VARIABILITY ANALYSIS OF SHEAR WALL STRUCTURES

By Jack R. Benjamin[1]

Abstract

Both the rigidity and the strength of shear walls are highly variable so that even when loads are known, the distribution of loads to the resisting walls as well as their response can only be studied using the methods of probabilistic structural mechanics.

This paper is directed toward developing techniques for analysis of shear wall structures under seismic loading from a probabilistic viewpoint in which the actual wall properties are employed.

Introduction

From 1952 to 1956 a large number and variety of shear walls of masonry and concrete were tested and studied at Stanford University. Summaries of the test results and comparison with theoretical analysis have been reported in four papers.[2,3,4,5] Despite this and other published literature on shear walls, the influence of one primary characteristic of shear walls has not been studied. The properties of shear walls are highly variable. The variability of strength and rigidity is not confined to the uncracked region but actually increases in the cracked region. Figure 1 illustrates this characteristic. The great variation in rigidity up to a shear load of 500 kips is evident. These load-deflection curves were obtained by using a linear scaling transformation on five walls modeling a prototype reinforced concrete wall 12 feet high, 21 feet center to center of columns, and 8 inches thick.

---

[1] Professor of Structural Engineering, Stanford University. Presented at State of the Art Synposium, Earthquake Engineering Research Institute, San Francisco, February 1968.

[2] "The Behavior of One-Story Reinforced Concrete Shear Walls," by Jack R. Benjamin and Harry A. Williams, Journal of the Structural Division, Trans. ASCE, Vol. 124, 1959, pp. 669-708.

[3] "The Behavior of One-Story Brick Shear Walls," By Jack R. Benjamin and Harry A. Williams, Journal of the Structural Division, Proc. ASCE, Paper No. 1723, ST 4, 1958.

[4] "Behavior of One-Story Reinforced Concrete Shear Walls Containing Openings," by Jack R. Benjamin and Harry A. Williams, Journal of the American Concrete Institute, No. 5, Vol. 30, No. 1958, pp. 605-618.

[5] "Reinforced Concrete Shear Wall Assemblies," by Jack R. Benjamin and Harry A. Williams, Journal of the Structural Division, Proc. ASCE, Paper No. 2566, ST 8, 1960.

The variability of shear wall properties has a very important influence on analysis and attendant design. The analysis problem is complicated by the effective (secant) rigidity being a function of loading once major cracking occurs. In view of the extremely large loads that may result from an earthquake, the actual physical characteristics of the primary shear wall elements must be used. The use of "conservative lower bound" rigidities and strengths is not acceptable owing to the fact that such procedures simply give the engineer wrong information for design.

Experimental Data

The comparison of observed strength and rigidity with theoretical calculated values discloses that the methods of elementary strength of materials are satisfactory for forecasting shear wall properties. The differences in calculated rigidities by various theoretical methods are small relative to the observed apparent inherent variability of shear wall properties. The deflection of a one-story contilever shear wall is given by

$$\delta = \frac{V}{E} \left[\frac{2.2H}{A} + \frac{H^3}{3I} \right] \qquad (1)$$

δ - Wall Deflection
E - Modulus of Elasticity
V - Wall Shear
H - Wall Height
A - Panel Area, A = (Panel Thickness) (Distance Center to Center of Columns), Neglect steel in calculation
I - Moment of Inertia of Wall Cross-Section, Neglect Steel

The equation assumes that $G = \frac{E}{2.2}$ and the shear shape-factor for deflection is unity. If one simply assigns all variation to E, the effective modulus of elasticity of the wall is,

$$E_e = \frac{V}{\delta} \left[\frac{2.2H}{A} + \frac{H^3}{3I} \right] \qquad (2)$$

The actual wall rigidity, $\frac{V}{\delta}$, can be determined experimentally and the quantities in the brackets can be calculated from the dimensions. The estimation of $\frac{V}{\delta}$ from load-deflection data is difficult in many cases owing to the very small deflections in the uncracked region.

The effective experimental E_e is shown in the histogram of Fig. 2 for the uncracked range of 55 reinforced and unreinforced concrete model walls. It is seen from Fig. 2 that the data are strongly skewed toward large values. The first asympotic distribution of largest values[6] provides a good fit to the data. The wall-panel reinforcing does not influence rigidity prior to major cracking. No correlation between cylinder strength at the time of the test and rigidity exists. This is illustrated by the scatter diagram of Fig. 3. The nominal design cylinder strength was 3000 psi at 14 days. The data represent samples from very similar but not identical mix designs. The probability distribution of cylinder strength is sensibly normal.

---

[6] "Statistics of Extremes," by E. J. Gumbel, Columbia University Press, 1958.

Analysis of the data on rigidity discloses that the coefficient of variation is approximately 0.5. In view of this large value, a very careful study was made of the data from 9 model walls having identical elastic properties and fitted with three sets of precision gages for measurement of deflections. They allow estimates of both the inherant variability of the walls and the apparent variability from measurement problems. The analysis of these data indicated that the true coefficient of variation is approximately 0.2 which is consistent with the coefficient of variation of deflection of reinforced concrete beams.

The average unit shear stress in the wall panel at first major cracking is given in the histogram of Fig. 4. The wall panel area was computed as the product of the wall thickness and the distance center to center of the columns. A reasonable fit of average shear stress to the normal probability law is indicated. The scatter diagram of Fig. 5 illustrates the apparent lack of correlation between average shear stress at first crack and concrete cylinder strength.

Variability Analysis

Once it is admitted that shear wall properties are highly variable, the next question is how to include this property in the analysis. One possibility is to search out upper and lower bounds, or bracket and possible solutions. The difficulties with this technique are obvious. What bounds do we use? How close to zero and infinite rigidity do we go or do we only consider the range of experimental results found in a small sample? This type of study does not indicate the most likely result or give a measure of likely variability.

Proper analysis of the structure on a probabilistic basis will not only yield realistic estimates of loading on the structural elements but also probability statements of the likelihood of cracking can be made. If the loading is probabilistic as well, it is a simple matter to consider the entire loading population providing rigidity is independent of loading.

The analysis of shear wall structures under seismic loading is accomplished using the requirements of equilibrium and consistent deformations. The floor diaphragms are assumed to be very rigid compared to the wall elements. Following conventional procedures,[7] the analysis of a one-story structure involves, first, the calculation of the shear center or center of twist, and, second, the calculation of forces on wall elements based on equilibrium and consistent deformations.

The following example illustrates the basic techniques of variability analysis of a one-story shear wall structure, Fig. 6.

Several techniques are available for the variability analysis of structures. The rigidities of the walls are independent random variables. Inasmuch as all three walls of Fig. 13 have the same dimensions and are of the same design, computations can be made in terms of E_e without calculating the actual rigidities. Probability statements about the E_e of a wall can be made, Table 1. The quantities indicated were obtained using a fitted extreme value probability model (First Asymptote) to the rigidity data which was then subdivided into ten intervals with equal probability of occurrence. The values shown are the medians of each such interval.

---

[7] Benjamin, J. R., "Statically Indeterminate Structures," McGraw-Hill, 1959.

Of the many available procedures for analysis of this type of problem, the method of enumeration is recommended as being the easiest to understand as well as the most accurate. The analysis reduces to recognizing that each of the three walls has ten possible independent rigidities. Thus 1000 possible combinations exist and the probability of occurrence of each of these combinations is equal to the product of their probabilities of occurrence, (0.1)(0.1)(0.1) = 0.001. The solution involves solving the usual analysis problem 1000 times, once for each set of rigidities, and then plotting the cumulative frequency diagram for the loads on the walls.

For example, if F_o equals unity and is in line with wall 2, and all three walls have identical E_e values, the load on each wall is 0.33. This solution can be found ten ways, once for each set of equal rigidities, so that the probability of a load of 0.33 on each wall is (10)(0.001) = 0.01. A computer can be used to solve the arithmetic problem as well as plot the cumulative frequency distributions. Fig. 7 contains a typical plot for Wall 2 with e = 0. The heavy line is the plot of the data and the light line that of a fitted normal distribution. Table 2 contains the means and standard deviations of loads on all walls for zero and 10 ft. eccentricity of F_o. All sets of data satisfactorily fit a normal distribution. Some round-off error is indicated in Table 2.

If no actual strength data are available, the design problem consists of setting an acceptable risk of major cracking and designing for this load level. For example, if F_o is 1000 kips in line with wall 2, the probability of the loading on Wall 2 being in excess of 333 kips is approximately 0.5.

The average shear stress at first major cracking is given by the histogram of Fig. 2. It is a simple matter to combine the analysis of loading on a wall with strength data to obtain estimates of probability of major cracking. Consider Wall 2 with an applied load of 1000 kips on the structure. What is the chance of major cracking if the design loading is 333 kips and a shear strength of 270 psi is used?

The analysis by enumeration proceeds by considering all combinations of loading and strength. The loading analysis is taken from the cumulative frequency diagram of Fig. 7 with, say, the probabilities of load levels for 10 equally likely intervals. The strength data are treated similarly assuming a normal distribution of strength with ten equally likely strengths. Now 100 equally likely possible combinations of loading and strength exist with the probability of each combination of (0.1)(0.1) = 0.01. The analysis proceeds rapidly by inspection comparing each strength level with each load level.

Alternatively, assume both load and strength are normally distributed and the sample estimates of the parameters are the true values. The probability distribution of strength minus load is also normally distributed. The mean of the combination is equal to the difference between the means, zero with the previous design assumptions. The probability of major cracking is 0.5 by the properties of the normal distribution.

If the design loading is taken as 333 kips but the assume strength is 185 psi, one standard deviation below the mean, the actual mean

Table 1

Values of E_e With Probabilities of Occurrence of 0.1

Mean = 2400 ksi and Coefficient of Variation = 0.2

Interval	E_e in ksi
1	1802
2	1966
3	2077
4	2177
5	2274
6	2377
7	2495
8	2639
9	2843
10	3254

Table 2

Loads on Walls

Wall	Eccentricity of F_Q	Mean	Standard Deviation	Coefficient of Variation
1	0	0.34	0.09	0.25
2	0	0.34	0.05	0.14
3	0	0.34	0.09	0.25
1	10	-0.17	0.03	0.18
2	10	0.34	0.06	0.18
3	10	0.85	0.19	0.22

strength is

$$(333)\left(\frac{270}{185}\right) = 486 \text{ kips}$$

The mean of strength minus loading is,

$$m = 486 - 333 = 153 \text{ kips}$$

The variance of strength minus loading is equal to the sum of the variances of strength and load,

$$\sigma^2 = \left[\left(\frac{85}{270}\right)(486)\right]^2 + (50)^2 = 25{,}800$$

The standard deviation of strength minus load is

$$\sigma = 160 \text{ kips}$$

The probability of strength minus loading being negative is the probability of major cracking. Divide m by σ,

$$u = \frac{m}{\sigma} = \frac{153}{160} = 0.96$$

Probability of Cracking = Probability ($u \geq 0.96$) = 0.17

Design with a mean strength of 486 kips instead of 333 kips approximately reduces the chance of major cracking from 0.50 to 0.17.

CONCLUSION

The variability characteristics of shear walls was discussed along with a technique for including this variability in analysis and design. It was seen that a deterministic analysis for extreme seismic loads is only one of many possible solutions and design even with the loading known always involves some risk of cracking as a consequence of the randomness of both rigidity and strength.

ACKNOWLEDGEMENT

The research reported in this paper was supported in part by National Science Foundation Grant GK 745.

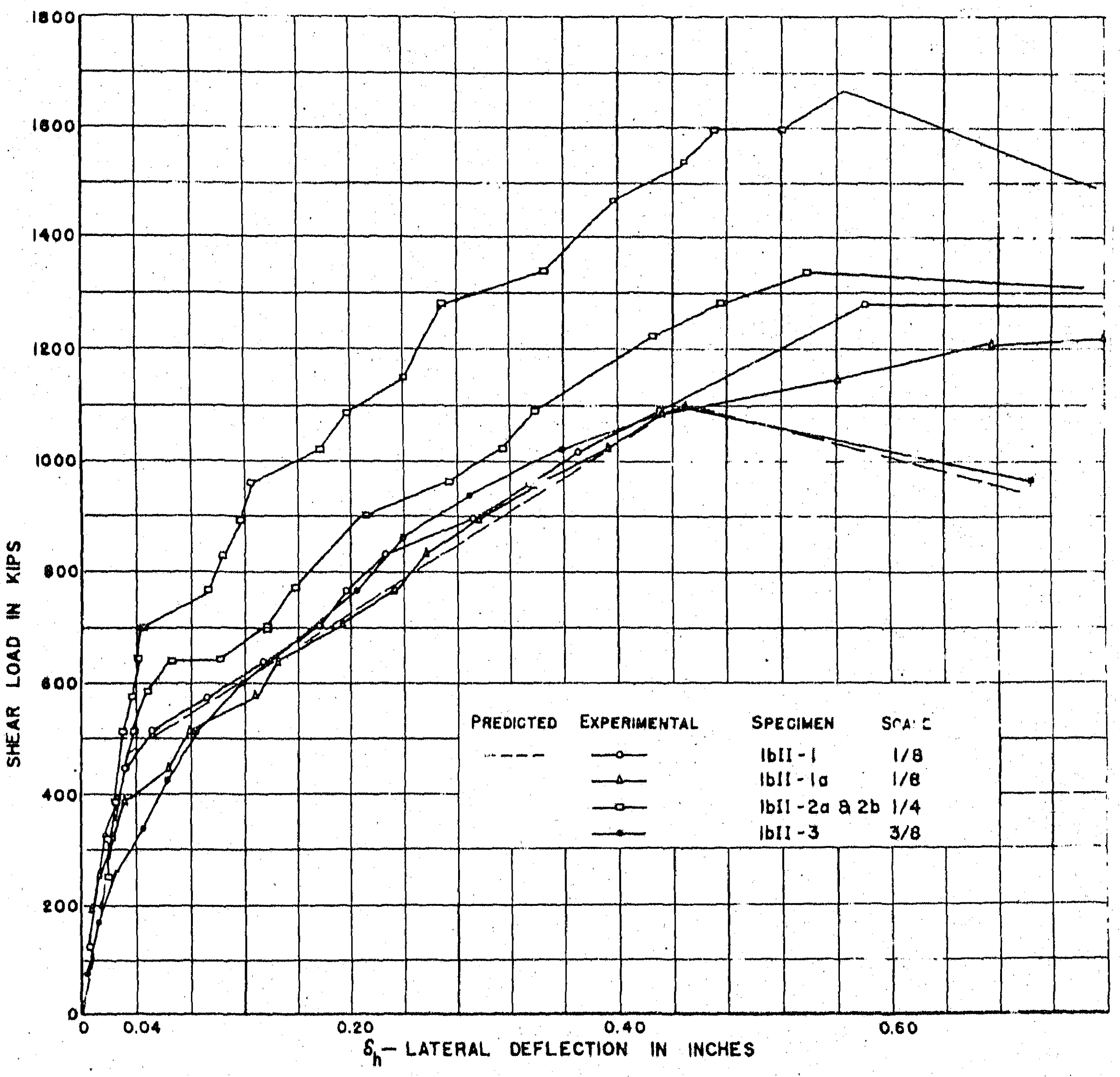

FIG 1 - LOAD-DEFLECTION CURVES FOR PROTOTYPE — SCALE EFFECT MODELS WITH 0.5 % REINFORCED PANELS

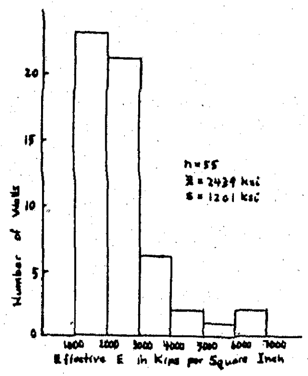

Fig. 2 Histogram of Effective E

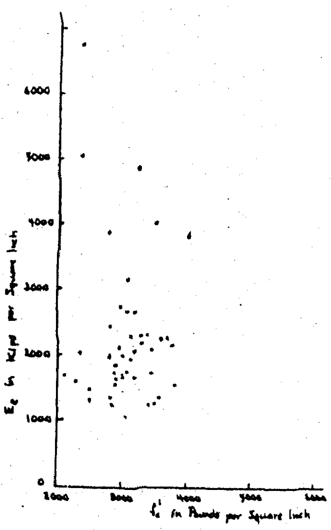

Fig. 3 Scatter Diagram of E_e Against f_c'

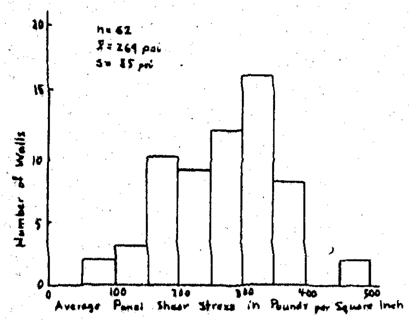

Fig. 4 Histogram of Average Unit Shear Stress at First Major Crack.

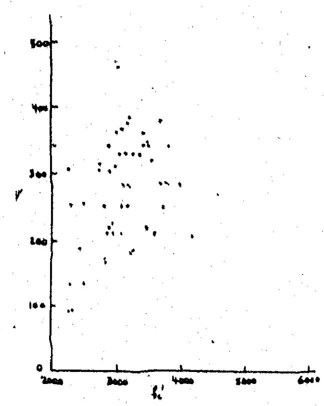

Fig. 5 Scatter Diagram of Average Panel Shear Stress Against f_c'

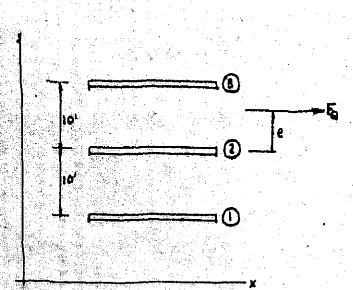

Fig. 6 One-Story Structure With Three Shear Walls

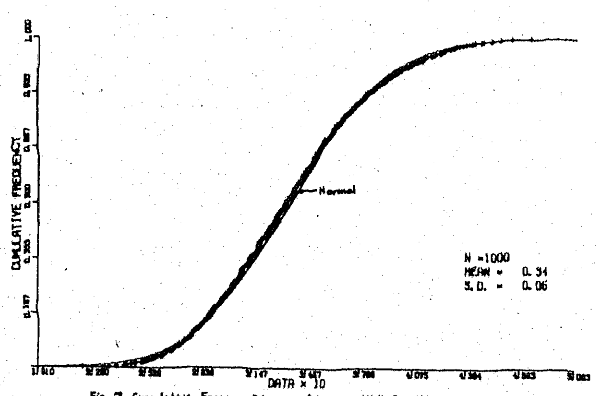

Fig. 7 Cumulative Frequency Diagram of Load on Wall 2 With e=0.