TORSIONAL PROBLEMS IN ASEISMIC DESIGN OF HIGH-RISE BUILDINGS

by

Tadaki Koh,[(i)] Hiromoto Takase[(ii)] and Tsunehisa Tsugawa[(ii)]

ABSTRACT

This paper describes the torsional effects due to irregularity of structural layout and eccentricity in mass distribution in high-rise buildings. The authors have been engaged in the design of many high-rise buildings in Japan and studied the torsional effect against severe earthquakes. A more rational approach and method to improve the design will be presented by investigating the response analysis results.

Assuming that floor slab is rigid in its plane, displacement at any point can be represented with the components translation x and y and rotation around the center of gravity in each floor. The fundamental vibration equation is as follows:

$$M \Delta \ddot{U} + C \Delta \dot{U} + K \Delta U = - M \Delta \alpha$$

where M: matrix of mass and moment of inertia, C: damping matrix, K: stiffness matrix, U: displacement vector of x, y and θ, and α: acceleration vector of ground motion

Computer programs are developed for evaluation of the fundamental vibration, distribution of maximum stress and deformation induced by a severe earthquake within elastic and plastic range. For simple and plain comparison of torsional effects, the same pattern of the El Centro 1940 earthquake accelerogram with various intensities is adopted and assumed to propagate in one direction.

The results of studies on four buildings actually designed in Japan are discussed. The buildings are analyzed first as a restrained system without torsional consideration, then as a coupled system with it. In the analysis, each building is characterized by its own structural factor for torsion. They are, eccentricity in mass distribution, eccentricity by additional rigidity, irregular layout of structural elements and unbalanced arrangement of walled frames.

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NOTATIONS

The following notations are used in this paper:

s(or t)	= subscript of s(or t) frame in X (or Y) direction
${}^{s}P_x$ (or ${}^{t}P_y$)	= lateral force vector of s(or t) frame in X (or Y) direction
${}^{s}G_x$ (or ${}^{t}G_y$)	= lateral stiffness matrix of s(or t) frame
${}^{s}\bar{L}$ (or ${}^{t}\bar{L}$)	= diagonal matrix of distance of s(or t) frame from center of gravity
$f_{x,y,\theta}$	= external force vector to center of gravity in directions X, Y and θ
x, y, θ	= displacement vector of center of gravity
F	= generalized force vector $= \{f_x, f_y, f_\theta\}^T$
U	= generalized displacement vector $= \{x, y, \theta\}^T$
K	= generalized stiffness matrix
M	= generalized diagonal matrix of mass
C	= generalized damping matrix
α	= acceleration vector of earthquake

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INTRODUCTION

Thanks to the development of digital computer in recent years, it has become possible to make detailed analyses of the dynamic behavior of high-rise buildings during earthquakes; in particular, it has now become possible to deal with torsional vibration considering not only translational movement but also plane rotation.

This paper represents a quantitative study of the increases in the shear force and drift caused by torsional effects in buildings, especially in the exterior frames. The study is undertaken by following a method of dynamic analysis in high-rise buildings which gives due consideration to rotational, as well as translational, movement.

That is, by first expressing a general equation of motion, which encompasses the 3 degrees of freedom on each floor of the structures (the displacement vectors X and Y and the rotational angle Θ), analyses can be made not only for linear, but also for non-linear, cases.

Next, this theory is used to analyze 4 types of structures which we have designed. They can be distinguished respectively by the following characteristics.

<u>Building A</u> The design calls for symmetrical rigidity and uniform mass distribution, but situations where live loads are not uniformly distributed, and lead to mass eccentricity, are taken into consideration.

<u>Building B</u> The design calls for symmetrical rigidity and uniform mass distribution but, taking into account that subsidiary elements such as staircases and room partitions may exert a rigidity effect, added rigidity is assumed for the exterior frame of the structure.

<u>Building C</u> Torsion is created inevitably because of the irregularity of the plane figure.

<u>Building D</u> Torsion is created inevitably because of the absence of uniformity in the disposition of earthquake-resistant walls.

In calculation of dynamic response, there was used the digital computer HITAC 5020 in Kajima Institute of Construction Technology.

ASSUMPTIONS AND LIMITATIONS

Assumptions in making the model

The following assumptions were made in deriving analytical models from these structures.

(a) The buildings are constructed from plane open frames or walled frames.

(b) The stiffness of the frames is determined, as the situation requires, by considering the bending, shearing, and axial deformation of members, as well as the shearing deformation of joint panels.

(c) Frames or members may have non-linear properties (such as bi-linear characteristics).

(d) Torsional rigidity of members is ignored.

(e) The floor slabs are rigid within their planes.

(f) The mass of each part of the building is concentrated on the floor slab.

Assumptions made in calculating response

(a) The earthquake wave pattern adopted for the response calculation was the El Centro 1940 NS component digitalized by Professor Berg. Only the pattern for the first 8 seconds was used, and various peak intensities were inserted into this pattern. Thereafter, transient and free vibration were generated over a sufficient and continuous period of time.

(b) The direction of the earthquake's input force was taken as being in one direction only (i.e., in direction X). The rotational component of the earthquake was ignored.

(c) In considering damping effects, a viscous type of damping caused by internal friction was assumed. In Building A, 0.03 was taken as the damping factor for the first mode, while in each of the other three buildings, the factor was 0.05.

METHOD OF ANALYSIS

Stiffness matrix

Utilizing the notations and assumptions just outlined, the generalized stiffness matrix is determined in order to express the equilibrium equation.

As shown in Fig.1, when the direction of each frame coincides with the X or Y coordinates, the displacement vector of each frame is shown in the form ($x - {}^{s}L\,\theta$) or ($y - {}^{t}L\,\theta$). The lateral force vector of each frame is then given as follows:

$$\{{}^{s}P_{x}, {}^{t}P_{y}\} = \{{}^{s}C_{x}\cdot(x - {}^{s}L\,\theta), {}^{t}C_{y}\cdot(y - {}^{t}L\,\theta)\} \quad (1)$$

The generalized force is expressed as the sum of the lateral forces of all the frames. Thus:

$$F = \left\{\sum^{S}{}^{s}P_{x}, \sum^{T}{}^{t}P_{y}, \left(\sum^{S}{}^{s}L\cdot{}^{s}P_{x} - \sum^{T}{}^{t}L\,{}^{t}P_{y}\right)\right\}^{T} \quad (2)$$

Substituting Equation (1) into (2), the generalized stiffness matrix may be obtained. Thus:

$$F = KU \tag{3}$$

where

$$K = \begin{bmatrix} \sum^{S} {}^{s}C_x & 0 & \sum^{S} {}^{s}C_x {}^{s}L \\ 0 & \sum^{T} {}^{t}C_y & -\sum^{T} {}^{t}C_y {}^{t}L \\ \sum^{T} {}^{s}L {}^{s}C_x & -\sum^{T} {}^{t}L {}^{t}C_y & \sum^{S} {}^{s}L {}^{s}C_x {}^{s}L + \sum^{T} {}^{t}L {}^{t}C_y {}^{t}L \end{bmatrix} \tag{4}$$

When the directions of the principal axes of a building's frames differ from X or Y coordinate, transfer operation is required; hence, the above equation is somewhat modified, so that the O(zero) factors in Equation (4) have expticit values.

Static analysis

The relationship between force and displacement in static analysis is shown in Fig.1 and Fig.2. For stress analysis, Equation (3) should be solved.

Dynamic equilibrium equation

In both linear and non-linear states, a differential equation of motion, corresponding to equilibrium conditions that vary with changes during extremely short time intervals (such as 0.01 second), may be expressed as follows:

$$M\Delta\ddot{U} + C\Delta\dot{U} + K\Delta U = -M\Delta\alpha \tag{5}$$

According to the trapezoidal rule,

$$\Delta\dot{U} = \frac{\tau}{2}(\ddot{U}_{t0} + \ddot{U}) = \tau\ddot{U}_{t0} + \frac{\tau}{2}\Delta\ddot{U} \tag{6}$$

$$\Delta U = \frac{\tau}{2}(\dot{U}_{t0} + \dot{U}) = \tau\dot{U}_{t0} + \frac{\tau^2}{2}\ddot{U}_{t0} + \frac{\tau^2}{4}\Delta\ddot{U} \tag{7}$$

where τ indicates the time interval and U_{t0} indicates displacement value at the beginning of the time interval.

Substituting Equations (6) and (7) in Equation (5), the differential equation of motion is given in the following form:

$$F^{*} = K^{*}\Delta\ddot{U} \tag{8}$$

where

$$K^{*} = M + \frac{\tau}{2}C + \frac{\tau^2}{4}K \tag{9}$$

$$F^{*} = -\tau C\ddot{U}_{t0} - \tau K(\dot{U}_{t0} + \frac{\tau}{2}\ddot{U}_{t0}) - M\Delta\alpha \tag{10}$$

EXAMPLES

Building A

Outline of the Building: The highest building in Japan, Building A is a 40-story office structure under construction in Tokyo. As shown in Fig.3 and Fig.4, the main steel frames are composed of wide flange H-shape columns of the 400mm series, as well as expanded H-shape (castellated) and full web H-shape beams of 800mm series. Precast reinforced concrete slitted walls are placed into the frames at the interior core position.

Analysis Model: The design calls for symmetrical rigidity and mass distribution on each floor; hence, this building has no eccentricity. However, in order to consider torsion effects, the center of gravity of each floor can be artificially shifted in the diagonal direction. Thus, eccentricities can be induced with magnitudes of, arbitrarily, 5% or 10% of the diagonal width of the building. In computer analysis, standard methods were used, taking into account all possible deformations of the composing elements of the open and walled frames.

Static Investigation: Figs.5 and 6 show the shear force and drift of the exterior frame when subjected to static force (design shear force). It is to be noted that shear force and drift increase in proportion to eccentricity.

Free Vibration: The natural periods, vibration modes and participation factors for the cases of the restrained system and the torsional systems with 5% and 10% induced eccentricity are shown in Table 1 and Fig.7. Note that in the restrained system, the first, second, and third periods are nearly the same in X and Y directions.

Earthquake Response: Results of bi-linear response for the earthquake wave with a maximum intensity of 0.1g are as follows. Fig.8 shows the displacement history curve of the top floor's center of gravity during and after the earthquake. It should be noted that the highest values of amplitudes in Y and θ directions appear in the free vibrations after the earthquake. Figs.9 and 10 indicate the maximum values of the total story shear force and story deflection of the center of gravity at each story level. The maximum values in X direction of the torsional system are nearly the same as those of the restrained system in the same direction, but it should be noticed that the maximum values in normal (Y) direction reach 40% (at most) of those in X direction. Figs.11 and 12 show maximum values of shear force and story drift in the exterior frames. It should be noted that the effects of eccentricity are remarkable from the 10th to 30th stories, compared to the restrained system, which has no rotational movement. That is, the increases of the maximum shear forces on the exterior frame are 13% at the 17th story and 18% at the 13th story in the case of 5% and 10% eccentricities respectively. The increase of maximum drift of exterior frames at the 14th story are 13% (for 5% eccentricity) and 20% (for 10% eccentricity).

Building B

Outline of the Building: Building B is a 16-story office building constructed in Tokyo. It is composed of open frames with 500mm box type columns and wide flange H-shape beams of 800mm depth.

Analysis Model: The design calls for symmetrical rigidity and mass distribution, but, taking into account that subsidiary elements, such as staircases and room partitions, may exert a rigidity effect, added rigidity is assumed for the exterior frame of the structure. As is shown in Fig.13, Model B-0 has zero eccentricity, exactly as assumed in design. Model B-1, B-2 and B-3 have, respectively, 4.5%, 10% and 17% eccentricity induced by adding 10%, 25% and 50% imaginary rigidity in X direction to the exterior end frame of the building.

Free Vibration: The natural periods and the vibration modes of these four models are indicated in Figs.14 and 15, and in Table 2.

Earthquake Response: Figs.17 and 18 represent the results of elastic response in each of the four models to an earthquake with maximum intensity of 0.1g. Maximum values of total story shear force in X direction are nearly the same, despite differences in eccentricity, but accompanying as eccentricity increases (in Models B-1, B-2 and B-3), maximum values of shear force and story drift of the exterior end frame also increase up to 35% above the amount of shear force and story drift in Model B-0.

Building C

Outline of the Building: Building C is a 15-story office structure constructed of steel and reinforced concrete open frames, arranged as shown in Fig.19.

Analysis Model: Because of the irregularity of the plane figure of this building, torsion is inevitably created. The torsional system of the actual building is studied, compared with the restrained system in X direction, shown in Fig.19. The stiffness of the frame, which is evaluated by the D-Value Method, is assumed to have bi-linear hysteresis. Fig.20 shows also that columns are assumed to yield at a drift of 1.0cm. Further, after they yield, the rigidity (in terms of D-Value) is assumed to be reduced to 1/5 the original.

Free Vibration: The natural periods of this building are shown in Table 3. There is no great difference between the two systems; the first period of the restrained system corresponds closely to the first and second periods of the torsional system, the second period of the restrained system closely matches the fourth and fifth periods of the torsional system, and the third period of the restrained system corresponds to the seventh and eighth periods of the torsional system. This observation remains valid for both X and Y directions in the restrained system.

Earthquake Response: Maximum response values are shown in Figs.21, 22 and 23. The building is elastic for earthquake with a maximum intensity of 0.1g, while at an intensity of 0.3g, it becomes partially plastic. The maximum story shear force of the torsional system in X direction is shown in Fig.21, it does not differ substantially from that of the restrained system. Figs.22 and 23 show the maximum shear force and story drift, respectively, of column No.1 and No.48, two identical columns. When the two columns are compared, it can be seen that there is considerable difference in the results of response at 0.3g intensity, and that No.48 becomes plastic from the increases of 1st to 11th floors while No.1 does so from the 1st to 7th floors.

Building D

Outline of the Building: Building D is a 23-story hotel structure, of which the transverse (X direction) is composed of open steel frames and walled frames with slitted walls, and the longitude (Y direction) is composed only of open frames. All frames consist of wide flange H-shape columns of 400mm series and beams of 700mm depth (See Figs.24 and 25).

Analysis Model: In this structure, torsion is inevitably created because of the absence of uniformity in the disposition of earthquake-resistant walls. Torsional effects in this building are studied in comparison with the vibration in X direction of the restrained system. Based on several experiments, shear rigidity of the slitted walls is determined to have bi-linear hysteresis (See Fig.26).

Free Vibration: The natural periods and the vibration modes of this building are indicated in Fig.27 and Table 4.

Earthquake Response: The distribution curve of maximum story shear force due to earthquake waves with the maximum intensities of 0.1g and 0.2g are shown in Fig.28. Shear force and story drift in the torsional system are remarkably greater than in the restrained system from the 1st to 8th floors and 22nd to 23rd floors. Shear force of walled frames is shown in Figs.30 and 31. The walled frames are in the elastic range for earthquake intensities of 0.1g, and in the elastic-plastic range for intensities of 0.2g. WF-1 frames, located near the center of gravity, do not sustain torsional effects, while WF-2 frames, near the exterior end, do. Moreover, a closer examination of the latter reveals that, in the torsional system. WF-2(a) frames bear more shear force, and WF-2(b) frames less, than their counterparts in the restrained system.

SUMMARY

In this paper, we have not discussed general methods of torsional design; nevertheless, with the development of large electronic computer, it has become possible to calculate accurately the elastic-plastic response of actual buildings. Hence, as the four examples herein illustrate, it is essential to give due consideration to torsional vibration in addition to translational vibration; for, in the torsional system, the magnitude of the shear force and story drift in exterior columns clearly differs significantly from that of the restrained system.

In the process of design, engineers have not yet reached perfect understanding on this torsional problem. It may be, however, that existing the SEAOC Code and A.I.J. Standards still lack an adequate underpinning of research and experiments and that henceforth, by repeated studies of torsional effects and careful organization of data, it will soon become possible to establish a practical method of calculation which gives due consideration to torsional effects. We hope, therefore, that this paper can be of some use as a small step in this direction.

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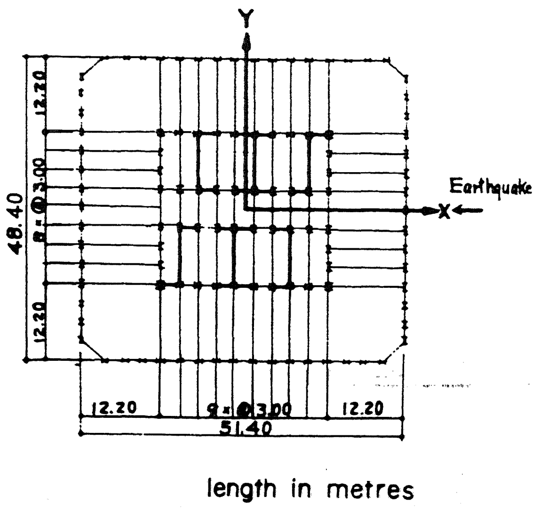


Fig. 3 Beam Plan of Building A

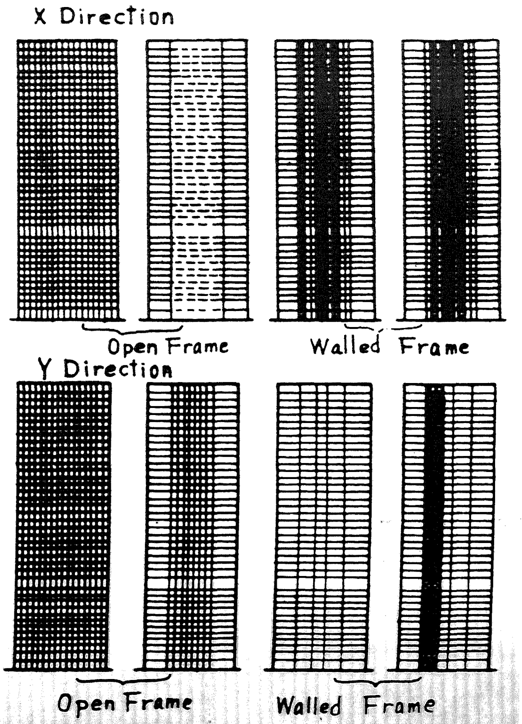


Fig. 4 Elevation of Each Frame

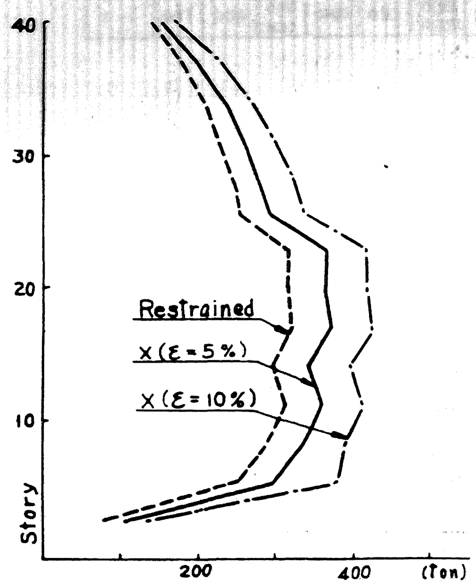


Fig. 5 Exterior Frame Shear by Design Force

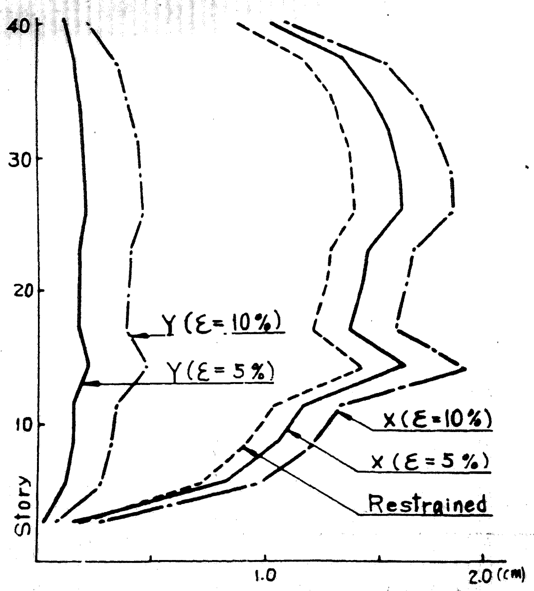


Fig. 6 Exterior Frame Drift by Design Force

Table 1. Natural Period and Participation Factor of Building A

	Coupled						Restraind				
	Eccentricity 5%			Eccentricity 10%				X		Y	
	Period	βx	βY	Period	βx	βY		Period	β	Period	β
1 st	4.482	0.424	-0.442	4.871	0.430	-0.452					
2 nd	4.161	0.743	0.700	4.161	0.745	0.700	1 st	4.165	1.407	4.159	1.403
3 rd	3.781	0.278	0.299	3.479	-0.272	0.291					
4 th	1.434	0.136	-0.129	1.561	-0.159	0.159					
5 th	1.290	0.327	0.288	1.290	0.315	0.290	2 nd	1.297	0.587	1.281	0.600
6 th	1.201	-0.167	0.212	1.103	-0.143	0.164					
7 th	0.772	-0.071	0.063	0.839	0.091	-0.090					
8 th	0.681	0.207	0.156	0.679	0.182	0.167	3 rd	0.687	0.327	0.667	0.340
9 th	0.636	-0.089	0.172	0.588	-0.041	0.078					

Table 3. Natural Period of Building C

Coupled		Restrained		
			X	Y
1 st 2 nd 3 rd	1.838 1.729 1.560	1 st	1.835	1.706
4 th 5 th 6 th	0.642 0.612 0.544	2 nd	0.640	0.603
7 th 8 th 9 th	0.397 0.377 0.334	3 rd	0.396	0.373

Table 2. Natural Period of Building B

	B - 0		B - 1			B - 2			B - 3		
			Coupled	Restrained		Coupled	Restrained		Coupled	Restrained	
	x	θ	x, θ	x	θ	x, θ	x	θ	x, θ	x	θ
1 st	1.93	1.63	1.87 1.47	1.86	1.73	1.83 1.33	1.73	1.41	1.81 1.15	1.58	1.27
2 nd	0.68	0.57	0.66 0.52	0.65	0.54	0.65 0.47	0.61	0.50	0.46 0.40	0.56	0.45
3 rd	0.41	0.35	0.40 0.31	0.40	0.33	0.39 0.28	0.37	0.30	0.38 0.27	0.34	0.27

Table 4. Natural Period of Building D

Coupled		Restrained	
1 st 2 nd	2.303 2.286	1 st	2.289
3 rd 4 th	0.760 0.724	2 nd	0.727
5 th 6 th	0.404 0.370	3 rd	0.373

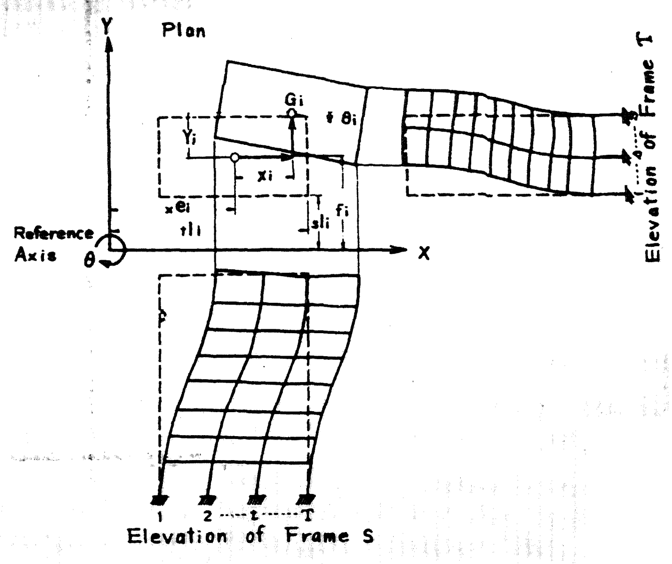
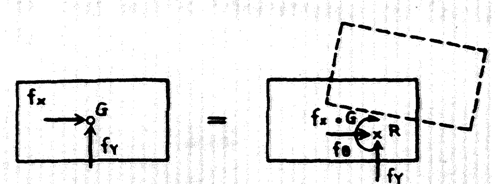


Fig. 1. Idealized Model of Structure



G: Center of Gravity
R: Center of Rigidity
fx, fY: Horizontal Force
fθ: Torsional Moment

Fig. 2 Statical Equilibrium State

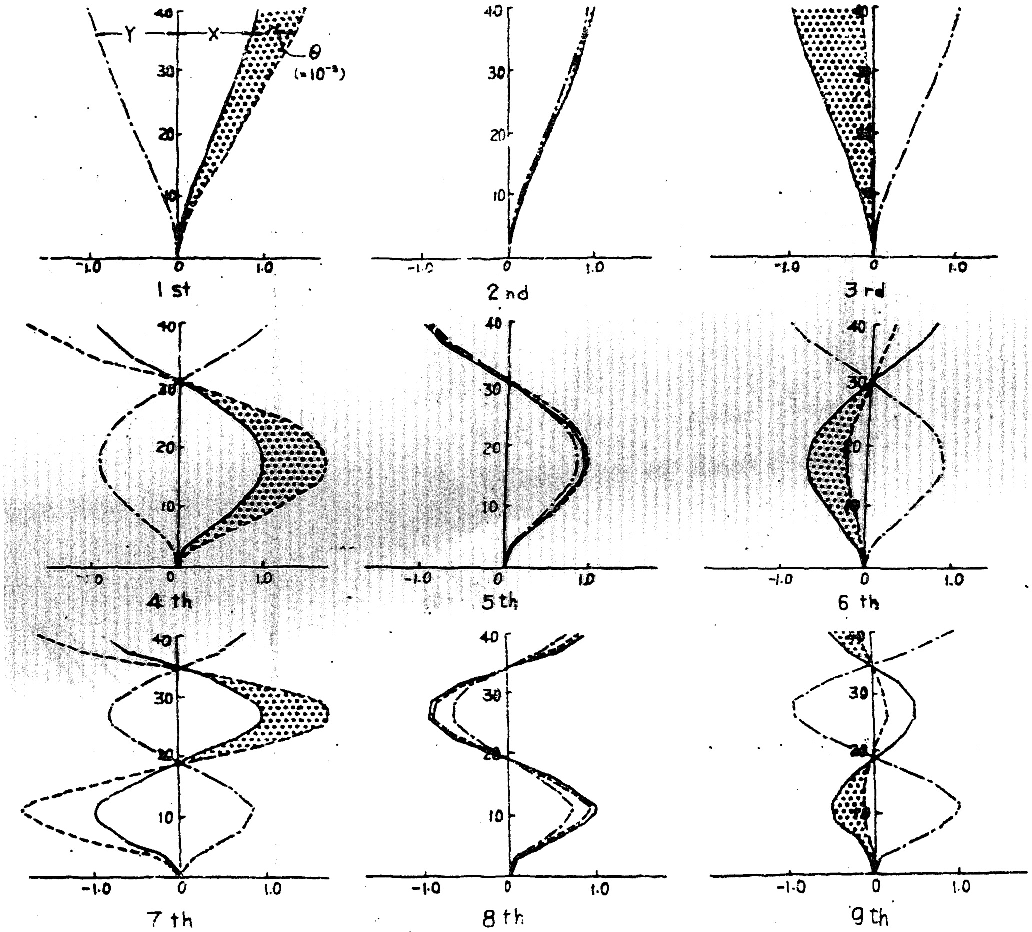


Fig. 7 Mode Shapes ($\mathcal{E}$ = 5 %)

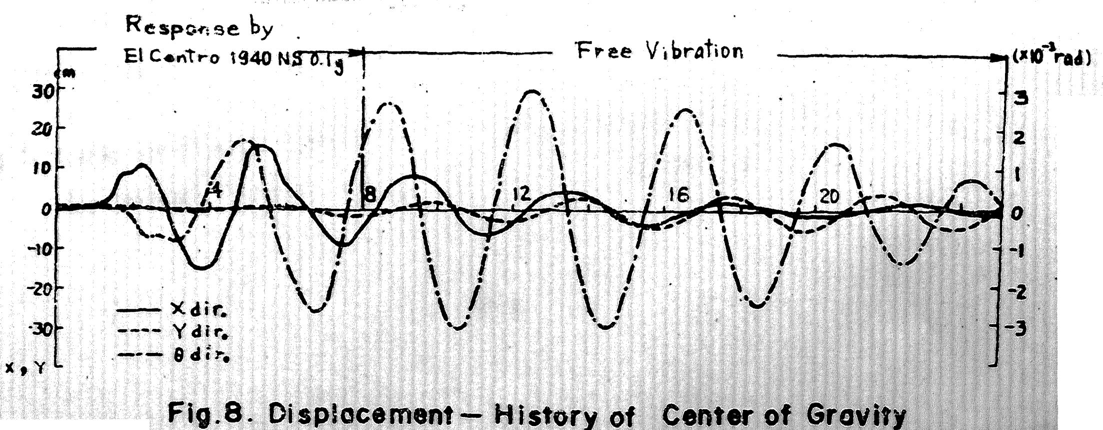


Fig. 8. Displacement — History of Center of Gravity at Top Floor ($\mathcal{E}$ = 5 %)

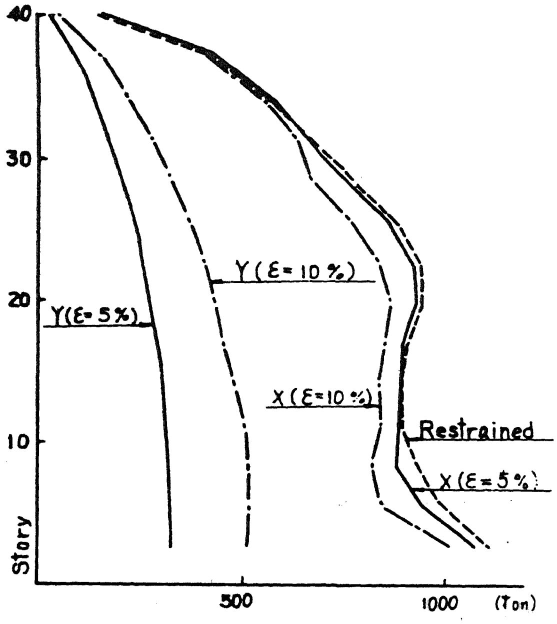


Fig.9 Max. Response Story Shear

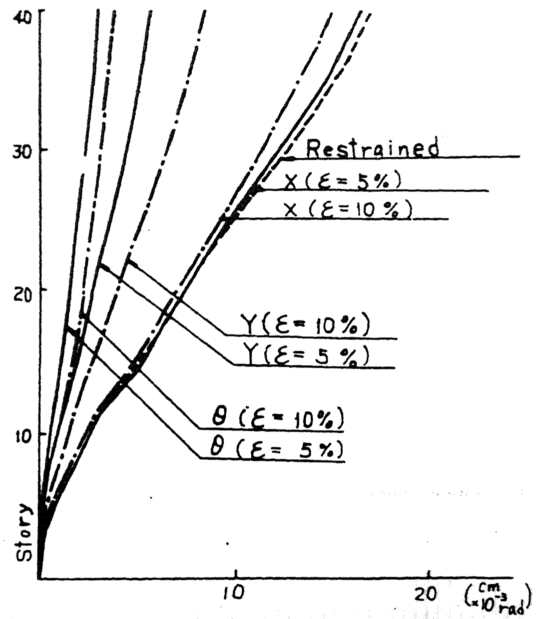


Fig.10 Max. Response Displacement

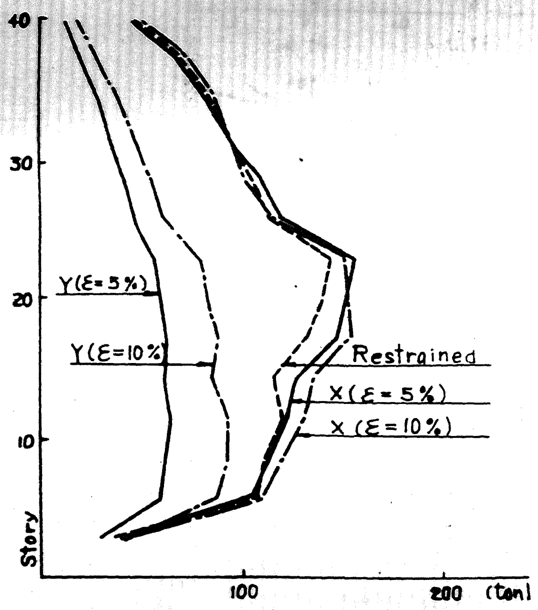


Fig.11 Max. Response Shear of Exterior Frame

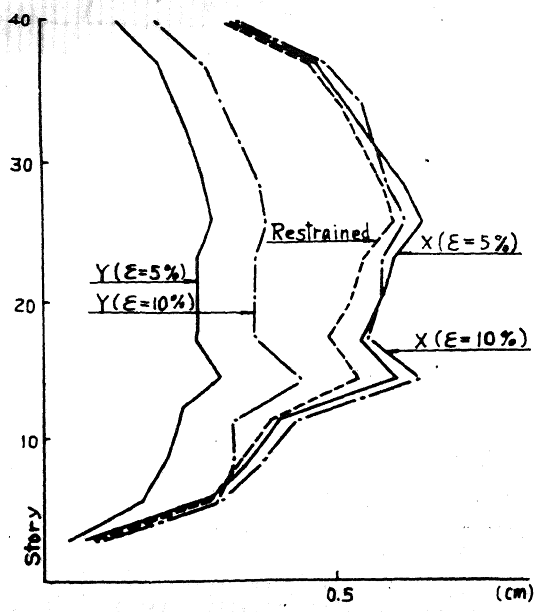
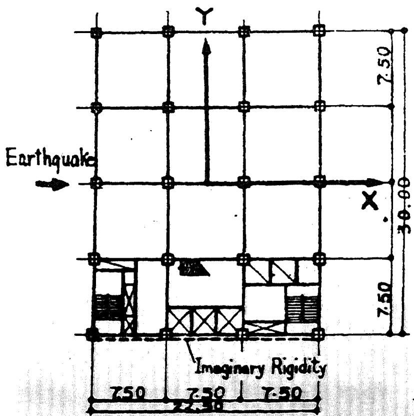


Fig.12 Max. Response Drift of Exterior Frame



length in metres

Fig.13 Beam Plan of Building B

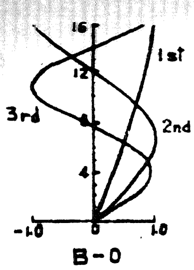


Fig.14. Mode Shapes of B-0

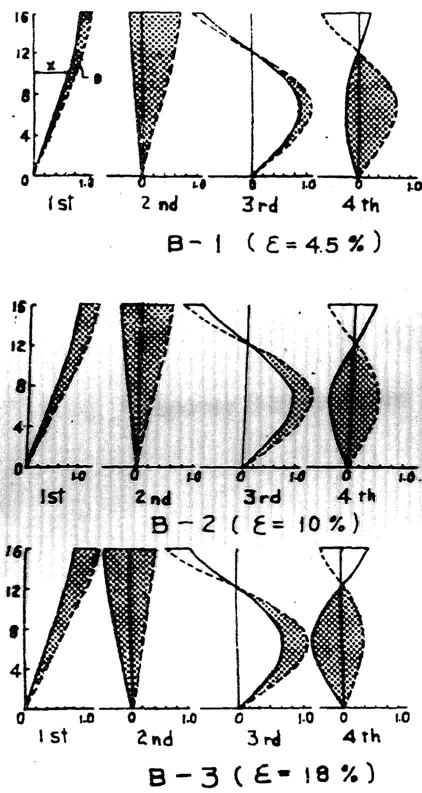


Fig.15 Mode Variations of Building B by Imaginary Rigidity

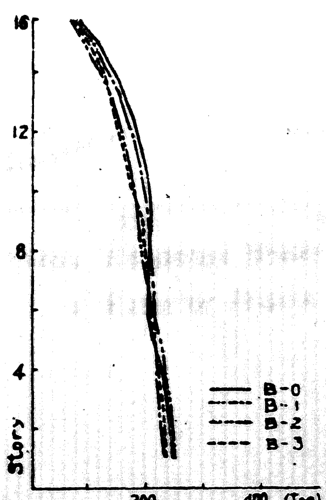


Fig. 16 Max. Story Shear

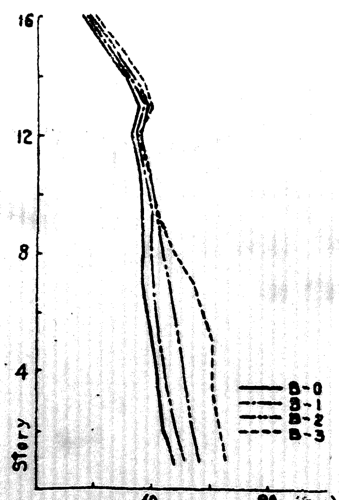


Fig.17 Max. Shear of Exterior Frame

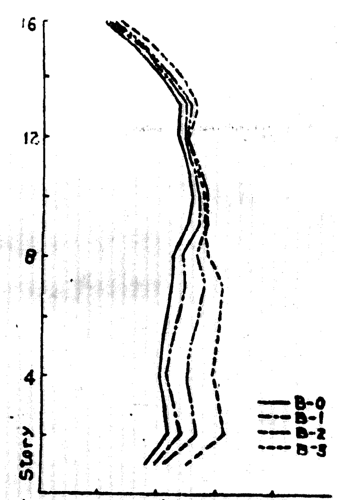


Fig.18 Max.Story Drift of Exterior Frame

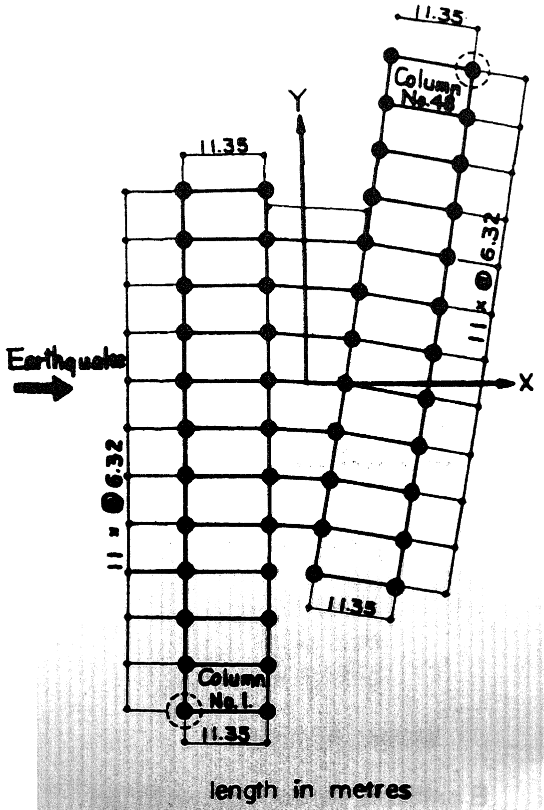
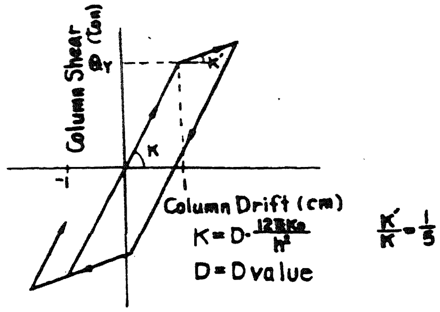


Fig.19 Beam Plan of Building C



$K = D \cdot \frac{12EK_0}{h^2}$ $\quad \frac{K'}{K} = \frac{1}{5}$

Fig.20 Assumed Restoring Force-Displacement Characteristic of Each Column

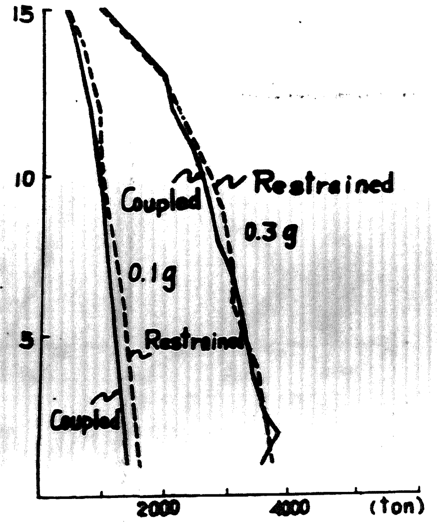


Fig.21 Max. Story Shear

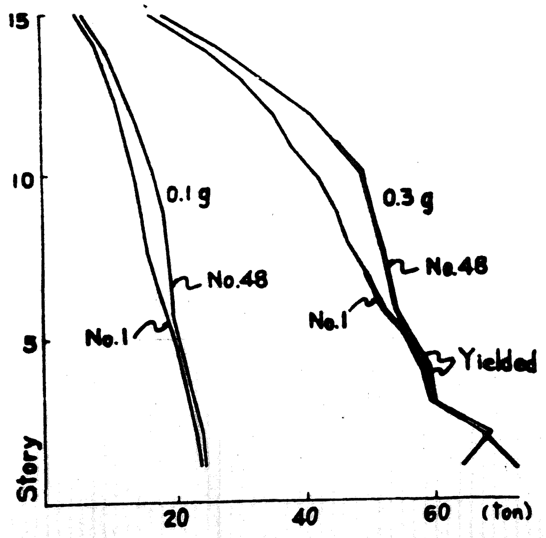


Fig.22 Max.Shear of Exterior Column

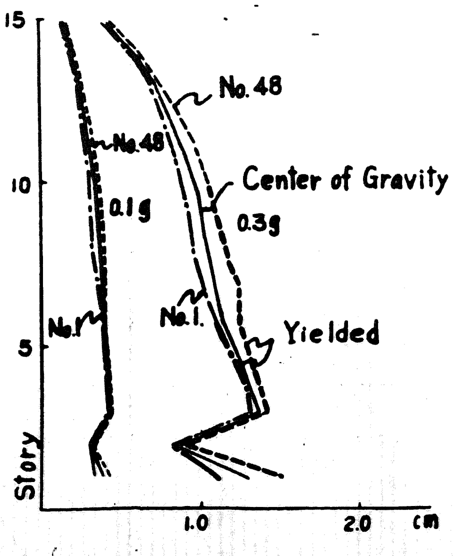


Fig.23 Max. Drift of Exterior Column

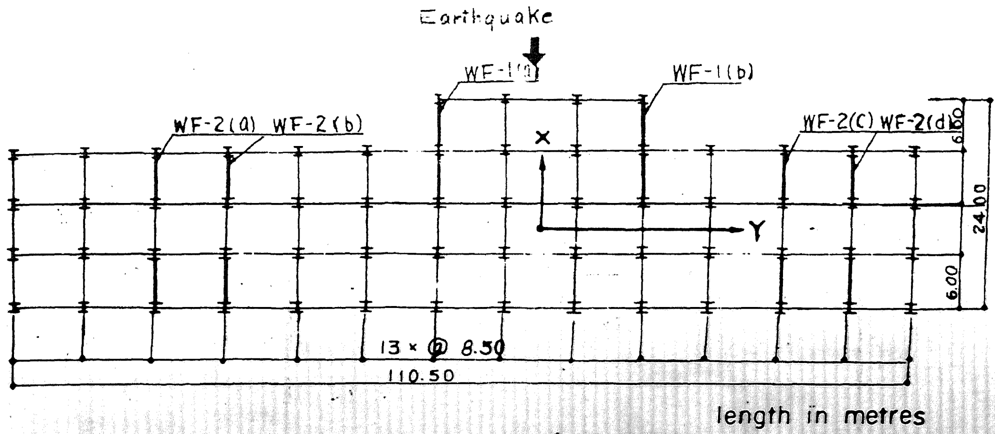


Fig. 24 Beam Plan of Building D

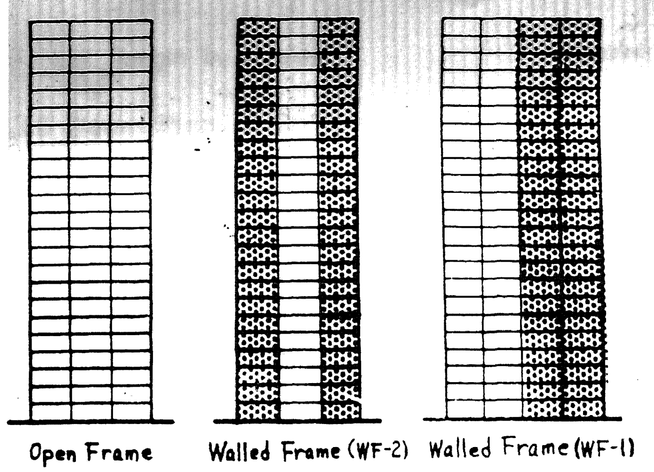


Fig. 25 Elevation of Each Frame

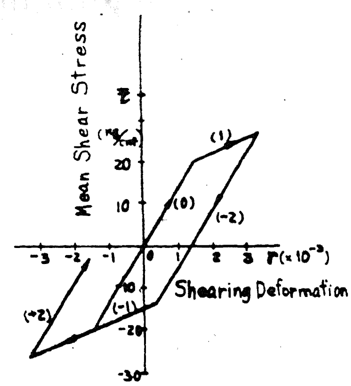


Fig. 26 Bi-linear Hysteresis of Slit Wall

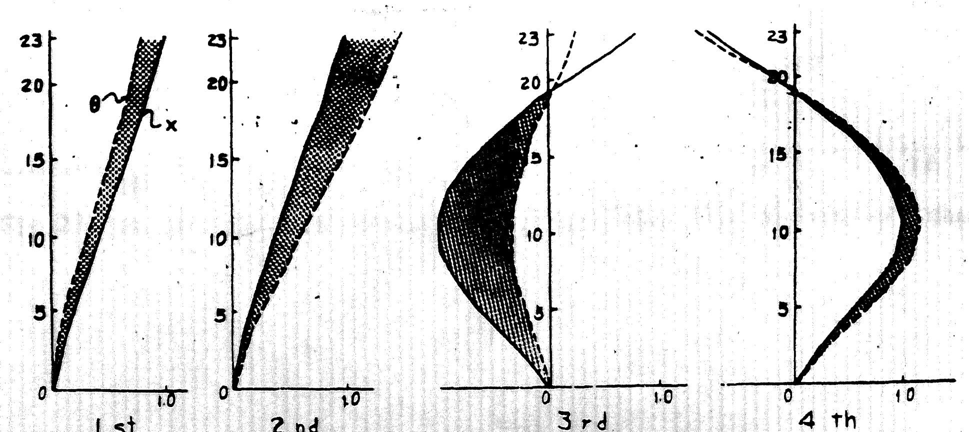


Fig. 27 Mode Shapes

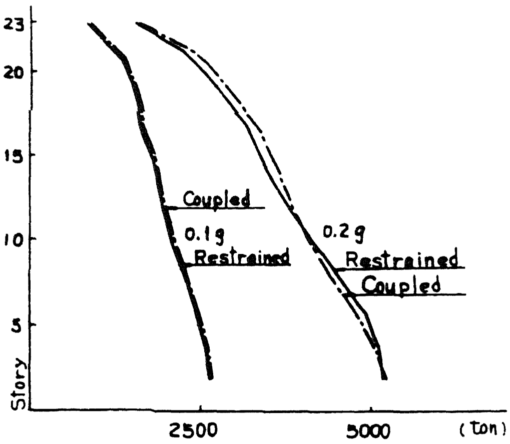


Fig. 28 Max. Response Shear

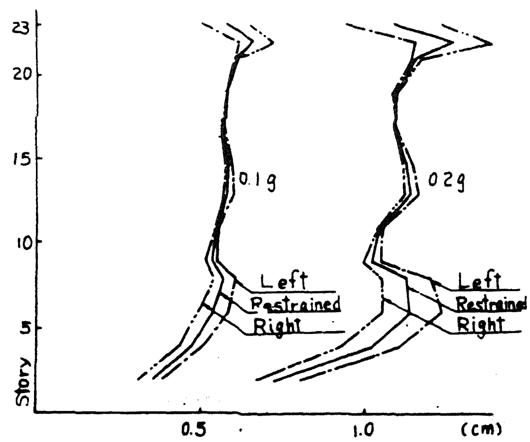


Fig. 29 Max. Drift of Exterior Frame

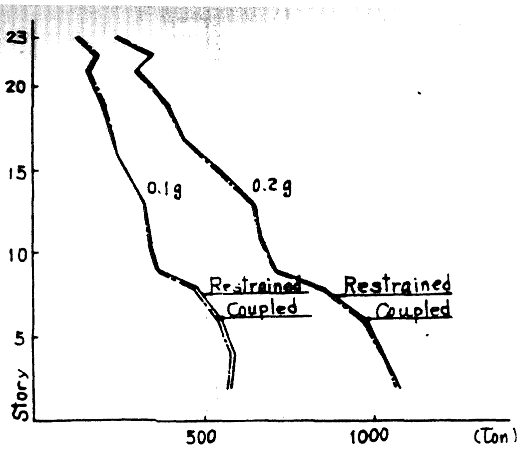


Fig. 30 Max. Shear of Walled Frame(WF-1)

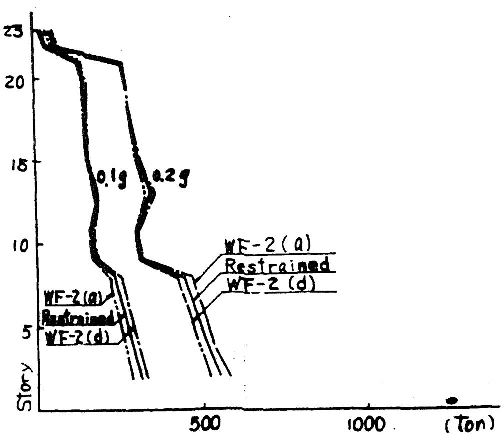


Fig. 31 Max. Shear of Walled Frame(WF-2)