AN APPROXIMATE METHOD OF STATIC AND DYNAMIC ANALYSES OF CORE-WALL BUILDINGS

By Sukenobu Tani*, Joji Sakurai** and Michio Iguchi***

ABSTRACT

Core-wall type buildings having box-shape walls extending over the entire height as shown in Fig. 2, are quite popular for high-rise buildings. The analyses of these buildings are complicated and it always requires much time and labour, unless digital computers are used.

Hitherto, many approximate analytical methods of solving stresses of core-wall type buildings have been presented and some of them have been used in actual design. In this paper a practical approximate method is developed based on the assumption of a continuous medium as suggested by L. Chitty. The connecting beams are replaced by an equivalent continuous medium, which is assumed to be rigidly attached to the wall and only capable of transmitting the action of the same type as discrete beams.

This paper is concerned with Fig. 2 type buildings. The basic equation was derived taking into consideration the axial deformations of the exterior columns and the flexural and the shear deformations of the core-wall. There is no assumption about the contraflexure points of the exterior columns or beams. The moment for a unit length of the beams was chosen as the redundant force and the equation derived was a simple ordinary differential equation of the second order. By solving this equation, the stresses of each member can easily be calculated. The sectional properties of all members and storey heights were assumed constant throughout the whole height. Modern high-rise buildings satisfy this condition and this method has general applicability to those buildings.

In static analyses, the stresses calculated from this method were compared with the exact solution and fairly good agreement between them was observed. Furthermore, buildings subjected to various types of lateral forces were solved and to facilitate calculations, suitable design charts have been prepared. The influenses of the axial deformations of the exterior columns, which is usually neglected in practical design, on the stresses of each member of the building is discussed and the conclusion reached that for buildings with ordinary sectional properties, this effect should be considerd for buildings more than 20 storeys.

In dynamic analyses, the basic equation was derived from the differential equation for the static analyses. Using this equation, dynamic characteristics of core-wall buildings were studied and the results examined. The significance of the influences of the axial deformations of the exterior columns and the shear deformation of the core-wall on the dynamic characteristics are also discussed by some examples.

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1. SYNOPSIS

Approximate analytical methods of solving the stresses and dynamic characteristics of core-wall type buildings, as shown in Figs. 1 and 2, are presented with special consideration given to the axial deformations of the exterior columns and the shear deformation of the core-wall. The influences of these deformations on the stresses and dynamic characteristics have been investigated and the results are presented.

2. INTRODUCTION

Core-wall type buildings, as shown in Figs. 1 and 2, are quite popular for high-rise buildings as a structural system to resist earthquake and wind loadings. However, analyses for stresses and dynamic characteristics of these buildings are complicated and time consuming. Therefore, a simple, good and precise analytical method is desirable and many approximate methods of stress analysis have been proposed. Among them, Chitty's method which is based on the differential equation, assuming that the discrete connecting beams may be replaced by a continuous medium, has been developed by Cardan, Beck, Dhillon et al. But some of these analyses have neglected the shear deformation of shear walls, or assumed the contraflexure points of beams or columns or neglected the axial deformations of columns. An approximate method of dynamic analysis of this type of buildings has been presented by Osawa which neglects the shear deformation of the shear wall and the axial deformations of columns.

This paper describes an approximate method of solving the stresses and dynamic analysis of core-wall type buildings and some examples are also given. In this approximate analysis of stress determination, Chitty's method is extended by taking into account the shear deformation and the axial deformations of exterior columns and no assumption is made as to the location of the points of contraflexure in the columns and beams. The core-wall type building shown in Fig. 2 is substituted by a continuous system as shown in Fig. 3. The basic differential equation is derived by choosing the bending moment for a unit length at the inner end of beams as the unknown function. Then, the unwinding bending moment of beams can be easily calculated. Influences of the shear deformation of the core-wall and of the axial deformations of the exterior columns on the stresses and the deflection curve of the buildings are examined by some examples.

In the dynamic analysis, on the other hand, the basic equation is derived from the differential equation used for the stress analyses. In this analysis the shear deformation of core-wall and axial deformation of the exterior columns are taken into account and influences of these two kinds of deformations on the dynamic characteristics are illustrated by some examples.

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3. APPROXIMATE ANALYSIS OF STRESSES

(3-1) DERIVATION OF BASIC EQUATION AND ITS SOLUTION

(a) ASSUMPTIONS

(1) The beams are replaced by an equivalent continuous medium which can transmit the same stresses as those of the discrete beams. (2) The sectional properties of each members and storey height are constant through the whole height. But the moment of inertia of the top beams is half of other beams. (3) For simplicity, all members are symmetrical with respect to the neutral axis of the core-wall, although this assumption is not always necessary. (4) The influence of members framing perpendicularly to the frame shown in Fig. 2 is neglected. (5) The shear deformation of the exterior columns and beams are neglected. (6) The exterior columns and the core-wall are rigidly connected to the foundation.

(b) DERIVATION OF THE BASIC EQUATION

In Fig. 4, let θ_i be the rotational angle at the point A, then θ_{i+1} at B and θ_{i-1} at C are given approximately by Eq. (3-1)

$$\theta_{i+1} \fallingdotseq \theta_i + \left.\frac{d\theta}{dx}\right)_i h \quad , \quad \theta_{i-1} \fallingdotseq \theta_i - \left.\frac{d\theta}{dx}\right)_i h \tag{3-1}$$

where h is storey height and x is the vertical co-ordinate. Further, the ratio of storey deflection to storey height of the i-th and the i+1-th storeys are given approximately by Eq. (3-2),

$$R_i \fallingdotseq \left.\frac{dy}{dx}\right)_i - \frac{1}{2}\left.\frac{d^2y}{dx^2}\right)_i h \quad , \quad R_{i+1} \fallingdotseq \left.\frac{dy}{dx}\right)_i + \frac{1}{2}\left.\frac{d^2y}{dx^2}\right)_i h \tag{3-2}$$

where y is the deflection curve of the core-wall.
Using Eqs. (3-1) and (3-2), the basic equations of the slope-deflection method for members AB, AC and AD are given by the following equations,

$$M_{AB}=K_c'\left(3\theta_i+\left.\frac{d\theta}{dx}\right)_i h-3\left.\frac{dy}{dx}\right)_i-\frac{3}{2}\left.\frac{d^2y}{dx^2}\right)_i h\right), \quad M_{BA}=K_c'\left(3\theta_i+2\left.\frac{d\theta}{dx}\right)_i h-3\left.\frac{dy}{dx}\right)_i-\frac{3}{2}\left.\frac{d^2y}{dx^2}\right)_i h\right) \tag{3-3}$$

$$M_{AC}=K_c'\left(3\theta_i-\left.\frac{d\theta}{dx}\right)_i h-3\left.\frac{dy}{dx}\right)_i+\frac{3}{2}\left.\frac{d^2y}{dx^2}\right)_i h\right), \quad M_{CA}=K_c'\left(3\theta_i-2\left.\frac{d\theta}{dx}\right)_i h-3\left.\frac{dy}{dx}\right)_i+\frac{3}{2}\left.\frac{d^2y}{dx^2}\right)_i h\right) \tag{3-4}$$

$$M_{AD}=K_B'\left\{2\theta_i+(1+3\lambda)\left.\frac{dy}{dx}\right)_i-\frac{3}{l_F}v_i\right\} \quad , \quad M_{DA}=K_B'\left\{\theta_i+(2+3\lambda)\left.\frac{dy}{dx}\right)_i-\frac{3}{l_F}v_i\right\} \tag{3-5}$$

where V_i is the axial deformation of the exterior column at the i-th floor (at the point A), $\lambda = l_w/2l_F$, $K_B' = 2EI_B/l_F$ and $K_c' = 2EI_c/h$ where I_B and I_c are moments of inertia of the beams and the exterior columns, respectively. From Eqs. (3-3) and (3-4), the shear stresses of the exterior columns at the i-th and the i+1-th storeys, $_cQ_i$ and $_cQ_{i+1}$ respectively, are given by the following equations.

$${}_cQ_{i+1}=-\frac{3K_c'}{h}\left(2\theta_i+\left.\frac{d\theta}{dx}\right)_i h-2\left.\frac{dy}{dx}\right)_i-\left.\frac{d^2y}{dx^2}\right)_i h\right) \tag{3-6}$$

$${}_cQ_i=-\frac{3K_c'}{h}\left(2\theta_i-\left.\frac{d\theta}{dx}\right)_i h-2\left.\frac{dy}{dx}\right)_i+\left.\frac{d^2y}{dx^2}\right)_i h\right) \tag{3-7}$$

Equation of equilibrium for moments at the point A is given by Eq. (3-8).

$$\theta_i=-\frac{6K_c'-(1+3\lambda)K_B'}{2(K_B'+3K_c')}\left.\frac{dy}{dx}\right)_i+\frac{3K_c'}{2(K_B'+3K_c')}\frac{v_i}{l_F} \tag{3-8}$$

At this stage, by letting the beams be distributed continuously throughout the whole height, then the stresses of all members are also distributed. The subscript i may then be substituted by x.

By substitution of Eq. (3-8) into Eq. (3-5), the bending moments for a unit length of the inner and the outer end of beams, m and m_o respectively, are given by the following equations.

$$m = \frac{M_{BA}}{h} = \frac{3K_B'(K_B'+6K_c')}{2(K_B'+3K_c')h}\left\{(1+\lambda)\frac{dy}{dx} - \frac{v}{l_F}\right\} \qquad (3\text{-}9)$$

$$m_o = \frac{6K_c'}{K_B'+6K_c'}m \qquad (3\text{-}10)$$

The shear stresses for a unit length of beams, q_B, are given by Eq. (3-11).

$$q_B = \frac{m+m_o}{l_F} = \frac{K_B'+12K_c'}{K_B'+6K_c'}\cdot\frac{m}{l_F} \qquad (3\text{-}11)$$

The axial deformation of the exterior column, v, can be expressed by Eq. (3-12).

$$v = \int_0^x \frac{1}{A_cE}dx\int_x^l q_B\,dx = \frac{1}{A_cE}\cdot\frac{K_B'+12K_c'}{l_F(K_B'+6K_c')}\int_0^x dx\int_x^l m\,dx \qquad (3\text{-}12)$$

By letting the shear stresses of the exterior column of the substitute system to be equal to the avarage value of cQi+1 and cQi, then the following equations are obtained,

$${}_cQ = -\frac{K_c'}{h}\left(\theta - \frac{dy}{dx}\right) = \frac{6K_c'}{K_B'+6K_c'}m \qquad (3\text{-}13)$$

$${}_cp = -\frac{6K_c'}{K_B'+6K_c'}\cdot\frac{dm}{dx} \qquad (3\text{-}14)$$

$${}_wp = p - 2\,{}_cp = p + \frac{12K_c'}{K_B'+6K_c'}\cdot\frac{dm}{dx} \qquad (3\text{-}15)$$

where p is the distributed lateral load, ${}_cp$ and ${}_wp$ are the distributed lateral force carried by the exterior column and the core-wall, respectively. Substitution of Eq. (3-12) into Eq. (3-9) leads to Eq. (3-16).

$$\frac{dy}{dx} = \frac{2(K_B'+3K_c')}{3(K_B'+6K_c')(1+\lambda)}m - \frac{1}{(1+\lambda)l_F^2A_cE}\cdot\frac{K_B'+12K_c'}{K_B'+6K_c'}\int_0^x dx\int_x^l m\,dx \qquad (3\text{-}16)$$

From above Eqs., the total bending moment of the core-wall is expressed by Eq. (3-17),

$${}_wM = \int_x^l {}_wp(\xi)(\xi - x)d\xi - 2\int_x^l m\,dx - l_w\int_x^l q_B\,dx = \int_x^l Q\,dx - (1+\lambda)\left(2 + \frac{12K_c'}{K_B'+6K_c'}\right)\int_x^l m\,dx \qquad (3\text{-}17)$$

where $Q = \int_x^l p\,dx$

Taking into account both the bending and shear deformations of the core-wall, the moment-curvature relationship for the core-wall are expressed by Eq. (3-18),

$$\frac{d^2y}{dx^2} = \frac{1}{(EI)_w}\cdot{}_wM - \frac{K}{(AG)_w}\cdot{}_wp \qquad (3\text{-}18)$$

where (EI)w and (AG)w/k are the flexual and shear regidities of the core-wall, respectively.
By substituting Eqs. (3-15) and (3-17) into Eq. (3-18), the integro-differential equation with respect to m is given by Eq. (3-19),

$$h^2\alpha^2\frac{dm}{dx} + \beta^2\int_x^l m\,dx = \int_x^l Q\,dx - \gamma h^2 p \qquad (3\text{-}19)$$

and by differentiating Eq. (3-19), Eq. (3-20) is obtained,

$$h^2\alpha^2\frac{d^2m}{dx^2} - \beta^2 m = -Q - \gamma h^2\frac{dp}{dx} \qquad (3\text{-}20)$$

where, $\gamma = \frac{K(EI)_w}{h^2(AG)_w}$, $K_F = \frac{12K_c'}{K_w + 6K_c'}$

$$\left.\begin{aligned} \alpha^2 &= \frac{(EI)_w}{6K_w'(1+\lambda)h}(4-K_F) + \gamma K_F , \quad \beta^2 = \frac{(2+K_F)(EI)_w}{2A_c E(1+\lambda)l_F^2} + (2+K_F)(1+\lambda) \end{aligned}\right\} \qquad (3\text{-}21)$$

(c) BOUNDARY CONDITIONS

(1) Eq. (3-19) contains one boundary condition with respect to m. That is wM=0 at the top, x=l, and hence Eq. (3-22) is obtained.

$$\alpha^2 \frac{dm}{dx}\Big)_{x=l} = -\gamma p\Big)_{x=l} \qquad (3\text{-}22)$$

(2) At the base, x=0, $dy_w/dx=0$ in which y_w is bending deflection of the core-wall, and hence,

$$\frac{dy}{dx}\Big)_{x=0} = \frac{K}{(AG)_w}\left(Q\Big)_{x=0} - 2_F Q\Big)_{x=0}\right) = \frac{K}{(AG)_w}\left(Q\Big)_{x=0} - K_F m\Big)_{x=0}\right) \qquad (3\text{-}23)$$

and from Eqs. (3-9) and (3-21)

$$\frac{dy}{dx}\Big)_{x=0} = \frac{h^2(\alpha^2-\gamma K_F)}{(EI)_w} m\Big)_{x=0} \qquad (3\text{-}24)$$

so that, from these two equations Eq. (3-25) is obtained.

$$\alpha^2 m\Big)_{x=0} = \gamma Q\Big)_{x=0} \qquad (3\text{-}25)$$

(d) CORE-WALL MOMENT, SHEAR, LATERAL DEFLECTION CURVE AND MOMENT AT INNER END OF BEAMS

Once the quantity m is obtained, these values can be calculated from the following equations.

$$_wM = \int_x^l Q\,dx - (1+\lambda)(2+K_F)\int_x^l m\,dx \qquad (3\text{-}26)$$

$$_wQ = Q - K_F m \qquad (3\text{-}27)$$

$$y = \frac{h^2(\alpha^2-\gamma K_F)}{(EI)_w}\int_0^x m\,dx + \frac{\beta^2-(1+\lambda)(2+K_F)}{(EI)_w}\int_0^x dx \int_0^x dx \int_x^l m\,dx \qquad (3\text{-}28)$$

$$_BM = h\,m \qquad (3\text{-}29)$$

(e) SOLUTION FOR UNIFORM LOAD (See Fig. 5)

In the case of uniformly distributed lateral loads, p(x)=p_o(const.), the solution of Eq. (3-20) can be obtained by considering the two boundary conditions of Eqs. (3-22) and (3-25). The solution is,

$$m = \frac{p_o n h}{\beta^2}(1-\xi) + \frac{p_o h \bar{\alpha}}{\beta^2}(\bar{\alpha}^2-\gamma)\left\{\frac{sh\,\bar{\alpha}n\xi}{ch\,\bar{\alpha}n} - n\bar{\alpha}\frac{ch\,\bar{\alpha}n(1-\xi)}{sh\,\bar{\alpha}n}\right\} \qquad (3\text{-}30)$$

where $\bar{\alpha} = \beta/\alpha$, $\xi = x/l$ and n is the number of storeys.
wM, wQ, y and $_B$M are given by the following equations.

$$\frac{_wM}{h^2 p_o} = \frac{n^2}{2}\left(1-\frac{(1+\lambda)(2+K_F)}{\beta^2}\right)(1-\xi)^2 - \left(\frac{1}{\bar{\alpha}^2}-\gamma\right)\frac{(1+\lambda)(2+K_F)}{\beta^2}\left\{\frac{ch\,\bar{\alpha}n - ch\,\bar{\alpha}n\xi}{ch\,\bar{\alpha}n} - n\bar{\alpha}\frac{sh\,\bar{\alpha}n(1-\xi)}{ch\,\bar{\alpha}n}\right\} \qquad (3\text{-}31)$$

$$\frac{_wQ}{h p_o} = \left(1-\frac{K_F}{\beta^2}\right)n(1-\xi) - \left(\frac{1}{\bar{\alpha}^2}-\gamma\right)\frac{\bar{\alpha}K_F}{\beta^2}\left\{\frac{sh\,\bar{\alpha}n}{ch\,\bar{\alpha}n} - n\bar{\alpha}\frac{ch\,\bar{\alpha}n(1-\xi)}{ch\,\bar{\alpha}n}\right\} \qquad (3\text{-}32)$$

$$\begin{aligned}\frac{(EI)_w}{h^4 p_o}y = {}& \frac{1}{\beta^2}\left\{\frac{(1+\lambda)(2+K_F)}{\bar{\alpha}^2} - \gamma K_F\right\}\left(\frac{1}{\bar{\alpha}^2}-\gamma\right)\left\{\frac{ch\,\bar{\alpha}n\xi - 1}{ch\,\bar{\alpha}n} + n\bar{\alpha}\frac{sh\,\bar{\alpha}n(1-\xi) - sh\,\bar{\alpha}n}{ch\,\bar{\alpha}n}\right\} + \frac{n^2}{\beta^2}\left\{\frac{(1+\lambda)(2+K_F)}{\bar{\alpha}^2} - \gamma K_F\right\}\left(1-\frac{1}{2}\xi\right)\xi \\ & + n^2\gamma\left\{1-\frac{(1+\lambda)(2+K_F)}{\beta^2}\right\}\left(1-\frac{1}{2}\xi\right)\xi + \frac{n^4}{24}\left\{1-\frac{(1+\lambda)(2+K_F)}{\beta^2}\right\}(\xi^2-4\xi+6)\xi^2\end{aligned} \qquad (3\text{-}33)$$

$$\frac{_BM}{h^2 p_o} = \frac{n}{\beta^2}(1-\xi) + \frac{\bar{\alpha}}{\beta^2}\left(\frac{1}{\bar{\alpha}^2}-\gamma\right)\left\{\frac{sh\,\bar{\alpha}n\xi}{ch\,\bar{\alpha}n} - n\bar{\alpha}\frac{ch\,\bar{\alpha}n(1-\xi)}{sh\,\bar{\alpha}n}\right\} \qquad (3\text{-}34)$$

(f) SOLUTION FOR CONCENTRATED LOAD AT THE TOP OF THE BUILDING (See Fig.6)

In case of a concentrated lateral load, Po, at the top of the building, the solution of Eq. (3-20) can be obtained by considering the two boundary conditions of Eqs. (3-22) and (3-25). The solution is,

$$m=\frac{P_o}{\beta^2}\left\{1-(1-\gamma\bar{\alpha}^2)\frac{ch\,\bar{\alpha}n(1-\xi)}{ch\,\bar{\alpha}n}\right\} \qquad (3\text{-}35)$$

(g) SOLUTION FOR TRIANGULAR LOAD (See Fig. 7)

In the case of triangular lateral load, $p(x)=(x/\ell)p_o$, and p_o is constant and the solution of Eq. (3-20) can be obtained by considering the two boundary conditions of Eqs. (3-22) and (3-25). The solution is,

$$m=\frac{\ell p_o\bar{\alpha}}{\beta^2}\left(\frac{1}{\bar{\alpha}^2}-\gamma\right)\left\{\frac{sh\,\bar{\alpha}n\xi}{ch\,\bar{\alpha}n}-\left(\frac{\bar{\alpha}^2}{2}-\frac{1}{n^2}\right)\frac{ch\,\bar{\alpha}n(1-\xi)}{ch\,\bar{\alpha}n}\right\}-\frac{\ell p_o}{n\beta^2}\left\{\left(\frac{1}{\bar{\alpha}^2}-\gamma\right)-\frac{n^2}{2}(\xi^2-1)\right\} \qquad (3\text{-}36)$$

(h) CALCULATION OF ACTUAL STRESSES

From the bending moment of inner end of beams, $_BM$, the bending moment of outer end of beams, $_BM_o$, can easily be calculated by using Eq. (3-37)

$$_BM_o=\frac{1}{2}K_F\,{}_BM \qquad (3\text{-}37)$$

The bending moment of the exterior columns can be calculated with sufficient approximation by distributing $_BM_o/2$ to the upper and to the lower columns at the panel point, respectively. (See Fig. 8a)
The bending moment of the core-wall should be given by considering the unwinding bending moment of beams and the method is illustrated in Fig. 8b. The unwinding bending moment of beams, $_BM_R$, is given by Eq. (3-38).

$$_BM_R=\left\{\lambda(1+\tfrac{1}{2}K_F)+1\right\}{}_BM \qquad (3\text{-}38)$$

(3-2) ACCURACY OF PROPOSED APPROXIMATE ANALYSIS

The stresses caluculated by the method developed in this paper are compared with exact solutions based on the slope-deflection method, taking into account the axial, bending and shear deformations and the rigid zone. The comparison is performed on a building having properties shown in Table 1 and the results are shown in Fig. 9.

(3-3) INFLUENCE OF SHEAR DEFORMATION OF CORE-WALL AND OF AXIAL DEFORMATION OF EXTERIOR COLUMNS

The method proposed takes into account specifically the shear deformation of the core-wall and the axial deformations of the exterior columns. In this paper the influences of these deformations on the bending moment of the core-wall, $_WM$, the bending moment of inner end of beams, $_BM$, and lateral deflection curve, y, have been investigated. The calculations have been performed on the buildings having properties shown in Table 2 and subjected to uniform lateral load. Stresses and deformations have been calculated by considering the following cases respectively; case (1) the bending and the shear deformations of the core-wall and the axial deformations of the exterior columns; case (2) the bending deformation of the core-wall and the axial deformations of the exterior columns; case (3) the bending and the shear deformations of the core-wall; and case (4) the bending deformation of the core-wall. In every case, the bending deformations of the beams and the exterior columns are taken into account. The results of calculations are shown in Figs. 10a – d and the numerals in Figs. 10a – d indicate the ratio of wM at the base, y at the top and the maximum values of $_{B}M$ of case (2), those of case (3) and those of case (4) to those of case (1).

(3-4) DESIGN CHARTS

To facilitate the calculations, suitable design charts for obtaining the bending moment of the core-wall, the bending moment of inner end of the beams and the lateral deflection curve due to uniform load are given by charts 1, 2 and 3. To use these charts, it is necessary to know the parameter $\bar{\alpha}n$, and from this value the coefficient $_{w}\bar{M}$, $_{B}\bar{M}$ and $\bar{Y}$ may be read at all storeys and wM, $_{B}M$ and y are obtained by the following equations.

$$\frac{_{w}M(\xi)}{h^2 p_0} = \frac{n^2}{2}\left(1-\frac{(1+\lambda)(2+K_F)}{\beta^2}\right)(1-\xi)^2 \cdot \left(\frac{1}{\bar{\alpha}^2}-\gamma\right)\frac{(1+\lambda)(2+K_F)}{\beta^2}\,{}_{w}\bar{M}(\xi) \tag{3-39}$$

$$\frac{_{B}M(\xi)}{h^2 p_0} = \frac{n}{\beta^2}(1-\xi) + \frac{\bar{\alpha}}{\beta^2}\left(\frac{1}{\bar{\alpha}^2}-\gamma\right){}_{B}\bar{M}(\xi) \quad , \quad \frac{_{B}M(1.0)}{h^2 p_0} = \frac{\bar{\alpha}}{2\beta^2}\left(\frac{1}{\bar{\alpha}^2}-\gamma\right){}_{B}\bar{M}(1.0) \tag{3-40}$$

$$\frac{(EI)_w}{h^4 p_0}y(\xi) = \frac{1}{\beta^2}\left\{\frac{(1+\lambda)(2+K_F)}{\bar{\alpha}^2}-\gamma K_F\right\}\left(\frac{1}{\bar{\alpha}^2}-\gamma\right)\bar{Y}(\xi) + \frac{n^2}{\beta^2}\left\{\frac{(1+\lambda)(2+K_F)}{\bar{\alpha}^2}-\gamma K_F\right\}\left(1-\frac{1}{2}\xi\right)\xi$$
$$+n^2\left\{1-\frac{(1+\lambda)(2+K_F)}{\beta^2}\right\}\gamma\left(1-\frac{1}{2}\xi\right)\xi - \frac{n^4}{24}\left(1-\frac{(1+\lambda)(2+K_F)}{\beta^2}\right)(\xi^2-4\xi+6)\xi^2 \tag{3-41}$$

4. DYNAMIC ANALYSIS

(4-1) DERIVATION OF DYNAMIC EQUATION AND ITS SOLUTION

(a) DERIVATION OF DYNAMIC EQUATION

Dynamic equation is derived by assuming items described in Section (3-1) (a) and by the further assumption of uniform distribution of mass. Integro-differential equation with respect to m can be reduced to integro-differential equation with respect to y by using Eq. (3-28) and the equation becomes as Eq. (4-1).

$$\frac{d^2y}{dx^2} - \frac{\alpha^2-\gamma K_F}{\alpha^2(EI)_w}\left\{\int_x^l dx\int_x^l p\,dx - \gamma h^2 p\right\} + \frac{\bar{\alpha}^2}{h^2}\left(y_{x=l} - y\right) - \frac{\beta^2-(1+\lambda)(2+K_F)}{h^2(\alpha^2-\gamma K_F)}\left.\frac{dy}{dx}\right)_{x=0}(l-x)$$
$$-\frac{\beta^2-(1+\lambda)(2+K_F)}{h^2\alpha^2(EI)_w}\int_x^l dx\int_0^x\left(\int_x^l dx\int_x^l p\,dx - \gamma h^2 p\right)dx = 0 \tag{4-1}$$

Lateral load function, p(x), can be transformed to inertia force and p(x) can be expressed by Eq. (4-2),

$$p = -\rho\frac{\partial^2 y}{\partial t^2} = \rho N^2 Y \qquad \left(y = Y(x)e^{iNt}\right) \tag{4-2}$$

where ρ is mass for a unit length, N is circular frequency and Y is the normal function.
By substitution of Eq. (4-2) into Eq. (4-1), the integro-differential equation with respect to the normal function Y is obtained by Eq. (4-3)

$$\frac{d^2Y}{dx^2} - \frac{\omega^2}{l^4}\left\{\int_x^l dx\int_x^l Y\,dx - l^2\bar{\gamma}Y\right\} + \frac{n^2\bar{\alpha}^2}{l^2}\left(Y_{x=l} - Y\right) - \frac{n^2\varepsilon}{l^2}\left.\frac{dY}{dx}\right)_{x=0}(l-x)$$
$$-\frac{n^2\varepsilon\omega^2}{l^6}\int_x^l dx\int_0^x\left(\int_x^l dx\int_x^l Y\,dx - l^2\bar{\gamma}Y\right)dx = 0 \tag{4-3}$$

where, $\omega^2 = \frac{(\alpha^2-\gamma K_F)\rho l^4}{\alpha^2(EI)_w}N^2$, $\varepsilon = \frac{\beta^2-(1+\lambda)(2+K_F)}{\alpha^2-\gamma K_F}$, $\bar{\gamma} = \frac{\gamma}{n^2}$

By differentiating Eq. (4-3), Eq. (4-4) is obtained.

$$\frac{d^6Y}{dx^6} + \frac{1}{l^2}\left(\bar{\gamma}\omega^2 - n^2\bar{\alpha}^2\right)\frac{d^4Y}{dx^4} - \frac{\omega^2}{l^4}\left(1+\bar{\gamma}n^2\varepsilon\right)\frac{d^2Y}{dx^2} + \frac{n^2\varepsilon\omega^2}{l^6}Y = 0 \tag{4-4}$$

(b) BOUNDARY CONDITONS

(1) Eq. (4-3) contains four boundary conditions with respect to Y. At the top, $x=\ell$,

$$\left.\frac{d^2Y}{dx^2}\right)_{x=\ell} + \left.\frac{\bar\gamma\omega^2}{\ell^2}Y\right)_{x=\ell} = 0 \tag{4-5}$$

$$\left.\frac{d^4Y}{dx^4}\right)_{x=\ell} + \frac{1}{\ell^2}(\omega^2\bar\gamma - \pi^2\bar\alpha^2)\left.\frac{d^2Y}{dx^2}\right)_{x=\ell} + \frac{\omega^2}{\ell^4}(1+\bar\gamma\pi^2\varepsilon)\left.Y\right)_{x=\ell} = 0 \tag{4-6}$$

$$\left.\frac{d^5Y}{dx^5}\right)_{x=\ell} + \frac{1}{\ell^2}(\omega^2\bar\gamma - \pi^2\bar\alpha^2)\left.\frac{d^3Y}{dx^3}\right)_{x=\ell} - \frac{\omega^2}{\ell^4}(1+\bar\gamma\pi^2\varepsilon)\left.\frac{dY}{dx}\right)_{x=\ell} = 0 \tag{4-7}$$

and at the base, $x=0$,

$$\left.\frac{d^3Y}{dx^3}\right)_{x=0} + \frac{1}{\ell^2}(\omega^2\bar\gamma + \pi^2\varepsilon - \pi^2\bar\alpha^2)\left.\frac{dY}{dx}\right)_{x=0} + \frac{\omega^2}{\ell^4}\int_0^\ell Y dx = 0 \tag{4-8}$$

(2) The other two conditions are at the base, $x=0$,

$$\left.Y\right)_{x=0} = 0 \tag{4-9}$$

$$\left.\frac{dY}{dx}\right)_{x=0} = \frac{\omega^2\bar\gamma}{\ell^2}\int_0^\ell Y dx \tag{4-10}$$

(c) DERIVATION OF FREQUENCY EQUATION

The general solution of Eq. (4-4) is expressed by Eq. (4-11), in which A_1 - A_6 are the arbitrary constants.

$$Y = A_1 \sin a\xi + A_2 \cos a\xi + A_3 \operatorname{sh} b\xi + A_4 \operatorname{ch} b\xi + A_5 \operatorname{sh} c\xi + A_6 \operatorname{ch} c\xi \tag{4-11}$$

where $\xi = x/\ell$ and $-a$, b and c are roots of the characteristics equation of Eq. (4-4).
By solving for the arbitrary constants by taking the boundary conditions of section (b) into account, A_1 - A_6 are given by the following equations.

$$\left.\begin{aligned}
A_1 &= B_c\{E\cos a \operatorname{ch} b + (B_t \operatorname{ch} b + A_c \cos a)\operatorname{ch} c\}A_0 \\
A_2 &= -B_c\{E(\tfrac{c}{a}\operatorname{sh} c + \sin a)\operatorname{ch} b + A_c(\sin a + \tfrac{b}{a}\operatorname{sh} b)\operatorname{ch} c\}A_0 \\
A_3 &= -\frac{b}{a}A_c\{E\cos a \operatorname{ch} b + (B_t \operatorname{ch} b + A_c \cos a)\operatorname{ch} c\}A_0 \\
A_4 &= A_c\{-E(\tfrac{c}{a}\operatorname{sh} c - \tfrac{b}{a}\operatorname{sh} b)\cos a + B_c(\sin a + \tfrac{b}{a}\operatorname{sh} b)\operatorname{ch} c\}A_0 \\
A_5 &= \frac{c^3(b^2+\omega^2\bar\gamma)}{ab^2(c^2+\omega^2\bar\gamma)}A_b\{E\cos a \operatorname{ch} b + (B_c \operatorname{ch} b + A_c \cos a)\operatorname{ch} c\}A_0 \\
A_6 &= -(A_2 + A_4)
\end{aligned}\right\} \tag{4-12}$$

where Ao is a constant which can be determined from the initial conditions,

and $A_b = \frac{b^2}{a^2} - \frac{a^2-\omega^2\bar\gamma}{b^2+\omega^2\bar\gamma}$, $A_c = \frac{c^2}{a^2} - \frac{a^2-\omega^2\bar\gamma}{c^2+\omega^2\bar\gamma}$, $B_c = \frac{c^2}{b^2} - \frac{b^2+\omega^2\bar\gamma}{c^2+\omega^2\bar\gamma}$ & $E = \frac{c^2(a^2-\omega^2\bar\gamma)}{a^2(c^2+\omega^2\bar\gamma)}A_b$

The frequency equation is given by Eq. (4-13).

$$\begin{aligned}
&B_t A_c\{2 + (\tfrac{b}{a} - \tfrac{a}{b})\sin a \operatorname{sh} b\}\operatorname{ch} c - \{B_c - A_c + Ec^2(\tfrac{B_c}{a^2} - \tfrac{A_c}{b^2})\}\cos a \operatorname{ch} b \operatorname{ch} c \\
&+ EB_c(\tfrac{c}{a} - \tfrac{a}{c})\sin a \operatorname{sh} b \operatorname{sh} c + EA_c\{2 + (\tfrac{c}{b} + \tfrac{b}{c})\operatorname{sh} b \operatorname{sh} c\}\cos a = 0
\end{aligned} \tag{4-13}$$

(4-2) INFLUENCES OF SHEAR DEFORMATION OF CORE-WALL AND THE AXIAL DEFORMATION OF EXTERIOR COLUMNS

Influences of the shear deformation of the core-wall and the axial deformation of the exterior columns on the dynamic characteristics have been investigated on the building shown in Table 2. The case (1) - case (4) are the same as those in Section (3-3). The results with respect to participating functions of each mode are shown in Fig. 11a - d and the natural periods of each mode are shown in Table 3a - d.

5. RESULTS AND CONCLUSIONS

From the examples given in this paper, the following results and conclusions are obtained.

(a) WITH RESPECT TO STRESSES ANALYSIS

(1) The stresses calculated by the proposed method are in good agreement with those obtained by the exact methods.
(2) Influence of axial deformations of the exterior columns on the stresses is small for buildings of about ten storeys, but this effect becomes rather large for buildings of about twenty storeys.
(3) Influence of the shear deformation of the core-wall is small and this effect becomes relatively smaller with increasing number of storeys.
(4) The maximum value of the bending moment of beams appears at the level of storey (0.5n - 0.3n), where n is the number of storeys and this level becomes lower relatively with increases of n.

(b) WITH RESPECT TO DYNAMIC ANALYSIS

(5) Influence of axial deformation of the exterior columns on the fundamental period and mode is small for buildings of about ten storeys, but this effect becomes rather large for buildings more than twenty storeys high. This effect decreases for the higher periods and modes.
(6) Influence of the shear deformation of the core-wall on the fundamental period and mode is small, but for higher periods and modes the effect increases. The effect decreases with increasing number of storeys.

6. ACKNOWLEDGEMENTS

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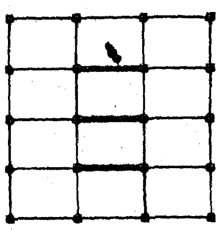


Fig. 1 TYPICAL PLAN OF CORE-WALL TYPE BUILDING.

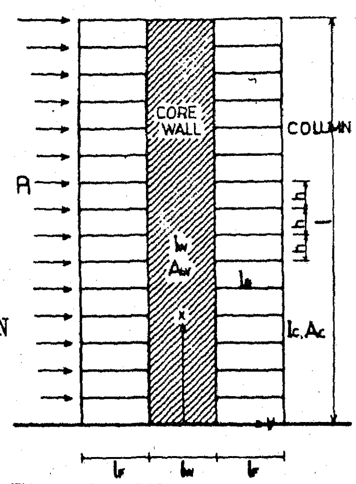


Fig. 2 CORE-WALL TYPE BUILDING.

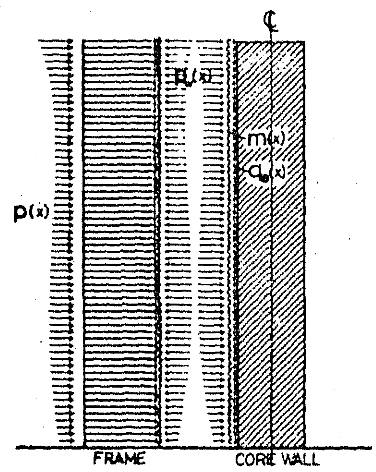


Fig.3 SUBSTITUTE SYSTEM.

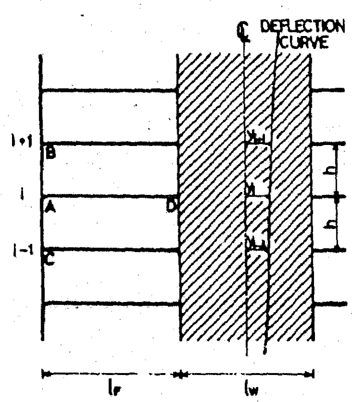


Fig. 4 DETAIL OF STRUCTURE.

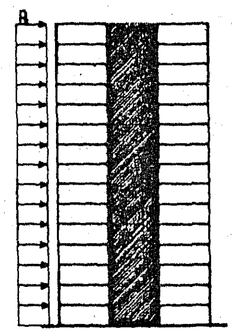


Fig. 5 UNIFORM LOAD.

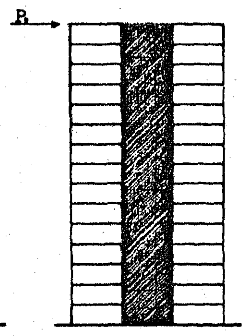


Fig. 6 CONCENTRATED LOAD.

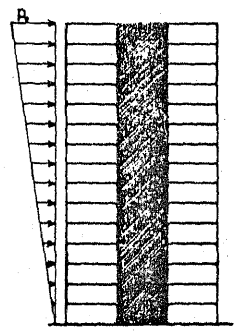
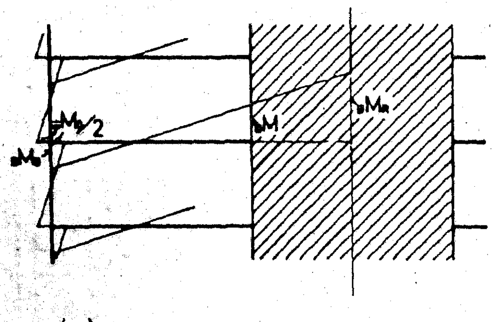
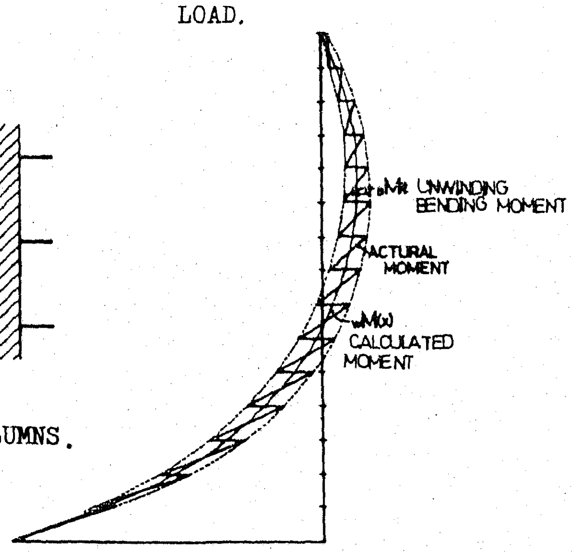


Fig. 7 TRIANGULER LOAD.



(a) MOMENT OF BEAMS & COLUMNS.



(b) MOMENT OF CORE-WALL.

Fig.8 CALUCULATION OF ACTUAL STRESSES.

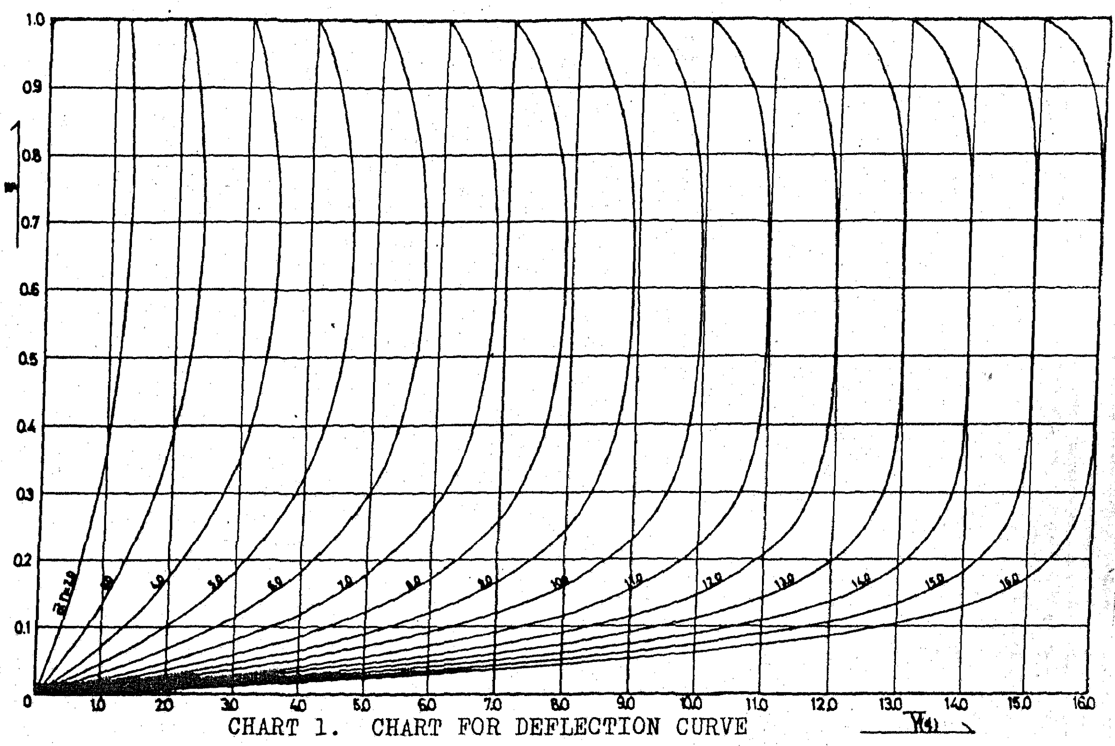
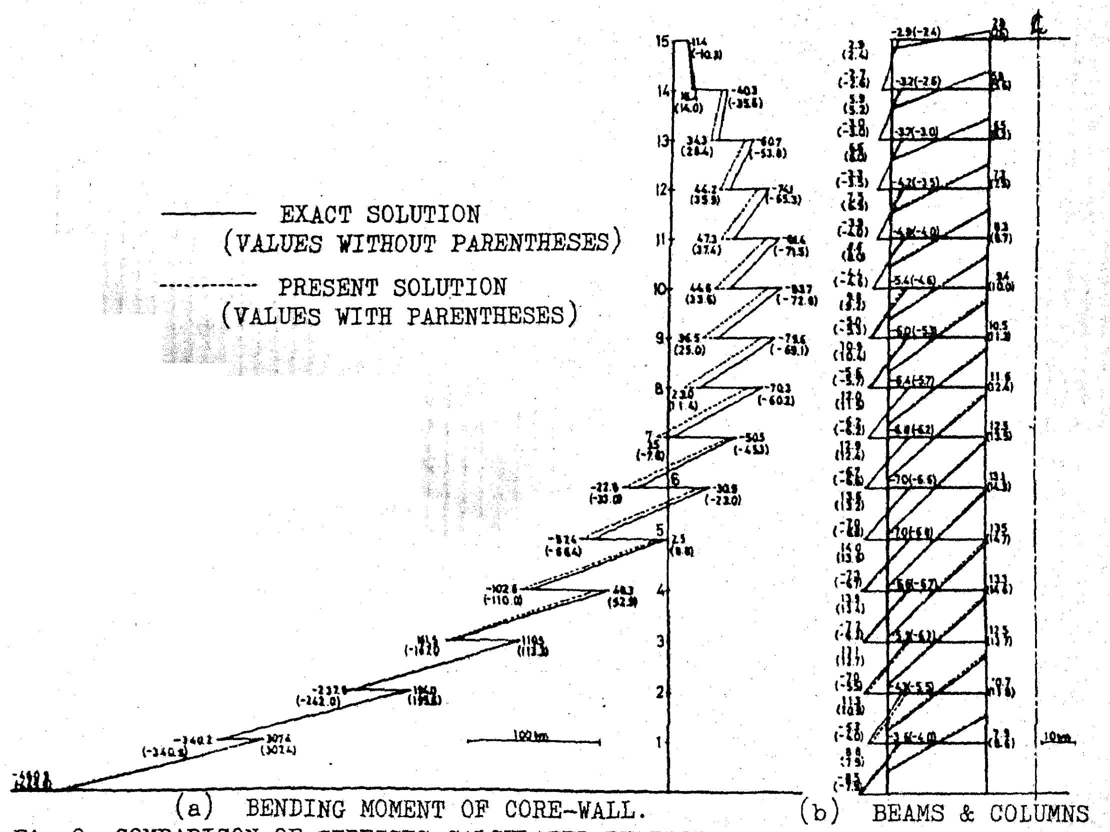


CHART 1. CHART FOR DEFLECTION CURVE



(a) BENDING MOMENT OF CORE-WALL. (b) BEAMS & COLUMNS

Fig.9 COMPARISON OF STRESSES CALCULATED BY PROPOSED METHOD WITH EXACT ONES.

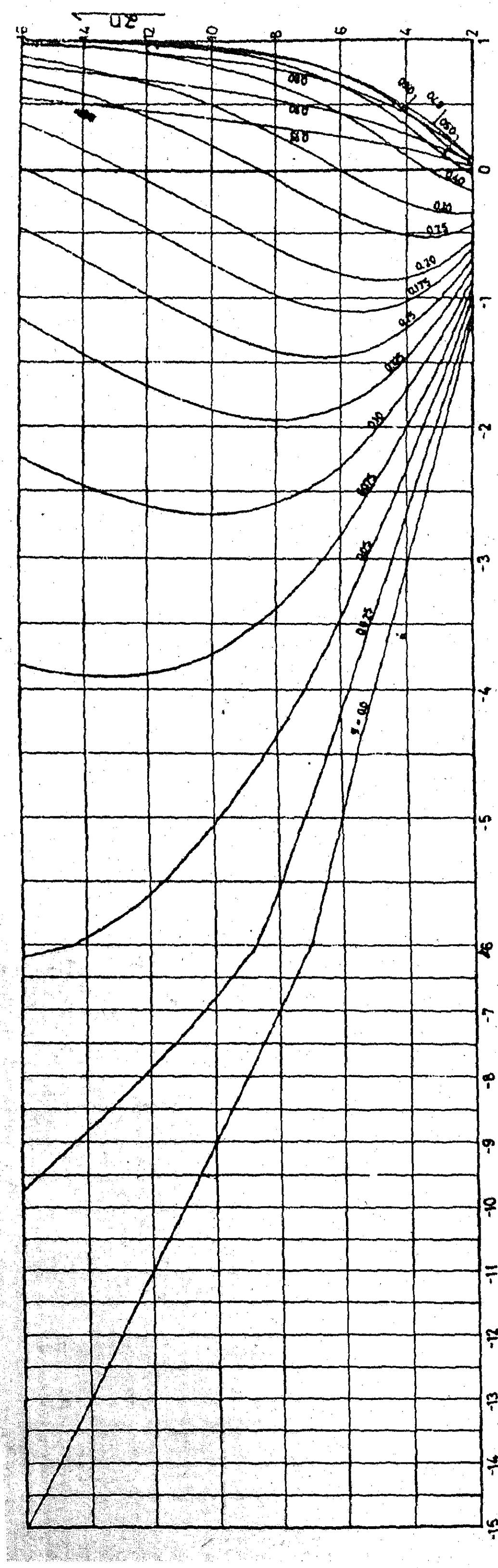


CHART 2. CHART FOR BENDING MOMENT OF CORE-WALL.

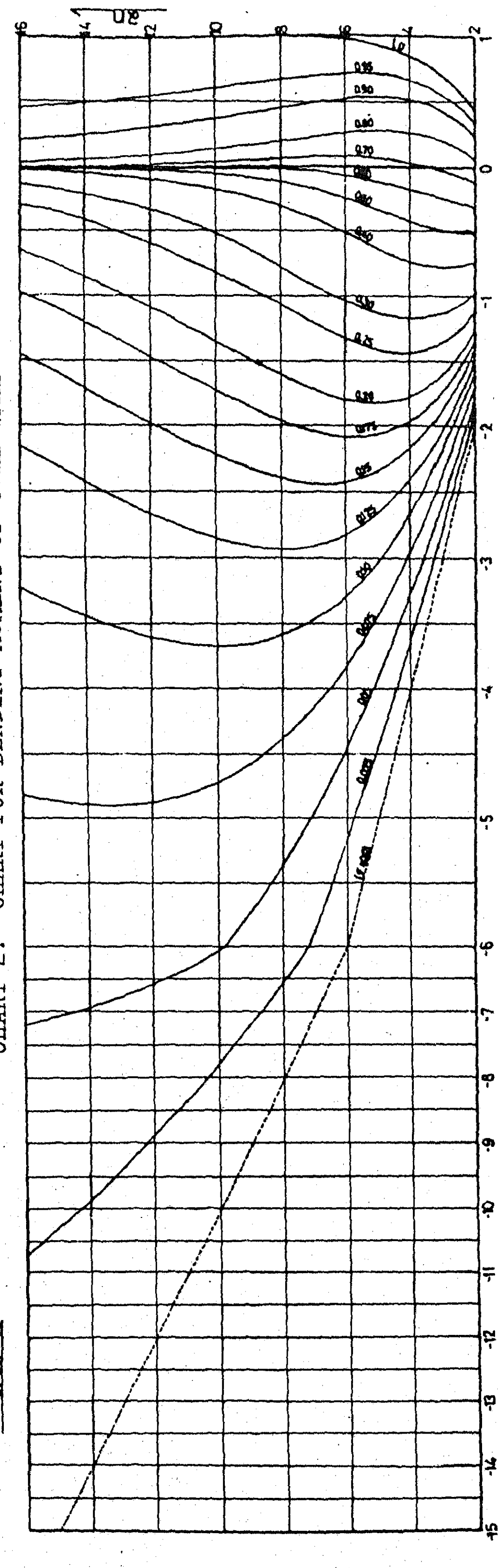


CHART 3. CHART FOR BENDING MOMENT OF INNER END OF BEAMS.

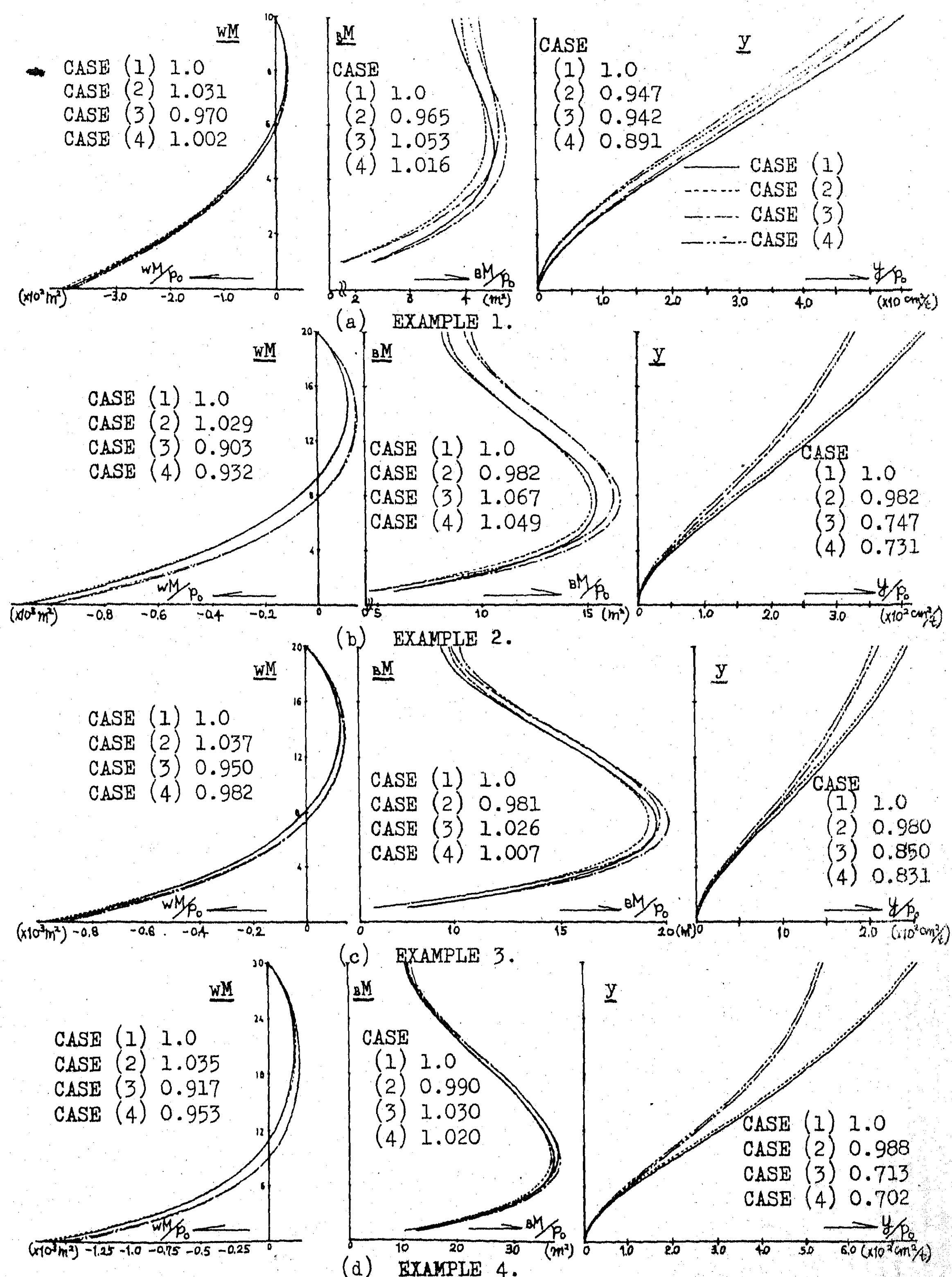


Fig. 10 STRESSES & DEFLECTION CURVES.

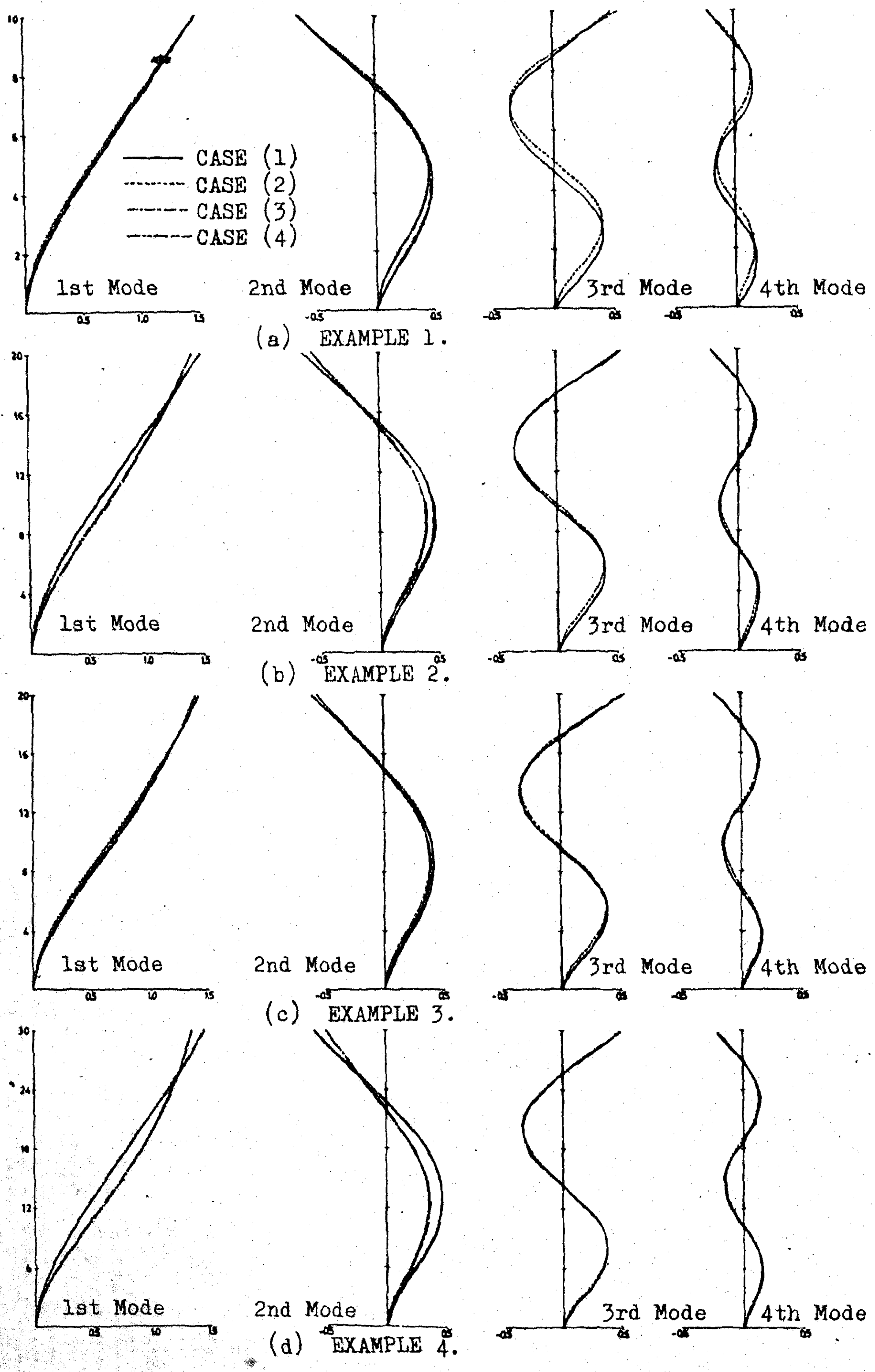


Fig. 11 PARTICIPATING FUNCTION OF EACH MODE.

Table 1

SYMMETRICAL BUILDING
$n=15$ $h=3.5m$ $\ell_F-\ell_w=7.0m$
EXTERIOR COLUMNS
80 cm×80cm
$I_c=3.41\times10^6\,cm^4$
$A_c=6.40\times10^3\,cm^2$
BEAMS $I_B=I_c$
CORE WALL 7.0m×30cm
$I_w=8.57\times10^8\,cm^4$
$A_w=2.10\times10^4\,cm^2$
LATERAL LOADS
ORIGINAL SYSTEM
$P_i=3.5t$ $P_n=1.75t$
SUBSTITUTE SYSTEM
$p(x)=1.0t/m$

Table 2

	n	WALL THICK.	COLUMN (cm)	ℓ_F	ρ
EX.1	10	30cm	60×60	7.0m	$4.29\,sec^2t/m^2$
EX.2	20	30cm	60×60	7.0m	$4.29\,sec^2t/m^2$
EX.3	20	40cm	80×80	10.5m	$5.72\,sec^2t/m^2$
EX.4	30	40cm	80×80	10.5m	$5.72\,sec^2t/m^2$
COMMON PROPERTIES TO ALL EXAMPLES. $\ell_w=7.0m$ $K_B'=2.0K_c'$, $h=3.5m$ $\gamma=1.15$ $E=2.1\times10^2\,t/cm^2$					

Table 3a

	NATURAL PERIOD (sec)			
	1st	2nd	3rd	4th
CACE(1)	0.7880 (1.0)	0.1823 (1.0)	0.0803 (1.0)	0.0489 (1.0)
CASE(2)	0.7615 (0.97)	0.1565 (0.86)	0.0591 (0.74)	0.0306 (0.63)
CASE(3)	0.7683 (0.98)	0.1809 (0.99)	0.0803 (1.0)	0.0489 (1.0)
CASE(4)	0.7420 (0.97)	0.1555 (0.85)	0.0590 (0.73)	0.0306 (0.63)

Table 3b

	NATURAL PERIOD (sec)			
	1st	2nd	3rd	4th
CASE(1)	2.3138 (1.0)	0.5732 (1.0)	0.2419 (1.0)	0.1391 (1.0)
CASE(2)	2.2688 (0.98)	0.5481 (0.96)	0.2197 (0.91)	0.1179 (0.85)
CASE(3)	1.9863 (0.86)	0.5321 (0.93)	0.2400 (0.99)	0.1[illegible] (1.0)
CASE(4)	1.9609 (0.85)	0.5104 (0.89)	0.2180 (0.90)	0.1174 (0.84)

Table 3c

	NATURAL PERIOD (sec)			
	1st	2nd	3rd	4th
CASE(1)	1.9935 (1.0)	0.5732 (1.0)	0.2419 (1.0)	0.1391 (1.0)
CASE(2)	1.9536 (0.98)	0.5032 (0.96)	0.2139 (0.91)	0.1162 (0.85)
CASE(3)	1.8340 (0.92)	0.5063 (0.96)	0.2340 (1.0)	0.1365 (1.0)
CASE(4)	1.8118 (0.91)	0.4857 (0.93)	0.2130 (0.91)	0.1159 (0.85)

Table 3d

	NATURAL PERIOD (sec)			
	1st	2nd	3rd	4th
CASE(1)	3.5914 (1.0)	1.0015 (1.0)	0.4486 (1.0)	0.2641 (1.0)
CASE(2)	3.5170 (0.99)	0.9800 (0.98)	0.4292 (0.96)	0.2442 (0.92)
CASE(3)	3.0319 (0.84)	0.9818 (0.89)	0.4410 (0.98)	0.2619 (0.99)
CASE(4)	3.0039 (0.86)	0.8740 (0.87)	0.4222 (0.94)	0.2420 (0.92)