ANTISEISMIC DESIGN OF MULTI-STORY STEEL FRAMES BY PLASTIC METHODS

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Synopsis

A method is presented for the determination of the behavior, in the plastic range, of multi-story steel frames under the combined action of vertical and horizontal loads. The method is applied to a study of the behavior of a frame, including the effect of composite action between girders and concrete slab. It is shown that the method can also be used to improve elastic designs of frames, and a discussion is included of the various factors which influence the behavior of steel multi-story frames under seismic forces.

Nomenclature

A	=	area of member
E	=	elastic modulus
I_c	=	column strong axis moment of inertia
I_g	=	girder strong axis moment of inertia
L_g	=	girder span length
M	=	bending moment
M_p	=	plastic moment
M_{pc}	=	reduced plastic moment
M_r	=	restraining moment
P	=	applied axial force
P_y	=	axial yield force = $A\ \sigma_y$
Q	=	shear resistance of sway subassemblage or story
h	=	story height
h/r	=	slenderness ratio
Δ	=	subassemblage or story sway deflection
$P-\Delta$	=	secondary story moment
θ	=	joint rotation
λ	=	shear distribution coefficient
σ_y	=	yield stress of material

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Introduction

A method for the plastic analysis and design of unbraced multi-story steel frames is presented in the first part of this paper. These frames depend only on their own strength and stiffness for supporting lateral loads and to avoid overall instability.

The purpose of the method is to check the girder and column sizes established in a preliminary design, in order to know if they are able to resist the action of the exterior loads or if they have to be strengthened (methods available to make the preliminary design will not be discussed here, because they have been described in the literature; see, for instance, refs 1 and 2); it is suited only for those stories of tall buildings whose design is controlled by the combination of vertical and horizontal loads.

The method is also useful to check elastic designs with the aim of exploring the behavior of the structures in the elastoplastic range and to determine their correct strength, stiffness and capacity to absorb energy.

Checking of the preliminary design is done by drawing the horizontal load-lateral deflection curve of the structure. To obtain this curve it is assumed that the horizontal loads are gradually applied to the structure, after the application of the complete vertical load (it is assumed that the complete vertical loads, multiplied by the load factor corresponding to the combination of vertical and horizontal loads, are first applied, and afterwards the horizontal loads are gradually applied until they reach their final values, multiplied also by the same load factor). The curve is plotted taking into account the P-Δ effect, due to the action of the vertical loads on the deflected columns.

The knowledge of the complete horizontal load-sway deflection relationship provides the designer with all the information he needs on the behavior of the structure, because the curve permits him to determine the maximum story strength or the strength corresponding to a given value of the sway deflection. It provides, also, a measure of the structure's ductility and energy absorption capacity.

The members selected from the preliminary design will be adequate if the load-deformation behavior of the structure is satisfactory; if a revision of member sizes is necessary, the first analysis will assist in the selection of the new members, and the second stage is carried through.

The method that will be employed for the construction of the horizontal load-sway deflection curve has been developed mainly at Lehigh University. Nevertheless, it has only been presentd, until now, in a form which requires using a set of charts to plot the curve[3,4], and it is not useful when the parameters of the structure lie outside the range covered by the charts.

This paper follows basically the Lehigh method, but the procedure used to plot the lateral load-sway deflection curves is modified in such a way that the charts are no longer necessary. The numerical work required to solve a given problem is, at the same time, considerably reduced, with little loss of accuracy in all cases of practical interest[5].

The second part of the paper is devoted to the application of the method to the design of a story of a multi-story frame and to the discussion of the behavior of the structure under several conditions.

Description of the design method

In this paper will be presented only the main features of the procedure to be followed to obtain the lateral load-sway deflection curves; a more detailed description has been presented elsewhere[5].

In order to simplify the method and to make it applicable to every-day design problems no attempt will be made to derive the complete frame lateral load-sway deflection curve (or Q-Δcurve), but to obtain the Q-Δ curve for each story independently; following this procedure it is possible to design any story without taking into consideration the rest of the frame, and in practical problems it is generally not necessary to check every story against sway, but only some of them, suitably choosen.

The first step consists of the isolation of the story which is going to be designed, including the girders of the n^{th} floor and the columns below them, by cutting through the inflection points of these columns and substituting the columns above with the bending moments, axial forces and shears that they transmit to the joints in the n^{th} floor. Fig 1 shows diagramatically the simplified structure, the loads acting upon it and the corresponding deflections: the columns between floors n-1 and n and the girders at the n^{th} floor are then to be designed to resist all the forces shown in the figure.

The following assumptions will be made

1. Story heights above and below the n^{th} floor are equal to each other

2. Inflection points in columns above and below the floor in question lie at mid-height of the stories

3. The shear distribution constants λ_A, λ_B , etc, are the same for both stories

4. Column axial loads remain constant during the application of the horizontal load.

The soundness of these assumptions is discussed in ref 3.

Further simplifications in the design procedure are obtained by dividing the structure shown in fig 1 in several individual subassemblages, each one composed by a column and the girder or girders adjacent to it. The four sway subassemblages at the n^{th} floor of the frame shown in fig 1 are shown in fig 2, in which it is assumed, as in the rest of the paper, that the lateral load is applied from left to right.

The structure shown in fig 1 has been reduced to four sway subassemblages; each of these has only one column which must resist the lateral load, aided by the girders rigidly connected to its upper end; these girders act as restraining members and allow the column to adopt a lateral load resisting con figuration.

To establish the behavior of each subassemblage under increasing lateral load it is first necessary to obtain a solution for the restrained column shown in fig 3. The restraining characteristics of the girders, shown in fig 3 as a spring, are assumed to be known and independent of the magnitude of the horizontal load (the restraining characteristics of real girders do change under increasing horizontal load, because they are affected by the succesive formation of plastic hinges in the structure; the restraining characteristics of girders and procedures to evaluate them will be discussed afterwards); the column can then be analized to obtain the relationship between the horizontal load acting on it and the lateral displacement of its upper end, including the effect of the P-Δ moment.

Equations 1 and 2 are derived from equilibrium conditions at the top of the column of fig 3, assuming conservatively that M_{n+1} is equal to M_n and using the slope-deflection equations to relate the column rotations and deflections with their geometry and moments acting upon it[5]:

$$\frac{\Delta}{h} = \frac{M_r h}{12EI_c} + \theta \qquad (1)$$

$$Q = \frac{M_r}{h} - P\frac{\Delta}{h} \qquad (2)$$

M_r is the restraining moment at the column top and I_c is the moment of inertia of the column.

To arrive at eq (1) has been assumed that the column behaves elastically; also, that its bending stiffness does not change under increasing axial load. Neither assumption is strictly true, but studying the M-θ curves of part III, ref 6, it can be seen that in the range $P/P_y \leq 0.6$ and $h/r \leq 60$ the column behavior can be idealized as linearly elastic until $M = M_{pc}$ without important loss of accuracy (M_{pc} is the reduced column plastic moment, computed with due consideration of its axial load); also the column has an adequate rotation capacity. If $h/r \leq 40$ it can be assumed that the foregoing conditions are met until $P/P_y = 0.8$ and, according to ref 3, for $h/r \leq 40$ the axial load effects in the columns are small, and can be neglected for practical calculations. Most of the columns of practical importance are then in the range in which eq (1) applies.

When M_r and θ are known, Δ/h is computed from eq (1), and Q from eq (2) and one point of the Q-Δ curve can be plotted. The complete curve will be known if it is possible to compute every value of M_r and Q in the complete process of loading.

The initial value of M_r is a function of the geometric and elastic properties of the adjacent girders whose far ends are, in turn, restrained by the remaining members in the story. A practical design method can not incorporate the influence of all columns and girders in a story on the restraining characteristics of a joint, especially when the frame contains many bays. Accordingly, in this first stage it will be assumed that every joint in each story rotates the same amount (this assumption simplifies considerably the calculations and it is sufficiently accurate for design purposes; a similar assumption permits the development of approximate methods for elastic analysis of multi-story frames and is frequently used[7]).

When the horizontal load is first applied, the bending moments in both ends of each girder are equal to $(6EI_g/L_g)\ \theta$.

Horizontal-load bending moments superimpose on those due to vertical load and, as horizontal load increases while vertical load remains constant, the girder plastic moment is eventually reached at a critical section and a plastic hinge develops there. As further horizontal load is added, this plastic moment value is maintained while the section rotates and moments increase elsewhere; eventually a second plastic hinge is formed, and the process continues until the structure becomes a mechanism.

The above process can be divided into several stages; the first one begins with the initial horizontal load application and ends with the formation of the first plastic hinge. Then begins the second stage, which ends with the formation of the second plastic hinge. The last stage ends with the formation of the plastic hinge which transforms the subassemblage into a mechanism, because the subassemblage ultimate load is then reached. Assuming that the structure behaves elastically between pairs of succesive plastic hinges, to plot the subassemblage complete horizontal load-sway deflection curve is only necessary to find the points which correspond to the formation of every plastic hinge and to connect them by straight lines, taking the shears Q as ordinates and the displacements Δ (or the ratios Δ/h) as abscissas.

Restraining moments M_r and the angle θ corresponding to the formation of every plastic hinge must be computed, and the column strength must be checked in order to see if it is sufficiently large to resist the moments that the girders apply to it; if the column moment M_{pc} is smaller than the sum of the girders end moments the plastic hinge will form at the column top and the girders will not develop their ultimate load capacity.

After the last plastic hinge has formed, the Q- Δ curve becomes the curve corresponding to the rigid-plastic mechanism: a descending straight line which starts in the point corresponding to the last plastic hinge, whose equation is $Q = M_r/h - (P/h)\ \Delta$.

The equation of the rigid-plastic mechanism curve being known, the complete subassemblage horizontal load-sway deflection curve can be plotted (see ref 5 for a thorough description of the procedure).

Fig 4 shows the mechanisms corresponding to a windward subassemblage and to an interior one, a possible sequence of plastic hinges formation and sketches of both Q- Δ curves.

The total shear resistance of the story is the sum of the shear resistances of the individual sway subassemblages in the story, and the load-deflection curve of the story is determined by graphically combining the subassemblages curves.

The sequence of plastic hinge formation in the story can also be determined, because the sway deflection and the θ angle corresponding to every plastic hinge are known.

Employing the horizontal load-sway deflection curve of the story it is possible to determine the maximum value of the shear force which can be resisted by the story, the shear force corresponding to the formation of the collapse mechanism or the shear force corresponding to a given value of the sway deflection Δ ; the latter value is important in earthquake resistant design because the lateral deformations of building frames must be maintained below such limits as permitted by partitions and other nonstructural elements in order to avoid damages under working seismic disturbances.

Members sizes selected from the preliminary design, which were the basis for the computations leading to the subassemblages and to the story Q-Δ curves, are adequate if the maximum shear strength of the story is equal or larger than the factorized earthquake horizontal load and if, simultaneously, the lateral deflections under working horizontal loads are admissible.

Examples

The method described can be applied in the plastic design of building frames. It can also be used for reviewing elastic designs, in order to determine the structure behavior in the plastic range and to obtain its true safety factor against failure. The method also gives a measure of the structure's energy absorption capacity, providing the designer with all the information he needs to make an overall plastic design or to improve the original elastic design, obtaining an adequate and constant safety factor in every story of the building.

The results of the application of the method to the story located between stories 11 and 12 of a 20-story building frame will now be presented. The building is under seismic forces computed by static methods, corresponding to a base shear coefficient of 4 percent, with accelerations which grow linearly from zero at the base of the building to a maximum at the top. The load factors considered were equal to 1.7 for vertical loading only and to 1.3 for vertical and horizontal loads acting simultaneously.

Fig 5 shows the dimensions of columns and girders in the story and the loads acting on them, multiplied by 1.3.

The frame to be studied is an interior one; other frames are located 6 m from it and the floor is a 10 cm concrete slab; the strength of the concrete is f'_c = 140 kg/cm^2.

The preliminary design was done twice, plasticaly and elasticaly. The plastic design used the method described in ref 2 including, aproximately, the P-Δ effect. The elastic design followed conventional approximate methods. In both cases the AISC 1963[8] specification was followed. The results arrived at are shown in table 1, columns 2 and 4, and are identified as structure I (plastic design) and II (elastic design). I shapes were employed in girders and H shapes in columns, made by welding together three ASTM A36 steel plates (yield point 2530 kg/cm^2). Plate dimensions were always chosen in order to get shapes able to develop plastic hinges and to sustain, under constant M_p moment, the rotations which are necessary to form the collapse mechanism.

In order to maintain the story horizontal displacement below a prescribed value (0.003 times the story height) it was necessary to increase the elastically designed members (structure II) beyond the sizes which were computed from strength considerations only.

Both structures were revised according to the method described in the first part of this paper. Results are shown in fig 6, curves IA and IIA. Both curves were obtained by considering only the strength and stiffness of the steel members, beams and columns, without taking into account the concrete slab influence on the structure's behavior.

By comparing these curves it is apparent that structure I (plastically designed) has neither strength nor stiffness adequate to resist the horizontal earthquake load. This is probably due to having assumed too small a displacement in the preliminary design and to the neglect of compatibility conditions at that stage. Structure II has an ultimate strength considerably greater than needed (its load factor is 1.96, against 1.3), but its stiffness under working load is smaller than required (the allowable displacement of 0.003 times the story height takes place under a horizontal shear equal to 38.4 ton, and the seismic design shear is 49.1 ton). Structure II weighs 28.6 percent more than structure I (table 1).

The amount of energy absorbed in the inelastic range is, in both cases, several times greater than the energy absorbed in the elastic range (for curve IIA is about 6.5 times greater).

Curves IB and IIB, fig 6, depict the behavior of structures I and II when the influence of the concrete slab is considered. To arrive at these curves, girder stiffnesses were increased up to the stiffness of a section composed by the steel shape and the concrete slab (following a common practice in reinforced concrete structures, the moment of inertia of the section was computed considering the uncracked slab section), and the M_p moment was raised to the ultimate moment of the composite section at every place where the bending moment was such that compression took place at the top of the section; the composite section properties were computed as recommended in refs 8 and 9.

The concrete slab influences the frame behavior in two ways, both favorable; first, the increase in girder stiffness produces a diminution in the value of the θ angles through which the column tops rotate and a corresponding reduction in the values of displacements (see eq 1), thus increasing the overall strength of the structure to resist horizontal loads; second, the strength of the structure is also increased because of ultimate moments in some sections of the beams are increased.

By recognizing that girders and slab work as a composite section more economic designs are obtained and, besides, a more realistic description of the behavior of the structure is arrive at. For instance, it is usual to increase, more or less arbitrarily, the moment of inertia of the girders when computing the sway displacements of a frame, invoking the aditional stiffness due to the girder-slab interaction, but it is clear that the increase in stiffness is accompanied by an increasing in the bending moment resistance in some sections of the girders, which can originate changes in the positions of the plastic hinges. Neglect of this effect can introduce important differences between the assumed and the actual behavior of the frame. Moreover, results arrived at by connecting slab and girders are uncertain, except when the connecting elements are designed to resist the complete shear force which develops between them.

Fig 7a shows the sequence of plastic hinge formation for structure I, considering only the steel members, and fig 7b shows the sequence for the same structure when the composite action between slab and girders is taken into account. In the first case, every plastic hinge that is necessary for the formation of the collapse mechanism appears in the girders. In the second case plastic hinges appear at the top of both interior columns because their strengths, large enough to resist the maximum moments applied to them by the bare girders, are insufficient to take the maximum moments corresponding to the composite action. The story shear resistance can be increased by reinforcing both intermediate columns, in order to cause the plastic hinges again to develop in the girders; results are shown in table 1, column 3, and in fig 6, curve IC.

Even when taking into account the slab-girder composite action the strength and stiffness of structure I is found to be inadequate. By reinforcing the interior columns enough strength is attained, but the working load deflections are still excessive (curves IB and IC, fig 6).

The load factor of structure II grows to 2.24 when considering composite action, and its strength corresponding to the maximum allowable displacement (0.003 h) increases to 64.2 ton, greater than the working earthquake shear force (curve IIB, fig 6). The maximum capacity to absorb energy is in every case much greater than the elastic capacity.

(Design seismic shears have not been modified when the frame stiffness changes because they have been computed statically.)

By comparing curves IC and IIB, fig 6, it is seen that it is impossible to obtain, for this particular example, a structure which satisfies both strength and stiffness requirements simultaneously; structure I, with interior columns reinforced, can resist a maximum shear force of 64.2 ton, practically equal to the design shear force (63.9 ton), but the relative displacement between floors above and below the story in question, due to working seismic load, is equal to 0.0078 h, much greater than the allowable displacement, while structure II has a correct stiffness but has a much greater strength than the factorized seismic shear force. It is thus impossible to adjust it to obtain the necessary strength without reducing its stiffness below the correct value.

The optimum solution lies between the two cases studied, but to keep the working load displacements below the maximum allowable value it is necessary to adopt an structure with ultimate strength appreciably greater than that needed.

The above discussion confirms an already known and interesting point relative to the behavior of steel building frames under seismic loads: their strength is generaly very satisfactory but they are in most cases too flexi ble. Their design is ordinarily controlled by deformation considerations and not by their strength. Hence, in order to achieve economic structures it is generaly convenient to provide them with x-bracing or shear walls in order to obtain adequate rigidity so that wind and moderate earthquakes do not cause expensive nonstructural damage, and to assign to the tough steel structure the task of resisting exceptionally intense earthquakes.

Summary

A method has been presented for determining the behavior of multi-story steel frames under the action of constant vertical loading and increasing horizontal loading, from the begining of the application of the horizontal loads to the failure of the frames, because of the formation of a collapse mechanism with plastic hinges. The method is simple enough and sufficiently accurate to be used in rutine design problems.

In cities where code does not permit use of plastic design, the method provides the necessary tools for reviewing the member sizes obtained elastically and can be used to improve the original elastic design, making a better use of the material employed.

It also allows easy modificationof girder depths (for instance, one bay of a frame can be designed for resisting only vertical loading, if the rest of the story can resist the complete horizontal load), and this can be very useful when there are special architectural or functional requirements.

The method permits also to include the composite slab-girder action, and this leads to more economic and rational structures. When beams and columns are encased in concrete, the additional rigidity and strength can also be incorporated.

The method makes clear the inconvenient behavior of frames whose members do not have adequate rotation capacity. It also brings out the advantages of employing structural shapes with low width-tickness ratios in structures to be constructed in seismic areas, instead of using beams with very thin webs or trusses, even if such members appear to be very efficient from an elastic point of view.

Acknowledgments

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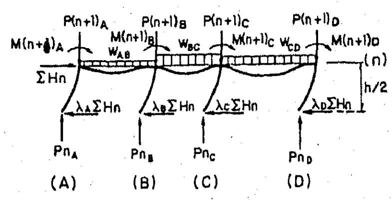

Fig 1 Reduced portion of level n

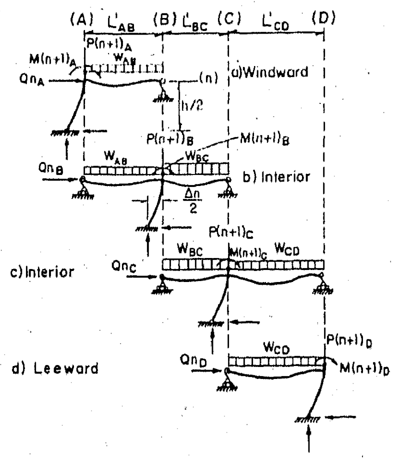

Fig 2 Sway subassemblages at level n

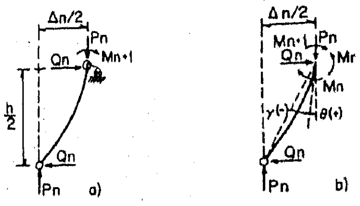

Fig 3 Restrained column

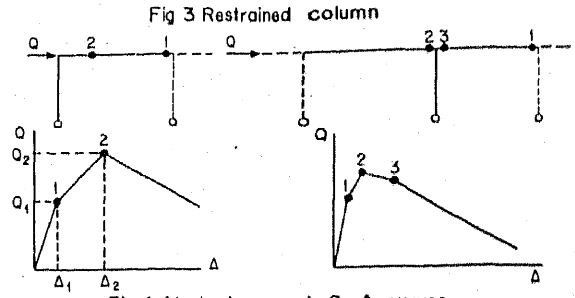

Fig 4 Mechanisms and Q-Δ curves

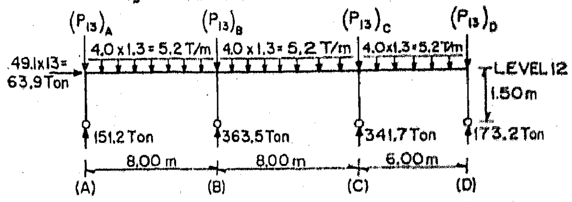

Fig 5 Frame analyzed in examples

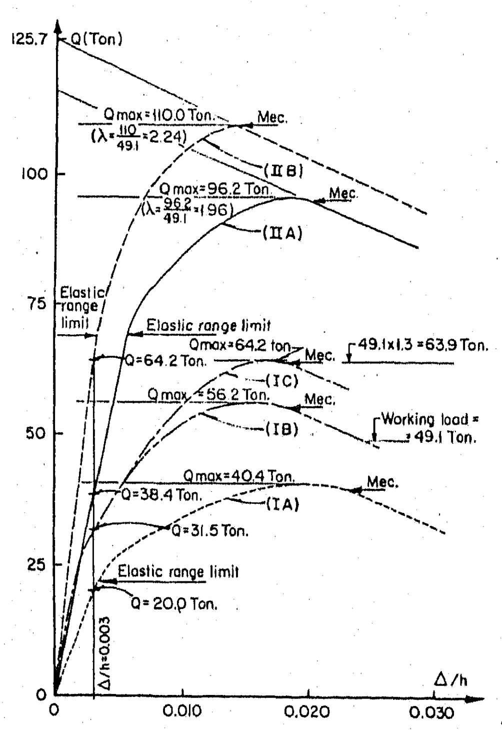

Fig 6 Load-deflection curves for the examples

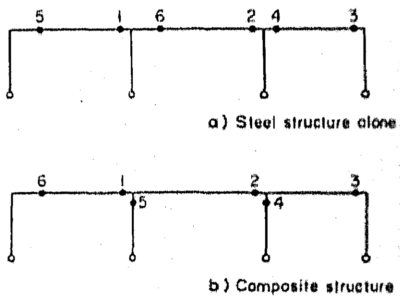

Fig 7 Sequence of plastic hinge formation in structure I

(1)	(2)	(3)	(4)
	STRUCTURE I (curves IA and IB)	STR. I REINFORCED (curve IC)	STRUCTURE II (curves IIA and IIB)
GIRDERS	16"x 8"x 70 kg/m	16"x 8"x 70 kg/m	18"x12"x 98 kg/m
COLUM. A, D	10"x10"x109 kg/m	10"x10"x109 kg/m	12"x12"x132 kg/m
COLUM. B, C	14"x14"x180 kg/m	14"x14"x198 kg/m	16"x16"x 215 kg/m
WEIGHT	3200 kg	3310 kg	4120 kg

Table 1