# ABSORBER SYSTEM FOR EARTHQUAKE EXCITATIONS

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## SYNOPSIS

Vibration absorbers have been so far effectively used for steady state sinusoidal type of excitations. This paper explores the possibility of their use for irregular motion of earthquakes by having a number of absorbers of different periods and dampings. Basic systems considered in the analysis were liner single degree of freedom systems. The absorber system had both linear and elasto-plastic restoring force characteristics and viscous type of damping.

## NOTATIONS

| | | |
|---|---|---|
| $C_p$ | - | viscous damping in parent system. |
| $C_{ai}$ | - | viscous damping in absorber i. |
| $[D]$ | - | damping matrix, defined after equation 3. |
| $m_p$ | - | mass of parent system. |
| $m_{ai}$ | - | mass of vibration absorber i. |
| n | - | number of vibration absorbers. |
| P.R. | - | range of periods of absorbers in a group. |
| $Q_{ai}$ | - | restoring force for absorber i at any time t. |
| $\{Q\}$ | - | restoring force vector. |
| $T_p$ | - | undamped natural period of vibration of parent system. |
| $T_{ai}$ | - | undamped natural period of vibration of absorber. |
| $\{V\}$ | - | velocity vector. |
| $x_p$ | - | displacement of parent system. |
| $x_{ai}$ | - | displacement of absorber i. |
| $\ddot{y}$ | - | ground acceleration. |
| $y_{Lai}$ | - | yield level for absorber i. |

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$Z_p$ - relative displacement of parent system.

$Z_{ai}$ - relative displacement of absorber i.

$\{Z\}$ - relative displacement vector.

$\alpha_{ai}$ - mass ratio- $m_{ai}/m_p$

$\beta$ - response reduction factor for parent system.

$\gamma_{av}$ - average of all ratios of shear force acting on absorber to that of parent system without absorber.

$\zeta_p$ - fraction of critical damping in parent system.

$\zeta_{ai}$ - fraction of critical damping in absorber i.

$\mu_a$ - ratio of sum of masses of all absorbers in a group to that of parent system mass.

## INTRODUCTION

Vibration absorbers have been commonly used for reducing forces on sinusoidally vibrating single degree of freedom systems. Such absorbers are effective at one frequency and hence they have been made use of in machines and machine foundations where it is required to reduce the peak resonance amplitude. Investigations[1] have been carried out on the efficacy of absorbers for earthquake type excitations using multiple absorbers with linear force deformation characteristics and viscous type of damping. In this paper absorber system having non-linear force deformation characteristics have been considered in the analysis.

Parent systems considered in the analysis are idealized single degree of freedom systems. Viscous type of damping has been taken for parent system as well as for absorbers. Several combinations of parameters involved in the problem have been tried to determine the influence of absorbers on the response of parent system to an actually recorded earthquake.

Investigations reveal that absorber systems are not that effective for earthquake type excitations as compared to sinusoidal excitations.

## EQUATIONS OF MOTION

A mathematical model of idealized single degree of freedom parent system along with multiple absorbers is shown in figure 1. The parent system has a mass $m_p$, spring constant $k_p$ and coefficient of viscous damping $c_p$. A vibration absorber which is also represented by equivalent single degree, has a mass $m_a$, restoring force at any time $Q_a$ and coefficient of viscous damping $c_a$.

The equations of motion for such system when it is subjected to ground motion $\ddot{y}(t)$, can be written as follows:

$$m_p\ddot{x}_p + c_p(\dot{x}_p-\dot{y}) + c_{a1}(\dot{x}_p-\dot{x}_{a1}) + c_{a2}(\dot{x}_p-\dot{x}_{a2})+\ldots+ c_{an}(\dot{x}_p-\dot{x}_{an})$$

$$+ k_p(x_p-y) + Q_{a1}f(x_p-x_{a1})+Q_{a2}f(x_p-x_{a2})+\ldots+Q_{an}f(x_p-x_{an}) = 0$$

$$m_{a1}\ddot{x}_{a1} + c_{a1}(\dot{x}_{a1}-\dot{x}_p) + Q_{a1}f(x_{a1}-x_p) = 0$$

$$m_{a2}\ddot{x}_{a2} + c_{a2}(\dot{x}_{a2}-\dot{x}_p) + Q_{a2}f(x_{a2}-x_p) = 0$$

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$$m_{an}\ddot{x}_{an} + c_{an}(\dot{x}_{an}-\dot{x}_p) + Q_{an}f(x_{an}-x_p) = 0 \qquad \ldots 1$$

Where,

$x_p$ = displacement of parent system.

$x_a$ = displacement of absorber.

$Q_a$ = restoring force for absorber spring which is a function of relative displacements.

Simplifying equation 1 and writing in terms of periods, fractions of critical damping, mass ratios and relative displacements,

$$\ddot{z}_p + 4\pi\left(\frac{\zeta_p}{T_p} + \frac{\alpha_{a1}\zeta_{a1}}{T_{a1}} + \frac{\alpha_{a2}\zeta_{a2}}{T_{a2}} + \cdots\cdots + \frac{\alpha_{an}\zeta_{an}}{T_{an}}\right)\dot{z}_p$$

$$- 4\pi\frac{\alpha_{a1}\zeta_{a1}}{T_{a1}}\dot{z}_{a1} - 4\pi\frac{\alpha_{a2}\zeta_{a2}}{T_{a2}}\dot{z}_{a2} \cdots\cdots - 4\pi\frac{\alpha_{an}\zeta_{an}}{T_{an}}\dot{z}_{an}$$

$$+ \frac{4\pi^2}{T_p^2}z_p - \frac{\alpha_{a1}Q_{a1}}{m_{a1}}f(z_{a1}-z_p) - \frac{\alpha_{a2}Q_{a2}}{m_{a2}}f(z_{a2}-z_p) \cdots\cdots$$

$$- \frac{\alpha_{an}Q_{an}f(z_{an}-z_p)}{m_{an}} = -\ddot{y}$$

$$\ddot{z}_{a1} - 4\pi \frac{\zeta_{a1}}{T_{a1}} \dot{z}_p + 4\pi \frac{\zeta_{a1}}{T_{a1}} \dot{z}_{a1} + \frac{Q_{a1} f(z_{a1} - z_p)}{m_{a1}} = -\ddot{y}$$

$$\ddot{z}_{a2} - 4\pi \frac{\zeta_{a2}}{T_{a2}} \dot{z}_p + 4\pi \frac{\zeta_{a2}}{T_{a2}} \dot{z}_{a2} + \frac{Q_{a2} f(z_{a2} - z_p)}{m_{a2}} = -\ddot{y}$$

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$$\ddot{z}_{an} - 4\pi \frac{\zeta_{an}}{T_{an}} \dot{z}_p + 4\pi \frac{\zeta_{an}}{T_{an}} \dot{z}_{an} + \frac{Q_{an} f(z_{an} - z_p)}{m_{an}} = -\ddot{y}$$

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Where,

$T_p$ = undamped natural period of parent system.

$\zeta_p$ = fraction of critical damping in parent system.

$T_p$ = undamped natural period of absorber.

$\zeta_a$ = fraction of critical damping in absorber.

$Z_p$ = relative displacement of parent system at any time t.

$Z_a$ = relative displacement of absorber at any time t.

$\alpha_a$ = ratio of absorber mass to parent system mass, $m_a/m_p$.

Expressing equation 2 in the matrix notation.

$$\{\ddot{Z}\} + [D] \{\dot{Z}\} + \{Q\} = - \{\ddot{y}\} \quad \ldots 3$$

Where,

$$[D] = 4\pi \begin{bmatrix} \left(\frac{\zeta_p}{T_p} + \frac{\alpha_{a1}\zeta_{a1}}{T_{a1}} + \frac{\alpha_{a2}\zeta_{a2}}{T_{a2}} + \cdots + \frac{\alpha_{an}\zeta_{an}}{T_{an}}\right) & -\frac{\alpha_{a1}\zeta_{a1}}{T_{a1}} & -\frac{\alpha_{a2}\zeta_{a2}}{T_{a2}} & \cdots & -\frac{\alpha_{an}\zeta_{an}}{T_{an}} \\ -\frac{\zeta_{a1}}{T_{a1}} & \frac{\zeta_{a1}}{T_{a1}} & & & \bigcirc \\ -\frac{\zeta_{a2}}{T_{a2}} & & \frac{\zeta_{a2}}{T_{a2}} & & \\ \cdots & & \cdots & & \\ \cdots & & & \cdots & \\ -\frac{\zeta_{an}}{T_{an}} & \bigcirc & & & \frac{\zeta_{an}}{T_{an}} \end{bmatrix}$$

$$\{Q\} = \begin{Bmatrix} 4\pi^2 \frac{z_p}{T_p^2} - \frac{\alpha_{a1}Q_{a1}f(z_{a1}-z_p)}{m_{a1}} - \frac{\alpha_{a2}Q_{a2}f(z_{a2}-z_p)}{m_{a2}} \cdots \frac{\alpha_{an}Q_{an}f(z_{an}-z_p)}{m_{an}} \\ \frac{Q_{a1}f(z_{a1}-z_p)}{m_{a1}} \\ \frac{Q_{a2}f(z_{a2}-z_p)}{m_{a2}} \\ \cdots \\ \cdots \\ \frac{Q_{an}f(z_{an}-z_p)}{m_{an}} \end{Bmatrix}$$

$$\{z\} = \begin{Bmatrix} z_p \\ z_{a1} \\ z_{a2} \\ \cdots \\ \cdots \\ z_{an} \end{Bmatrix} \quad \text{and} \quad \{\ddot{y}\} = \begin{Bmatrix} \ddot{y} \\ \ddot{y} \\ \ddot{y} \\ \cdots \\ \cdots \\ \ddot{y} \end{Bmatrix}$$

## SOLUTION OF EQUATIONS

Equation 3 consisting of several second order differential equations can be expressed as two first order differential equations as follows:-

$$\{\dot{V}\} = -(\{\ddot{y}\} + [D]\{V\} + \{Q\})$$

$$\{\dot{z}\} = \{V\} \qquad \ldots 4$$

A modified fourth order Runge Kutta procedure has been used for numerical solution of equation 4, since there is no explicit solution possible.

The restoring force term Q, which depends on the amount of relative displacement and sign of velocity, has been calculated every time with the help of force-deformations curve assumed for the absorber. Elasto-plastic restoring force characteristics have been taken for absorber spring. Ground motion having amplitude in either direction gives rise to reversal of stresses in absorber spring.

A digital computor program was written in FORTRAN language and was used for the solution of various cases of parent system with vibration absorbers.

## SPECIFICATIONS OF THE PROBLEM

The parent system is assumed to be represented by linear viscous damped single degree of freedom system. The response, with and without vibration absorbers, has been determined for two parent systems having undamped periods ($T_p$) of 0.5 and 1.0 second, and having same percentage of critical damping ( $\zeta_p$) of 5 percent. Digitalised data of Taft earthquake of July 21, 1952, S21 W component has been used in the analysis.

The parameters of vibration absorbers which influence the behaviour of parent system, have been extensively varied and they are, (a) Total mass ratio, $\mu_a$ (b) Damping of absorbers, $\zeta_a$ (c) Number of absorbers, n (d) Yield level for restoring force of absorbers, $Y_{La}$ (e) Range and undamped period of absorbers, P.R.

Distribution of various parameters in a range of periods of several absorbers has been taken as follows. Let the range be given as 0.50 $T_p$ to 1.50 $T_p$ and the number of absorbers be five. Then the periods will be 0.50 $T_p$, 0.75 $T_p$, $T_p$, 1.25 $T_p$ and 1.50 $T_p$. That is, the absorbers are equally spaced in the range. The mass ratio and yield level of individual absorber are directly proportional to the corresponding periods of absorbers and increases with them. From a preliminary analysis it has been found that this type of distribution of mass ratio and yield level is most efficient. Damping is same for all absorbers in a group.

In each case, response in shear of parent system as well as of absorbers were determined. The ratio of shear force on parent system with absorbers to that without it is denoted as response reduction factor, $\beta$ , of the parent system. The ratio of shear force on an absorber to that of the parent system without it is defined as response ratio and denoted as $\gamma$.

## DISCUSSIONS AND CONCLUSIONS

The influence of various parameters of absorber system, on the response of parent system is as follows:

### Effect of Mass Ratio

The total mass ratio, $\mu_a$, is defined as the ratio of sum of masses of all absorbers to that of the parent system mass. Figure 2 gives a plot of total mass ratio, $\mu_a$, versus response reduction factor $\beta$, for a parent system having one second period and 5 percent damping. The number of absorbers in this case are five and the average yield level is 0.15 where average yield level is defined as the average value of all yield level of absorbers. The plot shows that value of $\beta$ decreases with the increase in mass ratio, but beyond a value of 0.10, the decrease is not much. Hence for further analysis as well as in design the value of mass ratio may be taken as 0.10.

### Effect of Damping

With increase in damping of absorbers, the response of parent system increases, as shown in figure 3. Physical systems always have some inherent damping present in them even without external damping. Therefore, it is assumed in the analysis that absorbers have a damping of the order of two percent of critical.

### Effect of Number of Absorbers

A plot of number of absorbers versus response reduction factor is shown in figure 4. It is seen that $\beta$ changes little when number of absorbers are five and above. However, the response of absorbers themselves decreases with the increase in numbers as seen from figure 5.

### Effect of Yield Level

Figure 6 gives a plot of response reduction factor for various yield levels for absorbers. It shows that with the increase in yield level the value of $\beta$ remains unchanged.

Figure 7 gives a plot of yield level versus ductility corresponding to the cases given in figure 6. Curve 1 indicates maximum ductilities of a group of five absorbers with ascending type of yield level distribution. Curve 2 indicates minimum ductilities for the corresponding cases. From the two curves it is seen that with the increase in yield level, ductility decreases.

Figure 8 indicates the variation of average response ratio of absorbers with average yield level, where average response ratio, $\gamma_{av}$, is defined as the average of response ratios of all absorbers in a group. Difference in maximum response among absorbers decreases with increase in number. Since a number of absorbers are usually required, average value is taken as a representation of response of absorbers. The curve indicates that with decrease in yield level the value of $\gamma_{av}$ decreases. Linear case is obtained when yield level is one and is shown in figure 7. It is observed that force on absorber decrease if non-linearity is considered.

## Effect of Range and Periods of Absorbers

Figure 9 indicates the response reduction factor, for various period ranges. It is seen that there is not much change in the values of $\beta$. However, a smaller range is better as compared to larger range.

Average response ratio, $\gamma_{av}$, for various period ranges is shown in figure 10.

## Parent System Periods

The influence of various parameters on the response of parent system having a period of 1.50 seconds, is more or less similar as for 1.0 second period. Response reduction factor curves for various period ranges are shown in figure 9.

Vibration absorbers are not that effective in reducing the response of parent systems subjected to earthquake excitations. However, the force acting on an absorber is very small and could be kept at any desired low value by varying the yield levels or number of absorbers in a group.

## ACKNOWLEDGEMENTS

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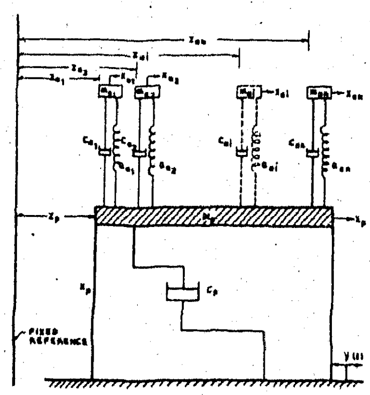

FIG. 1 _MODEL OF PARENT SYSTEM WITH ABSORBERS

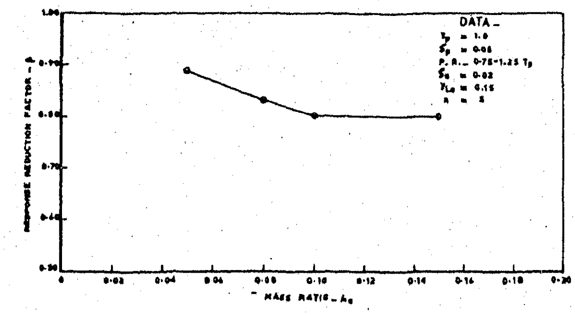

FIG. 2 _INFLUENCE OF MASS RATIO ON RESPONSE REDUCTION FACTOR

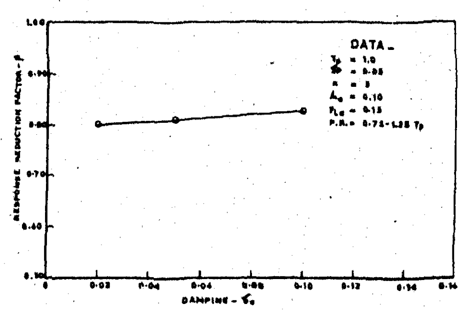

FIG. 3 _RESPONSE REDUCTION FACTOR VS DAMPING IN ABSORBERS

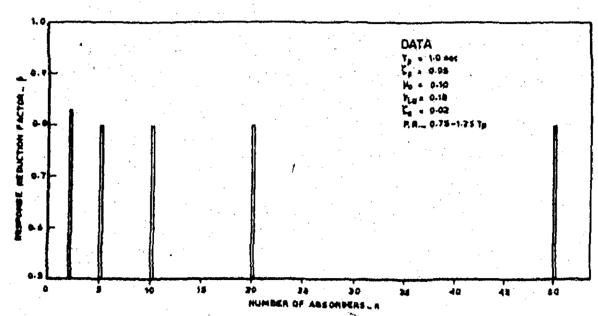

FIG. 4 _EFFECT OF NUMBER OF ABSORBERS ON RESPONSE REDUCTION FACTOR OF PARENT SYSTEM

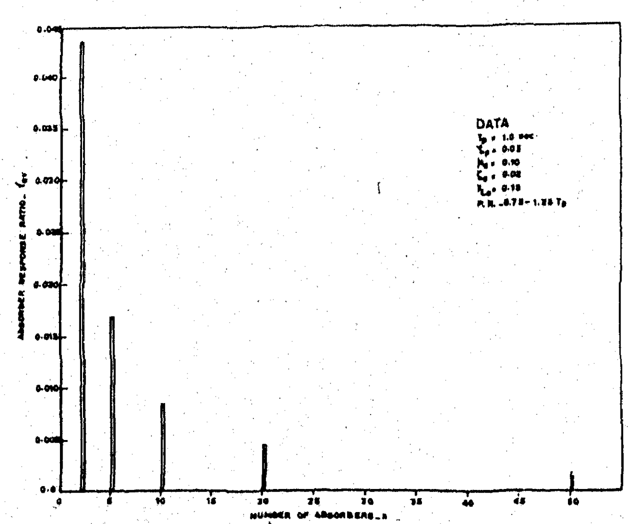

FIG. 5 _EFFECT OF NUMBER ON AVERAGE RESPONSE OF ABSORRERS

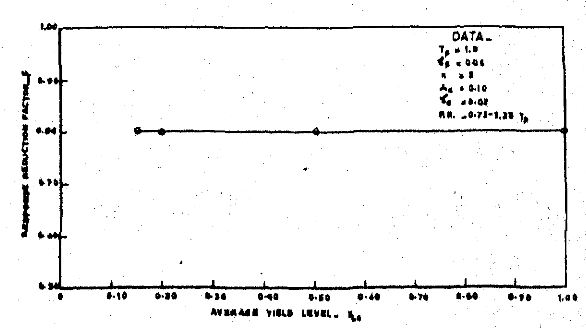

FIG. 6 _ INFLUENCE OF YIELD LEVEL ON RESPONSE REDUCTION FACTOR

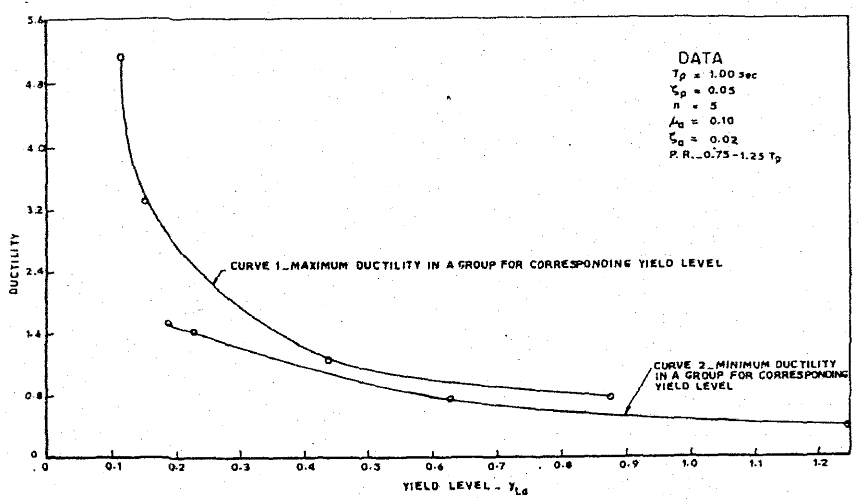


FIG. 7 _ RELATIONSHIP BETWEEN YIELD LEVEL AND DUCTILITY FOR ABSORBERS

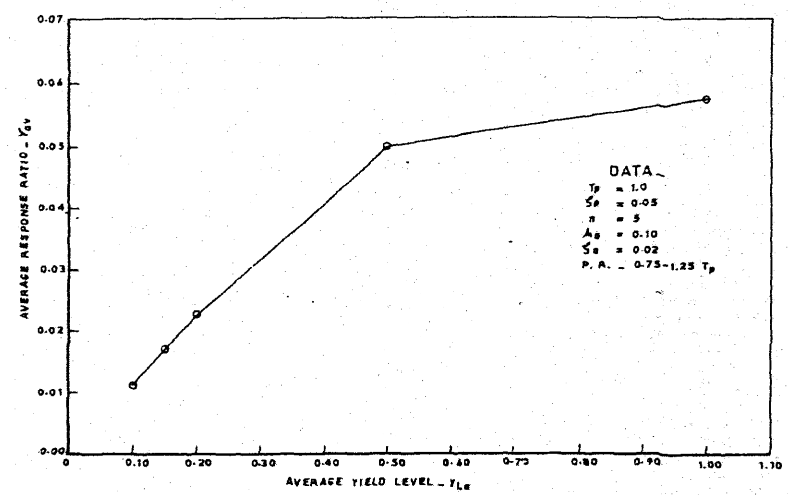


FIG. 8 _ EFFECT OF YIELD LEVEL ON AVERAGE RESPONSE RATIO FOR ABSORBERS

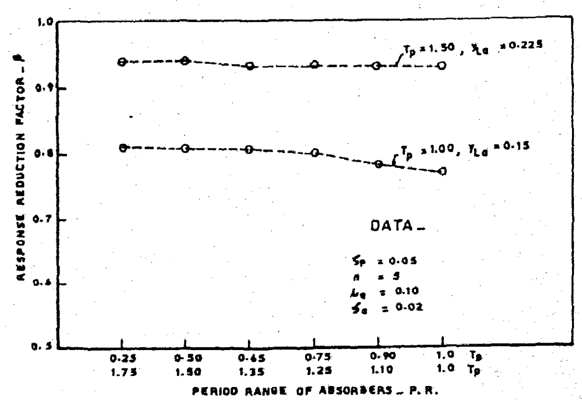


FIG. 9 _ RESPONSE REDUCTION FACTOR VS RANGE OF PERIODS OF ABSORBERS

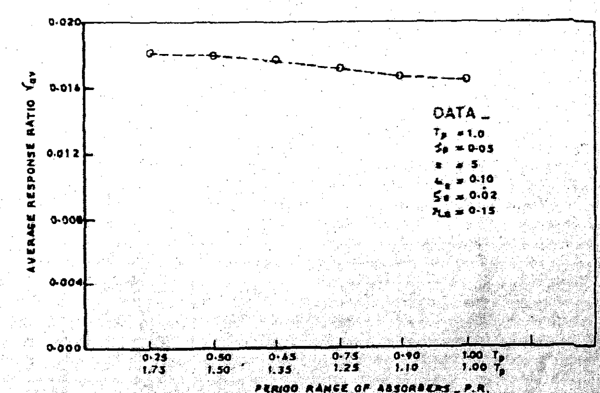


FIG. 10 _ AVERAGE RESPONSE RATIO VS RANGE OF PERIOD OF ABSORBERS