EARTHQUAKE ANALYSIS OF RESERVOIR-DAM SYSTEMS

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SYNOPSIS

The potential of the finite element method in analyzing the earthquake response of reservoir-dam systems is investigated. An exact solution is derived for the dynamic response of a rectangular reservoir with rigid walls, subjected to arbitrary excitations. The response of this system to unit step function excitation is computed by the exact analysis and also by the finite element method. The results demonstrate that the finite element solution compares satisfactorily with the exact solution. Results for response of the simple system to the El Centro 1940 earthquake obtained by the two methods are also compared. The application of the finite element method to a complete analysis of the earthquake response of a reservoir-dam system is demonstrated by an example.

INTRODUCTION

The analysis of effects of earthquakes on dams involves consideration of the dynamic behavior of the complete reservoir-dam system. The interaction between the dam and the stored reservoir of water presents a major difficulty in analysis of the total system. As a result, previous work has been concerned with two uncoupled problems: (1) Earthquake response of the dam ignoring hydrodynamic effects; and (2) Hydrodynamic pressures on a rigid dam during an earthquake. Treating these problems as two-dimensional, solutions have been obtained under rather general conditions. The earthquake response of a dam cross-section, without the reservoir, can be satisfactorily analyzed by representing it as an assemblage of finite elements[1]; and satisfactory techniques exist for determining hydrodynamic pressures acting on rigid dams during arbitrary earthquake ground motions[2]. If the interaction between the dam and the reservoir is negligible, the effects of the reservoir may be replaced by a set of external pressures equal to the hydrodynamic pressures that would develop if the dam were rigid. The problem then reduces to analyzing the response of the dam subjected simultaneously to the ground acceleration and the external pressures on the upstream face. This can be handled directly by the finite element method.

The above-mentioned procedure, however, ignores the interaction between the dam and the reservoir. A completely general analysis of the coupled reservoir-dam system is formidable. However, the problem has been solved under the restriction that the dam is constrained to deform in its fundamental mode[3,4]; the reservoir being treated as a continuum whose motion is governed by the two-dimensional wave equation. This study[4] has demonstrated that it is important to recognize the interaction between the dam and the reservoir in analyzing the response of dams to earthquakes.

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It appears that it is possible to extend the analysis to include the contribution of higher modes of the dam. However, the scope of the analysis in this approach will have to be restricted to dams with vertical upstream face and reservoirs of rather simple geometry in order that the hydrodynamic problem can be solved analytically.

An alternative approach which may be pursued is to represent the complete reservoir-dam system as an assemblage of finite elements. The finite element method has already proven to be a powerful tool for the dynamic analysis of two-dimensional solids[1,5]. It may appear that existing finite element programs for two-dimensional solids would be capable of treating water as it may be considered as a special case of an isotropic elastic solid. However, the capability of the finite element method to treat this class of problems remains to be demonstrated. The purpose of the present study is to investigate the potential of the finite element approach in analyzing the earthquake response of reservoir-dam systems.

FINITE ELEMENT METHOD FOR DYNAMIC ANALYSIS OF STRUCTURES

Finite Element Method

The finite element method[6] is a modern, computer-oriented approach to the analysis of continuous structures. The method is very effective in the idealization of complex structures of arbitrary shapes. The technique has been extensively applied to earthquake analysis of earth dams[1]. The problem was treated as two-dimensional and the dam cross-section was represented as an assemblage of constant strain triangular elements. The element used in the present investigation is a plane strain quadrilateral element composed of two four nodal point triangles. The development of the stiffness properties for this element is presented in another paper[5]. The element stiffness matrix is a function of the geometric and constitutive properties of the element. The stiffness matrix [K] of the complete structural assemblage may be obtained from the individual element stiffness matrices by direct stiffness assembly procedures. In the present study, a physical lumped mass approximation is used to represent the inertial properties of the system; one-fourth of the mass of each element is lumped at its nodal points. The diagonal mass matrix [M] for the complete structural assemblage is then easily obtained.

In this investigation, water is represented by the same type of finite element which has been used in the analysis of solids. If the constitutive properties are expressed in terms of the bulk modulus and the shear modulus, the water finite element has a zero shear modulus.

Analysis of Dynamic Response

The equations of motion for a two-dimensional structure, idealized as a plane strain finite element system, subjected to dynamic loads, may be expressed in matrix form as

$$[M]\{\ddot{r}\} + [C]\{\dot{r}\} + [K]\{r\} = \{R(t)\} \qquad (1)$$

In Eq. 1, [M] and [K] are the mass and stiffness matrices obtained by the finite element procedure described above and [C] is a viscous damping matrix for the finite element system. $\{r\}$ is the vector of 2N nodal point displacements where N is the number of nodal points in the finite element idealization; $\{\dot{r}\}$ and $\{\ddot{r}\}$ are respectively the nodal point velocity and acceleration vectors. In the case of earthquake excitation the load vector $\{R(t)\}$ is a function of the nodal point masses and ground accelerations.

Eq. 1 may be solved by the mode-superposition approach[1] and it has been possible to analyze large finite element systems by this method. The largest system that can be solved by this approach is limited basically by the computer storage requirements for solution of the characteristic value problem. Alternatively, the equations of motion (Eq. 1) may be directly solved by a step-by-step method of integration[5] and much larger systems can be analyzed. The step-by-step solution technique is employed in the present investigation. Once the displacement vector $\{r(t)\}$ has been determined, the stresses $\{\sigma(t)\}_p$ in element "p" at any instant of time may be obtained from the nodal displacements $\{r(t)\}_p$ for that element by

$$\{\sigma(t)\}_p = [S]_p \{r(t)\}_p \tag{2}$$

where the stress transformation matrix $[S]_p$ takes account of the assumed displacement field for the element as well as the constitutive properties.

Computer Program

A digital computer program has been developed to perform the analysis outlined above. The program is capable of analyzing plane strain structures of arbitrary shape and material properties. The basic input to the program is the geometry of the finite element system and the constitutive properties of all the elements. The excitation may consist of a horizontal and the vertical component of the earthquake ground acceleration. The program can analyze finite element systems with as many as 1000 nodal points, on a computer with 32K storage. The program listing and its usage is presented in a previous publication[7].

COMPARISON OF FINITE ELEMENT AND EXACT ANALYSES

In general, a reservoir-dam system is a three-dimensional non-homogeneous continuum of rather complicated geometry. In many practical situations the dam is rather long compared to the cross-sectional dimensions. It may therefore be reasonable to simplify the problem to earthquake analysis of the two-dimensional vibration of a cross-section of the reservoir-dam system shown in Fig. 1a. An exact analysis of the earthquake response of this system is not possible even if the dam were assumed to be rigid. The problem is much more complex if the flexibility of the dam is to be recognized. In case the dam is assumed to be rigid, difficulty arises mainly in treating the complicated geometry of the system. The reservoir-dam system of Fig. 1a may be idealized as an assemblage of finite elements and the flexibility of the dam, the arbitrary geometry and material nonhomogeneity of the system, can all be treated rationally. A simple system which can be analyzed exactly is chosen. This system is shown in Fig. 1b and consists of a rectangular reservoir of length L and depth H with rigid walls subjected to prescribed accelerations $\ddot{u}^l_g(t)$ and $\ddot{u}^r_g(t)$ at the left and right boundaries respectively. A finite element solution of this problem will be compared against the exact solution to demonstrate the effectiveness of the finite method.

Exact Solution

Disregarding the energy dissipation due to internal viscosity of water as well as other causes and considering the movements as limited to small amplitudes, the motion of water is governed by the wave equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{w}{gK} \frac{\partial^2 \phi}{\partial t^2} \tag{3}$$

where $\phi(x,y,t)$ is a velocity potential such that

$$\frac{\partial u}{\partial t} = - \frac{\partial \phi}{\partial x} \quad \text{and} \quad \frac{\partial v}{\partial t} = - \frac{\partial \phi}{\partial y} \tag{4}$$

where u and v are respectively the x- and y- components of displacement of water in the coordinate system of Fig. 1b, t is the time variable, w is the unit weight of water, g is the acceleration of gravity, and K is the bulk modulus of water. The hydrodynamic pressure $p = p(x,y,t)$ is given by

$$p = \frac{w}{g} \frac{\partial \phi}{\partial t} \tag{5}$$

and the velocity of sound in water is given by

$$C = \sqrt{\frac{gK}{w}} \tag{6}$$

Neglecting the effect of surface waves, the boundary conditions are

$$\begin{aligned} &\frac{\partial \phi}{\partial y} (x,0,t) = 0 \\ &\frac{\partial \phi}{\partial t} (x,H,t) = 0 \\ -&\frac{\partial}{\partial t} \left(\frac{\partial \phi}{\partial x}\right) (0,y,t) = \ddot{u}_g^l(t) \\ -&\frac{\partial}{\partial t} \left(\frac{\partial \phi}{\partial x}\right) (L,y,t) = \ddot{u}_g^r(t) \end{aligned} \tag{7}$$

Considering the reservoir to be at rest when the earthquake motion begins, the initial conditions are

$$\phi(x,y,0) = \frac{\partial \phi}{\partial t} (x,y,0) = 0 \tag{8}$$

A procedure suitable for solving Eq. 3 with boundary conditions of Eq. 7

and initial conditions of Eq. 8, for arbitrary excitations $\ddot{u}_g^l(t)$ and $\ddot{u}_g^r(t)$ is the use of an impulse response with the convolution integral. Consider the excitation for the system as

$$\ddot{u}_g^l(t) = \delta(t), \ \ddot{u}_g^r(t) = 0 \tag{9}$$

where $\delta(t)$ is the Dirac delta function. The response to the excitation of Eq. 9 is denoted by $h_\phi^l(x,y,t)$. This unit impulse response function is obtained by solving Eq. 3 by Laplace transform technique with separation of variables, and may be expressed as

$$h_\phi^l(x,y,t) = \frac{8}{\pi L} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^{m+n}}{(2m-1)(\alpha_n^2+\beta_m^2)} \cos \beta_m y \cos\{\frac{n\pi(L-x)}{L}\} \cos \omega_{mn} t \tag{10}$$

where $\beta_m = \frac{(2m-1)\pi}{2H}$, $\omega_{mn} = C\sqrt{\alpha_n^2 + \beta_m^2}$, and $\alpha_n = \frac{n\pi}{L}$. The response ϕ to the excitation

$$\ddot{u}_g^l(t) = 0, \ \ddot{u}_g^r(t) = \delta(t) \tag{11}$$

is denoted by $h_\phi^r(x,y,t)$ and may be obtained by solving Eq. 3 in exactly similar manner. This unit impulse response function is given by

$$h_\phi^r(x,y,t) = \frac{8}{\pi L} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^{m+n}}{(2m-1)(\alpha_n^2+\beta_m^2)} \cos \beta_m y \cos \frac{n\pi x}{L} \cos \omega_{mn} t \tag{12}$$

Using Eq. 5 the corresponding unit impulse response functions for the hydrodynamic pressure are given by

$$\begin{aligned} h_p^l(x,y,t) &= \frac{w}{g} \frac{\partial}{\partial t} h_\phi^l(x,y,t) \\ h_p^r(x,y,t) &= \frac{w}{g} \frac{\partial}{\partial t} h_\phi^r(x,y,t) \end{aligned} \tag{13}$$

The hydrodynamic pressure $p(x,y,t)$ due to arbitrary excitations can be expressed in terms of the unit impulse responses by the convolution integral. Thus,

$$p(x,y,t) = \int_0^t [\ddot{u}_g^l(\tau)\, h_p^l(x,y,t-\tau) + \ddot{u}_g^r(\tau)\, h_p^r(x,y,t-\tau)]\, d\tau \tag{14}$$

The total hydrodynamic force on the left boundary

$$F(t) = \int_0^H p(0,y,t)dy \tag{15}$$

Numerical Results and Comparison

The 'exact' total hydrodynamic force F(t) due to arbitrary excitations may be determined numerically from Eqs. 10, 12-14, and 15. The differentiations in Eq. 13 and integration in Eq. 15 can be performed explicitly, prior to the numerical computations. The total hydrodynamic force can also be determined by representing the system of Fig. 1b as an assemblage of finite elements (indicated by dashed lines) and analyzing the response of the finite element system by the computer program described above. The response to excitations $\ddot{u}^l_g(t)$ and $\ddot{u}^r_g(t)$ as determined by the two methods may be compared to evaluate the effectiveness of the finite element method. However, the results of such a comparison will depend on the selected excitations. Because the response to arbitrary excitations is a superposition of unit impulse responses, it is logical that the unit impulse responses form the basis for comparison. The determination of the unit impulse response by the finite element method, using the computer program described above, poses certain problems because of the nature of the Dirac delta function. However, the response to a unit step function can be conveniently obtained by the finite element method. The response to unit step function is the integral of the unit impulse response and is an equally basic response quantity; it may therefore be used instead of the unit impulse response, as the basis for comparison.

The properties selected for the system of Fig. 1b are C = 4720 fps, w = 62.5 pcf, and L = H = 300 ft. The excitation to the system is

$$\frac{\ddot{u}^l_g(t)}{g} = \frac{\ddot{u}^r_g(t)}{g} = 1, \ t \geq 0 \qquad (16)$$

The 'exact' hydrodynamic force due to this excitation is computed and shown in Fig. 2. It should be noted that the response has been normalized with respect to the hydrostatic force $wH^2/2$. The system of Fig. 1b is idealized as an assemblage of 100 finite elements, square in shape and equal in size. Because the viscosity of water is negligible, the finite element system is treated as undamped. The response to the excitation of Eq. 16 is determined by the computer program described above. The total hydrodynamic force response is computed using different values of the integration interval Δt. Results for two values of Δt = 0.0025 and 0.001 sec. are shown in Fig. 2. The results presented in Fig. 2 demonstrate that the amplitude of response obtained from the finite element method differs slightly from the 'exact' value. In addition, there is a phase shift in the finite element solution relative to the 'exact' solution. It is apparent that the finite element solution compares satisfactorily with the 'exact' solution. The results from the finite element method improve as Δt is reduced from 0.0025 to 0.001 sec. However, the results obtained with Δt = 0.0025 are quite satisfactory and the extra computational effort involved when Δt = 0.001 does not appear to be justified for the small improvement in results.

The comparison of unit step responses presented above demonstrates that the finite element method leads to results which compare satisfactorily with the 'exact' solution. It is of interest to determine the magnitude of error in the finite element solution during earthquake type excitations. The response of the simple system selected above is determined for $\ddot{u}_g^1(t) = \ddot{u}_g^r(t)$ = North-South component of ground acceleration during the El Centro 1940 earthquake. The 'exact' solution and the finite element solution with $\Delta t = 0.0025$ sec. are presented in Fig. 3. It is apparent that the finite element method leads to results in good agreement with the 'exact' solution for the first two seconds of the earthquake; there is significant discrepancy in the two solutions for the next one second and subsequently the two solutions are 'close' to each other. The finite element method, thus, leads to satisfactory results for the first few seconds of the earthquake. However, significant errors may be involved in the finite element solution as time progresses. It is quite likely that the errors will decrease appreciably if Δt was taken equal to 0.001 sec.

APPLICATION OF FINITE ELEMENT METHOD TO PRACTICAL PROBLEMS

The finite element method is used to analyze the earthquake response of a reservoir-dam system. The system is shown in Fig. 4, and consists of a 300 ft. high dam storing a 300 ft. deep, 300 ft. long reservoir. The simple geometry selected for the dam and reservoir is not essential, and arbitrary geometry can be treated without additional effort. The properties selected for mass concrete are modulus of elasticity, $E = 4 \times 10^6$ psi, Poisson's ratio $\nu = 0.167$, and unit weight, $\gamma = 150$ pcf. For water, C is taken equal to 4720 fps and unit weight, γ equal to 62.5 pcf. The dam as well as the water is assumed to be undamped in this analysis. The finite element idealization for the structure, with the appropriate geometric boundary conditions is shown in Fig. 4. In addition, it is required that the x-component of displacement of the upstream face of the dam be the same as the x-component of displacement of water particles in contact with it; whereas, relative displacement in the y-direction is permitted. This is achieved by introducing fictitious, horizontal, one-dimensional finite elements at the upstream face of the dam.

The response of this reservoir-dam system subjected to the North-South component of the El Centro 1940 earthquake is determined by the computer program described above; Δt being taken as 0.0025 sec. The time-history of the total hydrodynamic force on the upstream face of the dam is shown in Fig. 5. The corresponding force on a rigid dam is also presented; this result having been reproduced from Fig. 3. It is apparent that the flexibility of the dam has significant effect on the hydrodynamic force. The *increase* in natural period of the reservoir system, due to the flexibility of the dam is also evident from Fig. 5. The results of analysis include the complete time-history of deformations and stresses in the dam; however, these results are not presented here.

CONCLUDING REMARKS

The investigation reported above demonstrates that the finite element method offers a promising approach for the earthquake analysis of reservoir-dam systems. At present, certain errors exist in the finite element solution, and work is underway to study the source of these errors and to improve the analysis technique. The finite element method, by its very nature, is ideally suited for analyzing dam cross-sections of arbitrary geometry and material properties storing reservoirs of arbitrary configurations. The damping in the dam can be included, and effects of vertical component of ground motion can be considered in the analysis without any additional effort. The foundation interaction effects may be included in the analysis by extending the finite element mesh into the foundation, and the method may be readily extended to treat spatial variations in earthquake ground motion[1].

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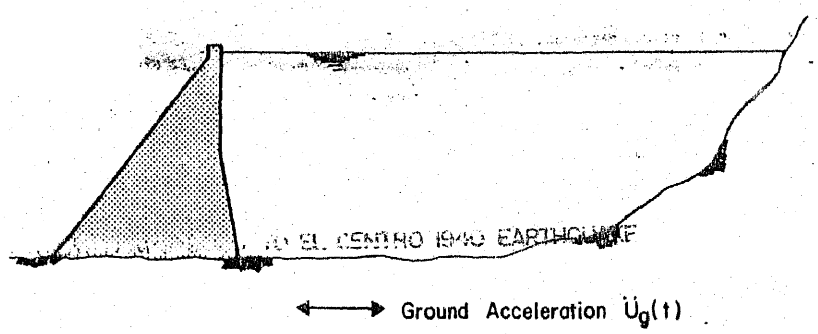

FIG. Ia. RESERVOIR- DAM SYSTEM

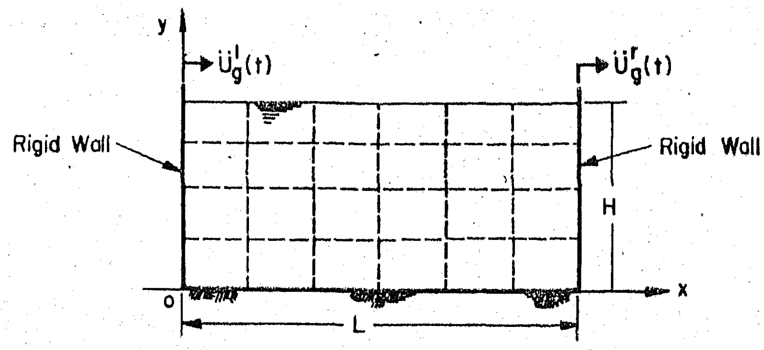

FIG. Ib. SIMPLE RESERVOIR - DAM SYSTEM

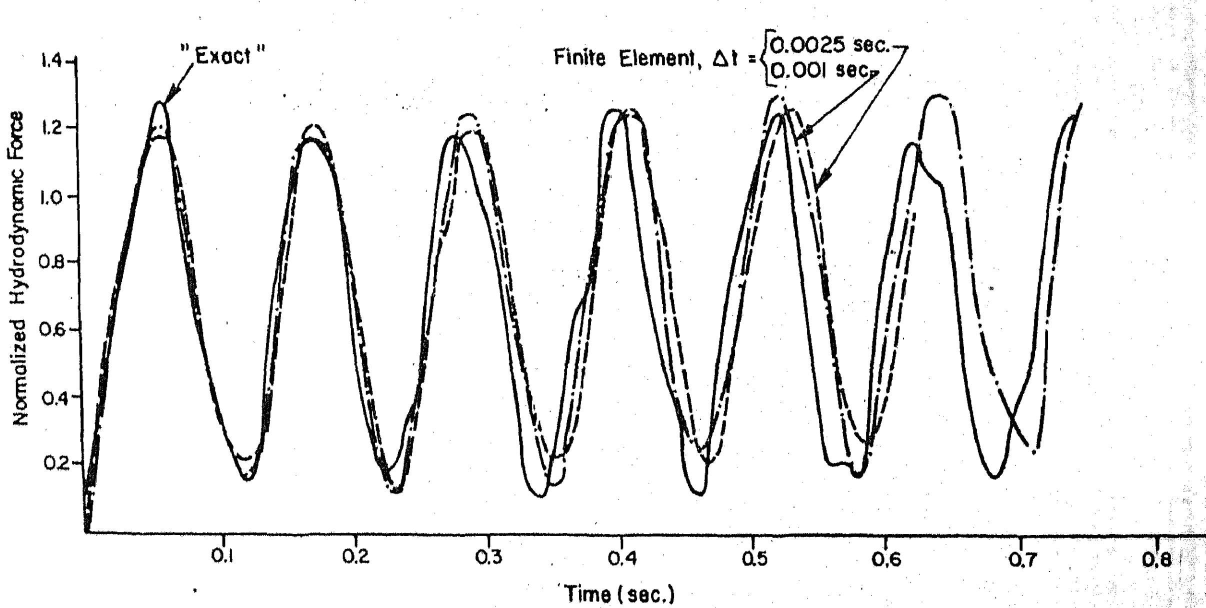

FIG. 2 UNIT STEP RESPONSES

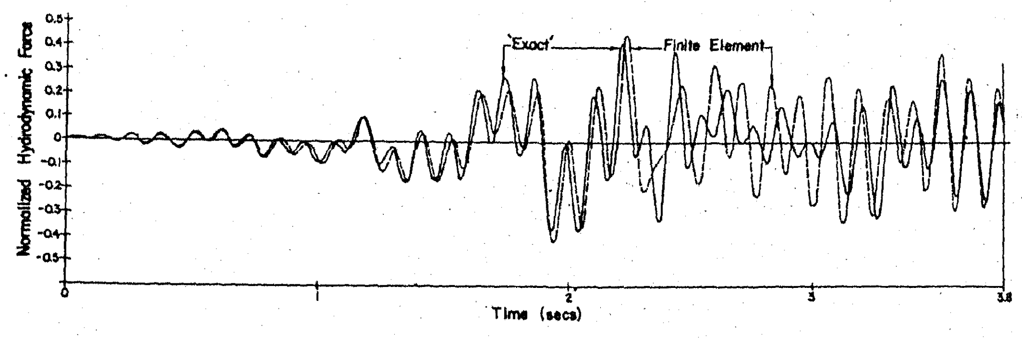

FIG. 3 . RESPONSE TO EL CENTRO 1940 EARTHQUAKE

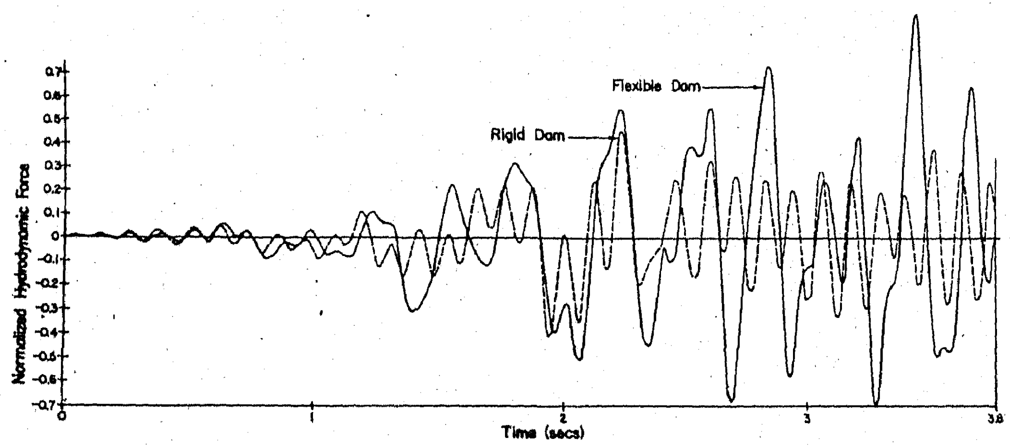

FIG. 5 RESPONSE OF RESERVOIR-DAM SYSTEM TO EL CENTRO 1940 EARTHQUAKE

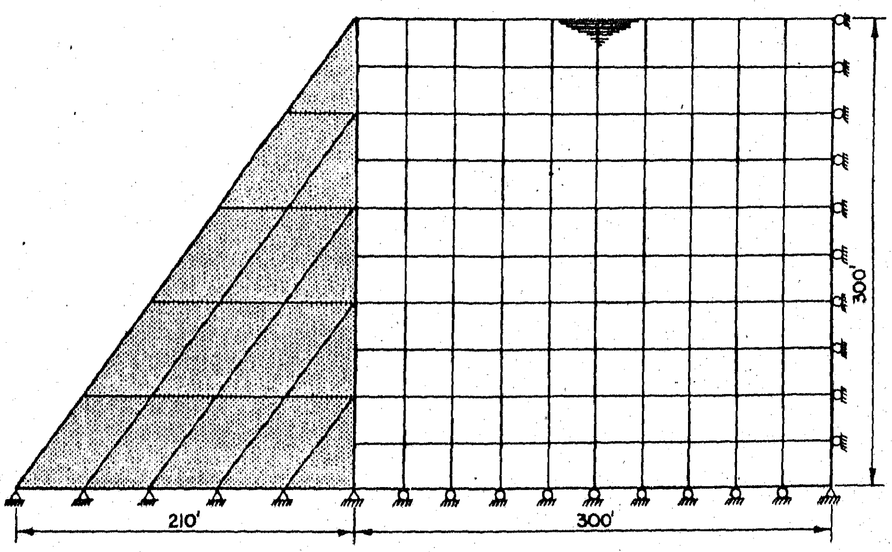

FIG. 4 . FINITE ELEMENT IDEALIZATION OF RESERVOIR-DAM SYSTEM