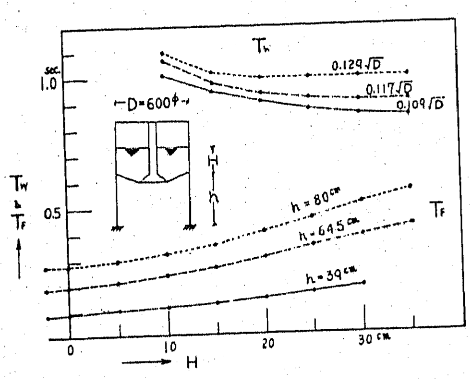


Fig.3 Periods of Cylindrical Model

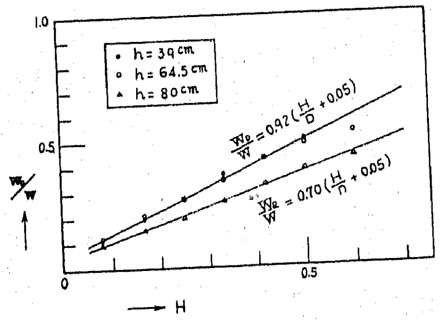


Fig.5 Dead Water of Cylindrical Model

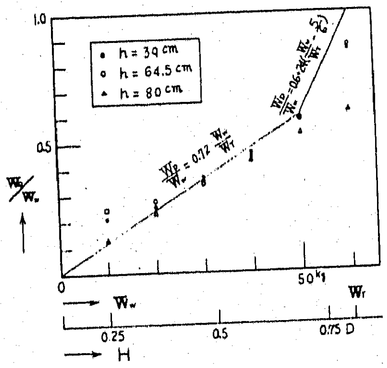


Fig.6 Dead Water of Spherical Model

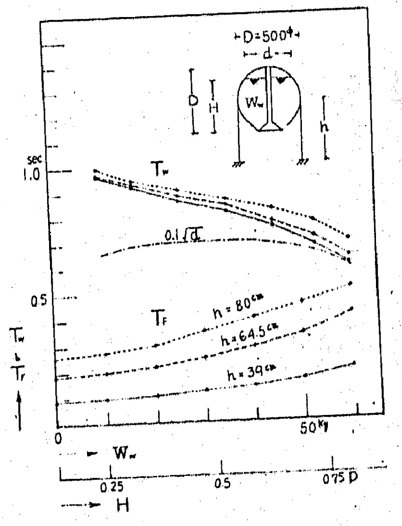


Fig.4 Periods of Spherical Model

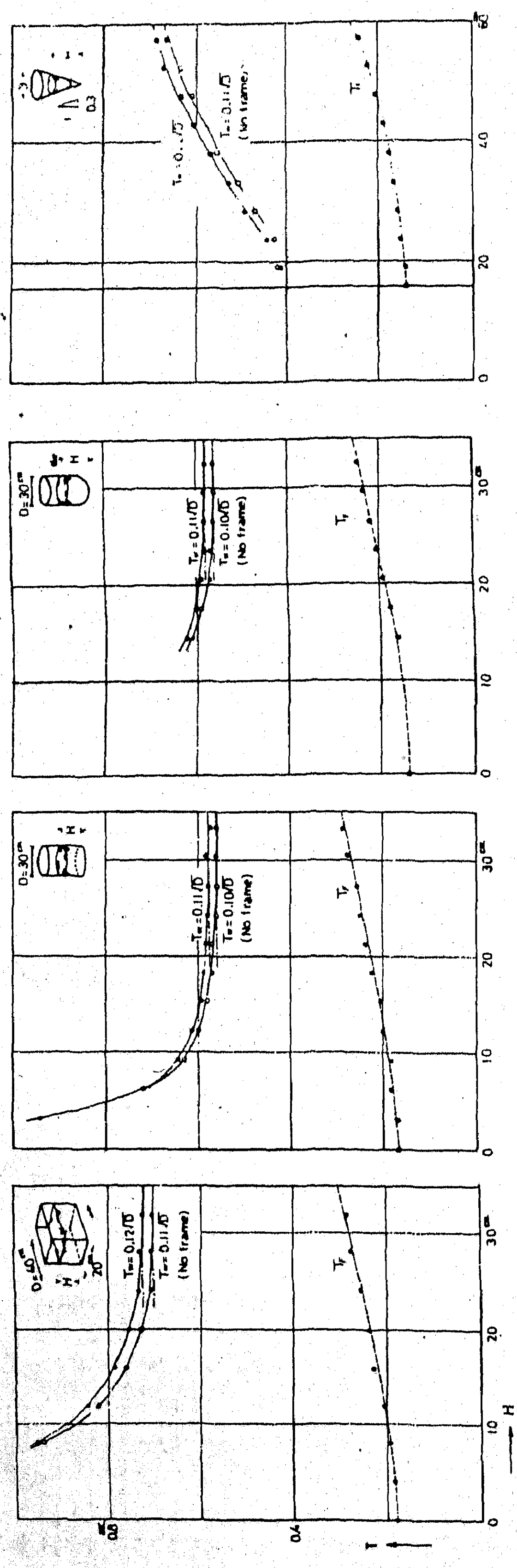


Fig.7 Periods of Other Type Models

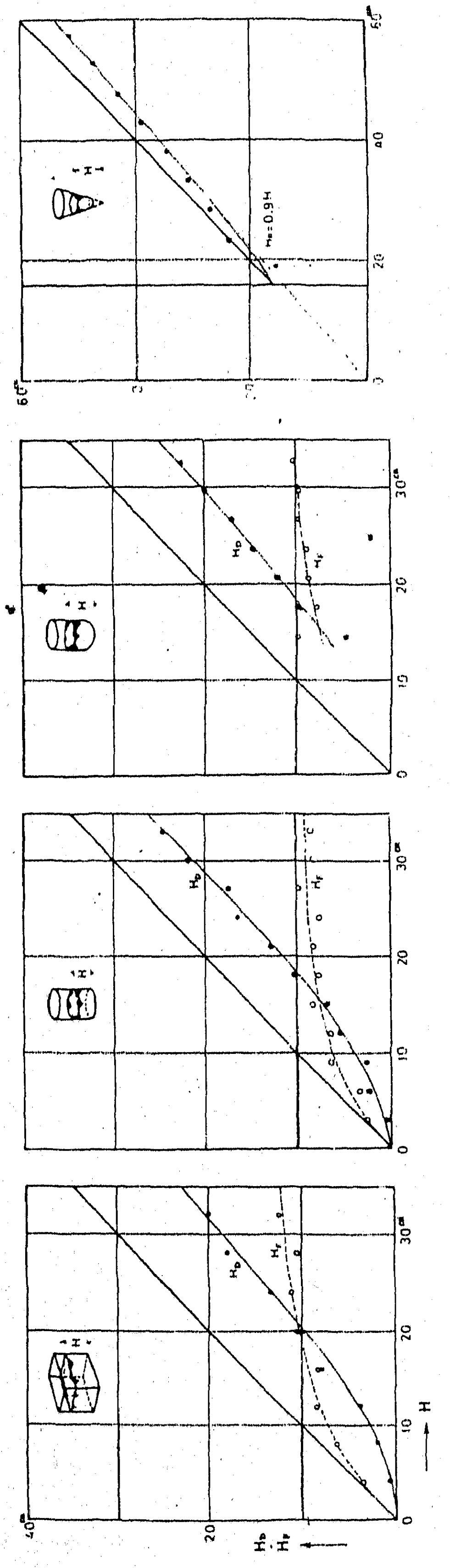


Fig.8 Dead Water of Other Type Models

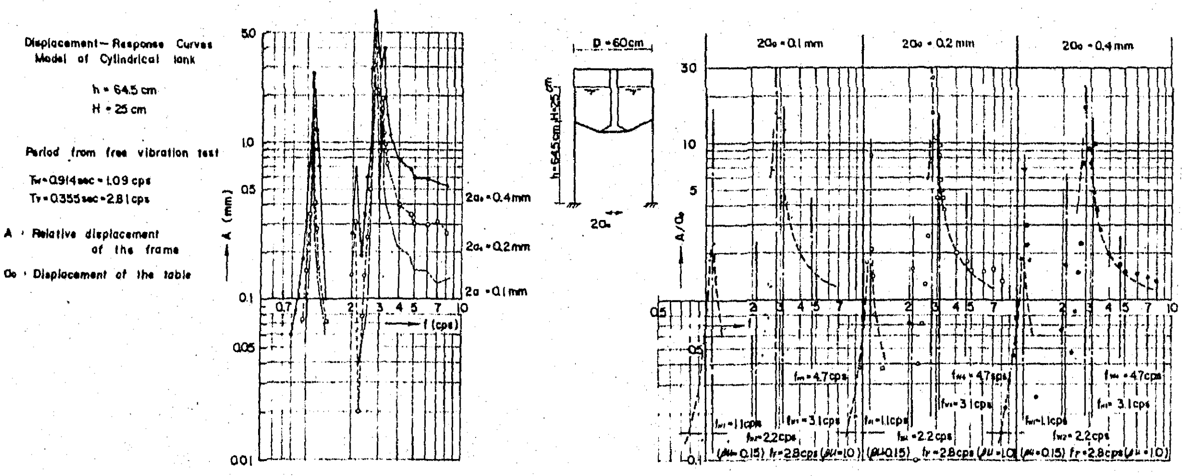


Fig.9 Resonance Curve

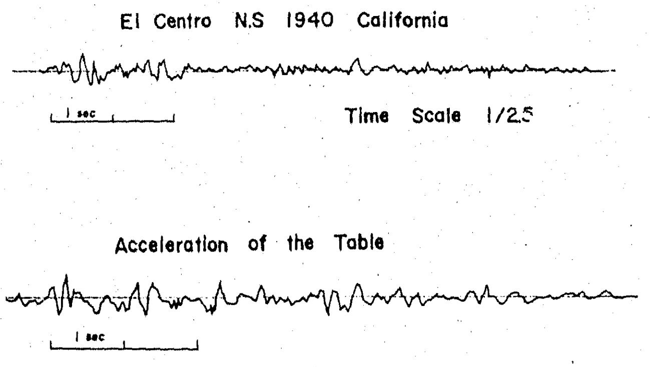


Fig.10 Input

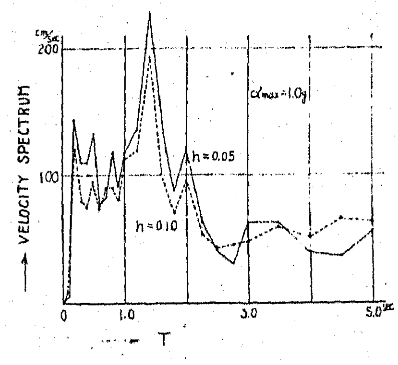


Fig.13 Velocity Spectrum of the Table Motion

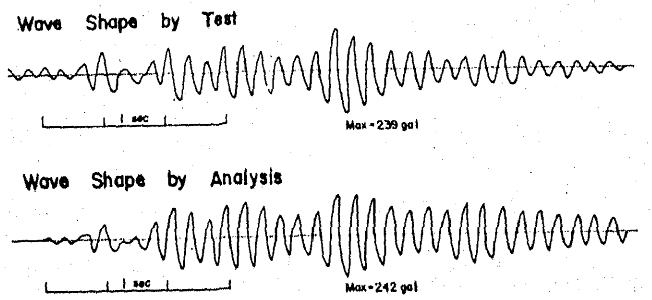
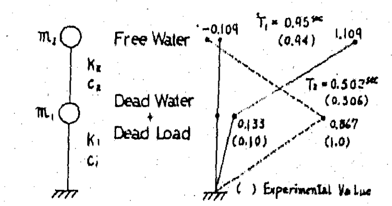


Fig.11 Response Acceleration



a) 2 Mass System b) Modes and Periods

Fig.14 Replaced Vibratory System

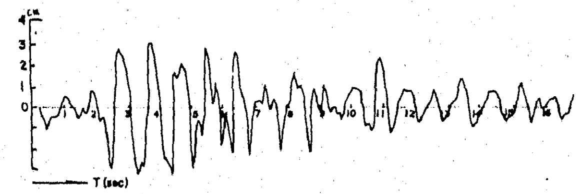


Fig.12 Response Fluid Height at the Tank Wall

Table 1 Spring Constant of Model Frame (Kg/cm)

Frame Height	Theory	Test(secant modulus)		
		$\delta = 0.4^{cm}$	$\delta = 0.6^{cm}$	$\delta = 0.8^{cm}$
h = 39 cm	76.5	80.0		
h = 64.5	16.8	16.8	16.5	
h = 80	8.86	8.86	8.55	8.40

note : δ = Frame Sway

Table 2 Period of Model without Water

Frame Height	Theory		Test
	Cal.Spring Const.	Test Spring Const.	
h = 39 cm	0.090 sec	0.088 sec	0.094 sec
h = 64.5	0.195	0.195	0.197
h = 80	0.283	0.283	0.283

Table 3 Damping Factor h of 1st Mode Vibration of Water

Frame Height	Water Level	h
64.5 cm	20 cm	0.50 %
	25	0.65
80	10	0.29
	20	0.50
	25	0.58