THE ULTIMATE STRENGTH OF THE STEEL STRUCTURES SUBJECTED TO EARTHQUAKE

by

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1. SYNOPSIS

The ultimate state of steel frames when subjected to severe earthquake is investigated. This is done by equating the deformation capacity of a steel frame when subjected to horizontal forces to the total amount of the horizontal sway of the frame when subjected to earthquake which may be predicted by the response analysis. The deformation capacity of steel frames may be predicted reasonably by considering the strain hardening of the material. As a conclusion, it can be said that the ultimate strength of the steel frames subjected to earthquake is highly affected by the post yielding properties of the material.

2. INTRODUCTION

The designer should have due concept on the safety of the structure. In this point of view, it will be insufficient only to observe the ductility factor obtained from the dynamic response analysis in the performance of the design. The object of this paper is to estimate the ultimate strength of steel frames subjected to severe earthquake.

In this problem, as the seismic force does not act independently of the stiffness of the structure, reasonable estimation of the load-deflection characteristics of it over elastic and inelastic range is indispensable. The load-deformation path of the steel frame until its collapse state can be predicted by the analysis allowing for the strain hardening of the material, and as the result of this study, it is shown that the total amount of the inelastic deformation of a frame until its collapse has its proper value independently of its loading history.

The method of response analysis has been established and many worthy results were obtained(1), but these results have not yet been united with the ultimate strength concept.

The procedure described here to get a criterion on the collapse of the steel frame is to compare the total amount of the horizontal sway of the frame predicted by inelastic response analysis with the deformation capacity of it. When the magnitude of the deformation predicted by the response analysis attains the deformation capacity of the frame, the collapse may occur. In such a way, the ultimate strength of the lower parts of the multi-story frames will be sought after.

3. INELASTIC BEHAVIOUR OF THE FRAMED STRUCTURE

For the simplification of the analysis, the framed structure is translated to the one shown in Fig.1, namely the framed structure is thought to be the assembled body of the unit frames jointed together with pin-connecti at the corners. In elastic range, it can be shown that this procedure coincides with the "D-method"(approximate method to analyse the frame subjected to horizontal forces)(2), and also can be shown in inelastic range that the fundamental characteristics of the frame behaviour may not be lost

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and this will always give the safe side prediction[3]. Hence,it may be essential to study the inelastic behaviour of this unit frame,where the distributed loads on the beams may be negligible compared with the large compressive loads acting on the columns because the frame considering is the lower parts of the multi-story building.

3.1. BEHAVIOUR OF BEAM-COLUMNS

When the inelastic behaviour of the cantilever beam-column as shown in Fig.2 is known,that of the unit frame may be derived easily by the assembly of these cantilever beam-columns considering the compatibility at their ends.

It has been shown that the inelastic behaviour of such cantilever beam-columns under alternating bending could be well analysed by using the simplified model[4]. In this model the wide flange shape is replaced by the two parallel plates section neglecting the contribution of the web plate as shown in Fig.3,and the stress is assumed to be concentrated on the center of gravity of each flange. The results of the study performed on this model are summarized as follows,

1) The moment-curvature diagram under alternating load is obtained as shown in Fig.4(a). Comparing this with the diagram for monotonous loading [Fig.4(b)] ,it can be seen that the figure (ABC) + (CE) in Fig.4(a) is identical with the figure (ABCE) in Fig.4(b). That is,with regard to the bending moment in definite direction,the moment-curvature diagram under alternating load is identical with that of under monotonous loading.

2) The monotonous load-deflection curve of the member is readily obtained by the integration of the curvature due to the bending moment produced by the external loads as shown in Fig.5(a), where the broken lines show the exact solution obtained in ref.(4). It may be interesting to compare this with the curve obtained from the simple plastic theory ignoring the strain-hardening of the material as is shown in Fig.5(b).

When the bending stress is determined uniquely by the external load, correlation between the load-deflection curve under monotonous loading and that of under alternating load is similar to the correlation as was seen above in moment-curvature curves. That is,the load-deflection diagram under alternating load is obtained from that under monotonous loading as shown in Fig.6(a). With regard to the bending moment in definite direction,the load-deflection curve in Fig.6(a) has the same configuration as that of the monotonous loading [Fig.6(b)] .

In the discussion so far,the additional bending effect produced by the combination of the axial thrust and the residual plastic deformation of the member was not considered. When the shear force Q is removed after having produced some extent of inelastic deformation in one direction,the plastic deformation may reside as shown in Fig.7,and this causes the residual moment by the axial thrust P. To remove this residual moment at the fixed end,the additional end load $-P\delta/\ell$ must be applied reversely. Though the residual moment along the length of the member is not removed perfectly as shown in Fig.7,this error seems to be negligible.

Thus finaly the load-deflection curve under alternating load may be derived as shown in Fig.8,where the broken line which represents the effect of the residual moment is the basic line,and with regard to the load-deflection relation in one side of this line,the curve under alternating load has the same configuration as that under the monotonous loading. From above study,it can be seen that the summation of the plastic deformation in one direction until the collapse of the member does not exceed the plastic deformation capacity which the member can exhibit under the condition of monotonous loading.

To verify the theoretical prediction obtained above, a test was conducted using the test specimen as shown in Fig.9. In this specimen, the column is subjected to constant axial thrust and alternating end moment conducted through beams. The deflection of the middle height point of the column from a~a line is compared with the theoretical prediction in Fig.10. It can be seen that the test result agrees with the theoretical prediction fairly well, and the effect of the strain hardening upon the inelastic behaviour is remarkable.

3.2. RESTORING FORCE CHARACTERISTICS OF THE UNIT FRAME

The load-deflection curve of cantilever beam-columns is further simplified by the straight lines as shown in Fig.11, according to the objective of this paper to investigate the ultimate state of the structure. The ductility factor at the collapse state μ_u and the slope of the load-deflection curve in inelastic range are expressed by this simplification as follows,

$$\mu_u = 3\left(\frac{\alpha_B - 1}{2}\frac{D}{D_{st}} + \beta\right)\left(\frac{\alpha_B^2 - 1}{2\alpha_B^2}\right) \qquad (1)$$

$$K = \tan\theta_p = \frac{D_{st}}{\left(\frac{\alpha_B - 1}{2} + \frac{\beta D_{st}}{D}\right)\left(\frac{\alpha_B + 1}{2\alpha_B^2}\right)\ell^3} - \frac{2\alpha_B}{\alpha_B + 1}\frac{P}{\ell} \qquad (2)$$

where

$\alpha_B = M_B / M_P$

$\beta = \phi_{st}/\phi_y$

$M_B = Z_P \cdot \sigma_B$: the ultimate bending capacity under axial force P.

M_P :the fully plastic moment under axial thrust P.

ϕ_{st} :the curvature at the starting point of strain hardening.

$\phi_y = M_P / D$

$D = EI$:the flexural rigidity in the elastic range.

D_{st} :the flexural rigidity in the strain hardening range.

Z_P : plastic section modulus.

The inelastic behaviour of the unit frame is easily analysed using these results. The simple case, as shown in Fig.12 is treated for an example, where the moment of inertia of both beams is I_B, and that of columns is I_C. The ductility factor of the unit frame ${}_F\mu_u$ at the collapse state and the slope of the load-deflection curve in inelastic range ${}_FK$ are expressed as follows considering the compatibility as shown in Fig.12.

The case when beams only deform plastically:

$${}_F\mu_u = {}_B\mu_u \frac{1}{1 + k} \qquad (3)$$

$$_F K = \frac{D_{st}}{\left(\frac{\alpha_B - 1}{2} + \frac{\beta D_{st}}{D}\right)\left(\frac{\alpha_B + 1}{\alpha_B^2}\right)\ell^2 b} - \frac{P}{2\ell} \qquad (4)$$

The case when columns only deform plastically:

$$_F\mu_u = {}_c\mu_u \frac{k}{1 + k} \qquad (5)$$

$$_F K = \frac{D_{st}}{\left(\frac{\alpha_B - 1}{2} + \frac{\beta D_{st}}{D}\right)\left(\frac{\alpha_B + 1}{\alpha_B^2}\right)\ell^3} - \frac{\alpha_B}{\alpha_B + 1}\frac{P}{\ell} \qquad (6)$$

where $k = \frac{I_B \ell}{I_c b}$: relative stiffness of the beam to the column.

$_B\mu_u$: ductility factor at the collapse state when beams only deform plastically

$_c\mu_u$: ductility factor at the collapse state when columns only deform plastically

In above equations, α_B , β , D and D_{st} are the properties of members which deform plastically,and the term of the elastic deformation is neglected.

4.RESPONSE ANALYSIS OF THE FRAMED STRUCTURE

4.1. SUBSTITUTION METHOD FOR MANY MASS SYSTEM

To simplify the analysis,many mass system is used as a substitution for the actual complicated structure. This abstracted model is characterized by the distribution of masses and stiffness. To take up all combinations of these factors is extremely difficult. Hence in this paper further simplification as shown in Fig.13 is made by substituting the many mass system into 5 mass system in which the lower 4 masses remain unchanged and the upper parts are concentrated into 1 mass at the center of gravity of the upper masses,and the elastic spring constants of the original system which distribute between 4th and this fictitious mass are replaced by the mean value of them.

Using this model,the response of the lower parts of the frame may be easily analysed. With regard to the original many mass system,the next assumptions are made.

(1) Each mass is identical.

(2) The yield shear coefficient f of the lower parts is constant.
where, $f = F_{yi} / M_i g$, g :acceleration of gravity
F_{yi} :yield shearing force of i-th story
M_i :total mass above i-th mass

(3) Elastic spring constant varies linearly along the height as shown in Fig.14,where the ratio of spring constants at the bottom to that at the top is $\mathcal{H}$.

(4) Each story height is identical.

Denoting natural period of the original system by T,the spring constant at the top K_o is obtained approximately as,

$$K_0 = \frac{8\pi^2 m\, n(n+1)}{\left(\frac{3n-1}{n}\varkappa + \frac{n+1}{n}\right) T^2} \tag{7}$$

Then the spring constants of the simplified model become as following,

$$K_i = K_0 + (\varkappa - 1)\left(\frac{n-i}{n}\right) K_0 \quad , \quad (i = 1,2,3,4)$$

$$K_5 = \frac{K_0}{2}\left(\frac{3n'-1}{2n'}\varkappa' + \frac{n'+1}{2n'}\right) / \frac{n'+1}{2} \tag{8}$$

$$\varkappa' = 1 + (\varkappa - 1)\frac{n-4}{n} \quad , \quad n' = n-4$$

where, m: mass
n: number of masses

Fundamental vibration equation becomes,

$$-m(n-4)(\ddot{y}_i + \ddot{y}_0) - F_5 = 0$$

$$-m(\ddot{y}_i + \ddot{y}_0) - F_i + F_{i+1} = 0, \quad (i = 1,2,3,4) \tag{9}$$

where, $F_i = K_i(y_i - y_{i-1})$, in elastic range
y_o : ground motion
y_i : deflection of i-th mass relative to the ground
$(\ddot{\ }) = d^2(\ )/dt^2$

The adequacy of such substitution is examined. Comparison is made between the response of 5 mass system and that of 2 mass system which is substituted by the method mentioned above. The response of the deflection of the 1st mass relative to the ground is observed. Yield shear coefficient of the original 5 mass system is given in Fig.15 and the other parameters are given in Table 1. Results of the analysis are given in Fig.16 in which abscissa μ means the ductility factor(the maximum deflection divided by the initial elastic deflection limit). From this figure it can be seen that the substitution method enables us to know the outline of the response.

In above and the following studies, the record of the ground motion of NS component of the EL CENTRO earthquake, May 1940 is used.

4.2. RESPONSE OF THE MANY MASS SYSTEM

Using these models, the response of the lower parts of the structures is obtained. Parameters are T,n, $\varkappa$, f and η which represents the slope of the inelastic range of the restoring force characteristics as shown in Fig.17. Of these parameters, difference of $\varkappa$ has few effect upon the response, and is fixed arbitrary to be 5.0. The effect of η was observed using the parameters shown in Table 2. The results presented in Fig.18 show that the inelastic deformation increases as η decreases, and when η becomes negative, inelastic deformation grows too large to be considered as practical. At present study, the value of η is assumed to be zero so that the upper limit of the inelastic deformation of structures, η of which be non-negative, may be calculated.

Thus finally, the analysis is made within the range of T and n given in Table 3. The results are shown in Fig.19. As was mentioned in section 3, the collapse of the structure occurs when the summation of the inelastic deformation in one direction reaches the critical value, so such an accumulated inelastic deformation is calculated for each of lower four stories, and the maximum value of them are showed in this figure.

5. THE ULTIMATE STRENGTH OF THE FRAMED STRUCTURE

To know the collapse state of the frame, the relation between yield shear coefficient f and the maximum ductility factor μ_u which the frame can develop must be obtained. When μ in Fig.19 agrees with μ_u, the collapse occurs by the earthquake.

The frames dealt in this study are composed of unit frames having the following properties,

(1) Beams are elastic and columns only deform plastically.

(2) The relative stiffness of the beam to the column is equal to unity (k = 1).

(3) The value of Y comprises three varieties as $Y = \sigma_Y/\sigma_B = 0.64$, 0.80 and 0.91. Where Y is the yield ratio of the material, σ_y is the yield point and σ_B is the maximum strength of the material.

(4) Axial force is below 0.7 P_y , P_y is the yield axial force of the column.

(5) Slenderness ratio of the column comprises three varieties as $\lambda = 20$, 25 and 30.

It is convenient to express the relation between the yield shear force and the maximum deformation capacity of the structure in the form of $f \sim \mu_u$ relation.

From the equilibrium of the unit frame shown in Fig.12,

$$2M_p = F_y \ell = \frac{F_y}{P} P \ell = f P \ell$$

where,

F_y : yield shearing force of the unit frame.

Then, yield shear coefficient is expressed as,

$$f = \frac{2M_P}{P\ell} = \frac{a\ M_P}{P\lambda} \qquad (10)$$

where,

a : coefficient proper to the sectional shape.

λ : slenderness ratio of the column.

From eqs.(5),(6) and (10), $f \sim \mu_u$ relation is obtained as shown in Fig.20. The collapse state is expressed by the intersecting points of the corresponding curves of Fig.19 and Fig.20, and the results thus obtained are shown in Fig.21. In these figures, f is interpreted as the minimum yield shear coefficient required to prevent the failure of the frame.

From this figure next features are pointed out,

(1) Y has large effect on the ultimate strength of the frame. For the frame composed of steel which has higher value of Y, design load for seismic force must be larger.

(2) Design load may be reduced for the frame which has long natural period.

(3) Slenderness ratio of the column has large influence on the strength of the frame.

6.CONCLUSION

A procedure to determine the ultimate strength of steel frames subjected to severe earthquake was presented,and factors which will affect the ultimate strength of the frame was pointed out.

Though it is necessary to investigate the ground motions as well as frame types more widely to obtain the generalized conclusion,the present objective of this study is to emphasize the necessity of establishing the concept on the safety in the seismic-resistant design and to point out the important factors which affect the ultimate strength of steel frames.

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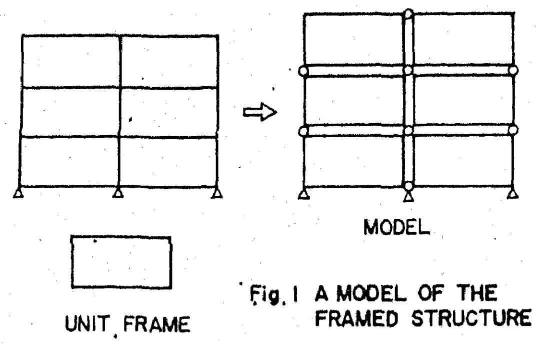


Fig. 1 A MODEL OF THE FRAMED STRUCTURE

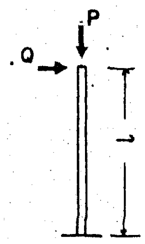


Fig. 2 BEAM-COLUMN

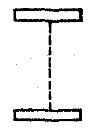


Fig. 3 MODEL OF THE SECTION

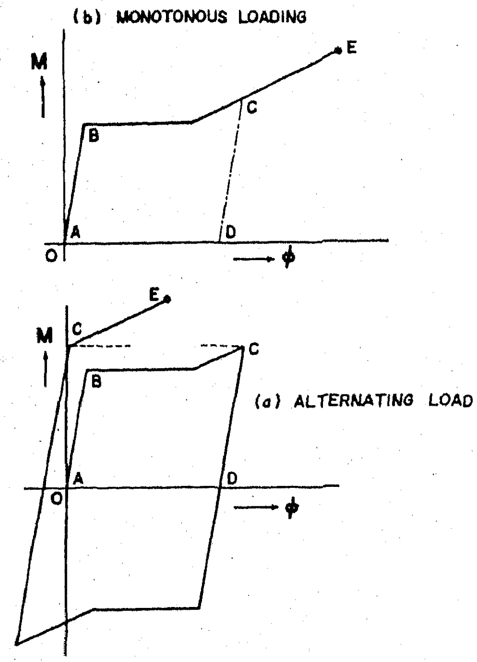


Fig. 4 MOMENT-CURVATURE DIAGRAM

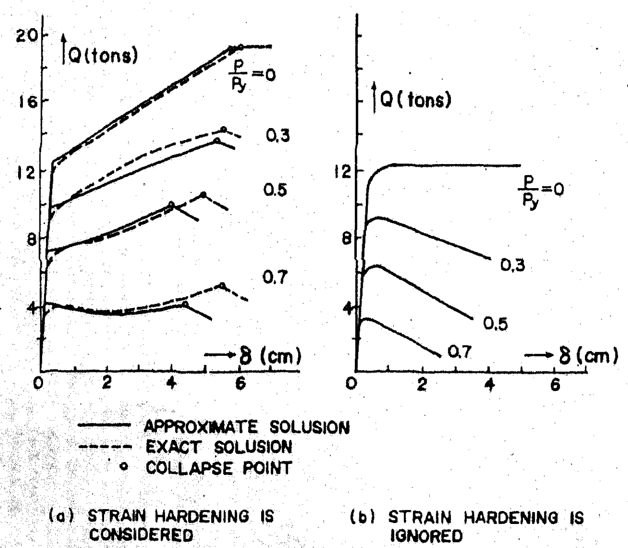


Fig. 5 THEORETICAL PREDICTION OF LOAD-DEFLECTION RELATION, $l = 75$cm $\lambda = 22$

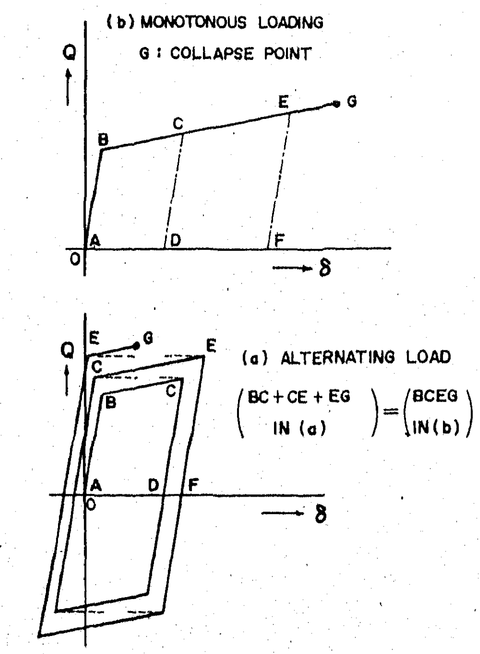


Fig. 6 LOAD-DEFLECTION DIAGRAM

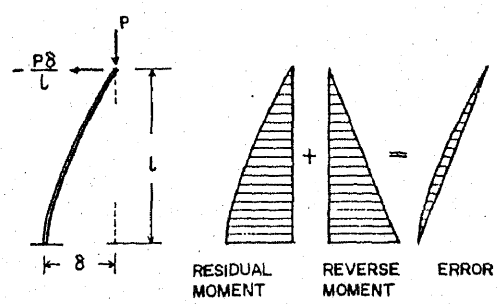


Fig.7 RESIDUAL MOMENT DUE TO PLASTIC DEFORMATION

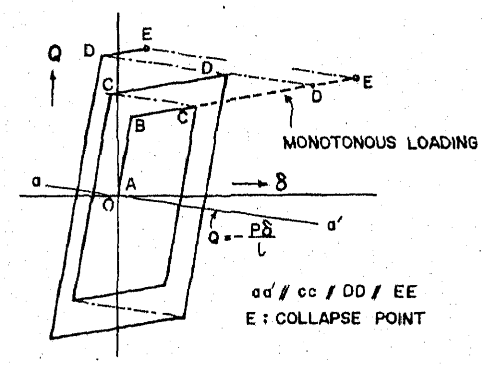


Fig.8 Q-δ DIAGRAM CONSIDERING SECONDARY BENDING EFFECT

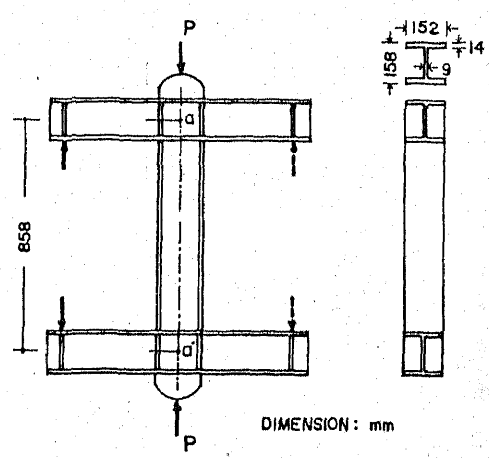


Fig.9 TEST SPECIMEN

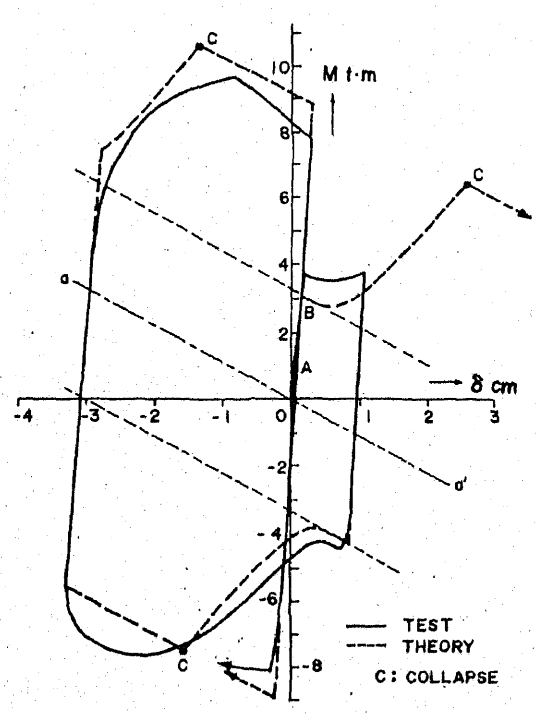


Fig.10 COMPARISON OF THE M-δ CURVE

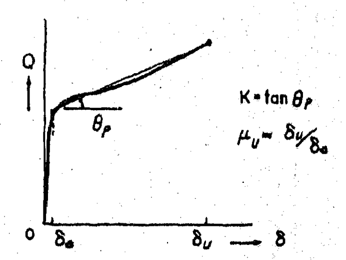


Fig.11 SIMPLIFIED Q-δ CURVE

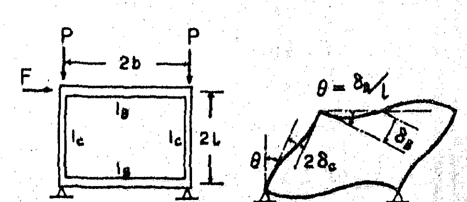
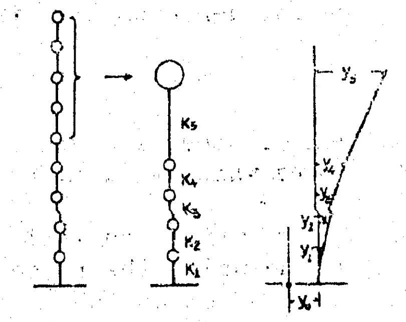


Fig.12 UNIT FRAME AND ITS DEFORMATION



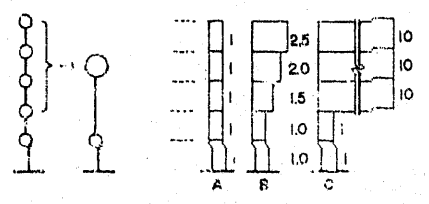
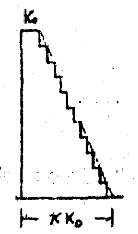


Fig. 13 SUBSTITUTION OF MANY MASS SYSTEM

Fig. 15 DISTRIBUTION OF YIELD SHEAR COEFF. (f_i/f_1)



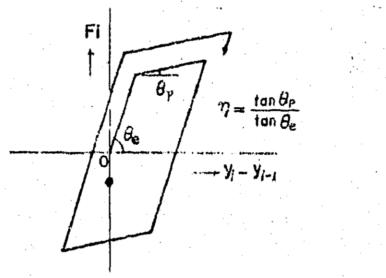


Fig. 14 DISTRIBUTION OF SPRING CONSTANT

Fig. 17 RESTORING FORCE CHARACTERISTICS

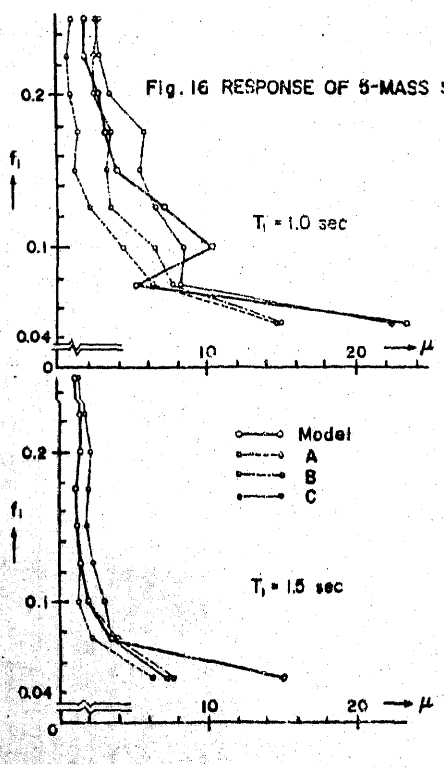


Fig. 16 RESPONSE OF 5-MASS SYSTEM

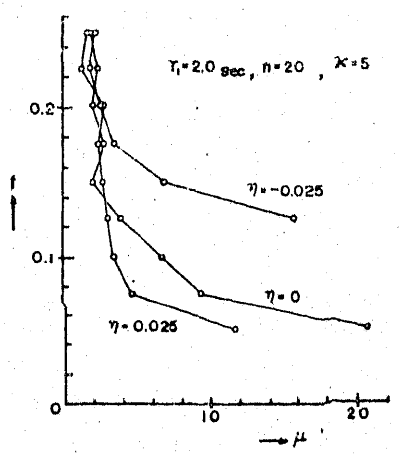


Fig. 18 EFFECT OF η

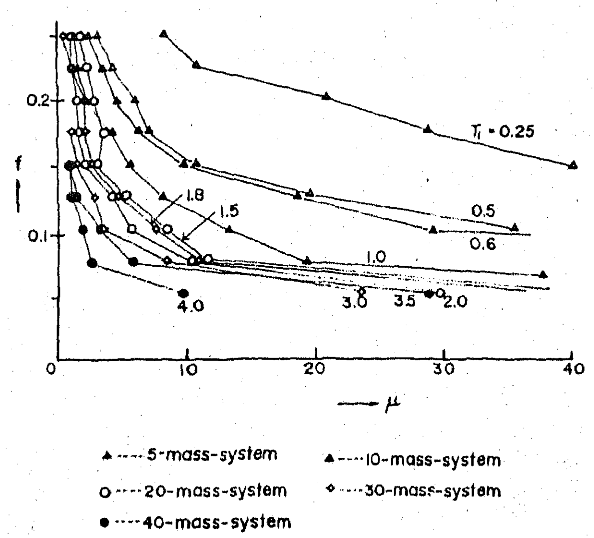


Fig.19 RESPONSE OF MANY-MASS SYSTEM

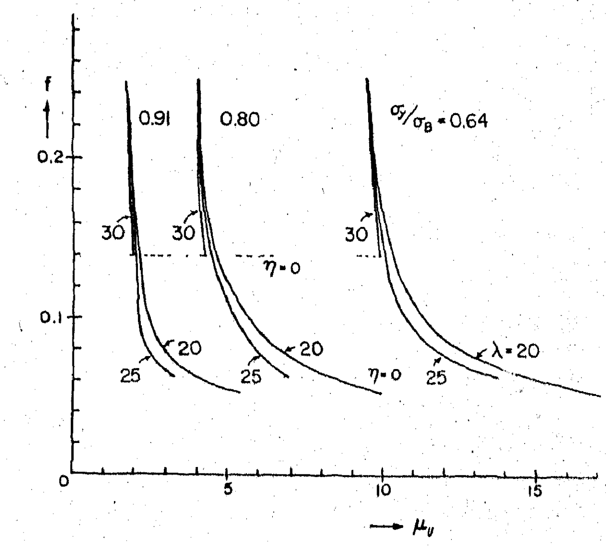


Fig. 20 F-μ_u RELATION OF THE FRAME

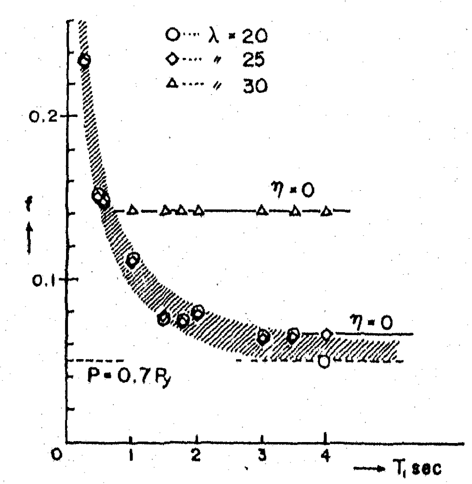


Fig.21 (a) Y = σ_y/σ_B = 0.64

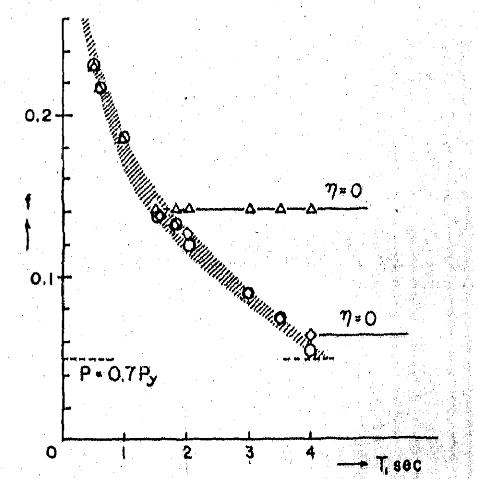
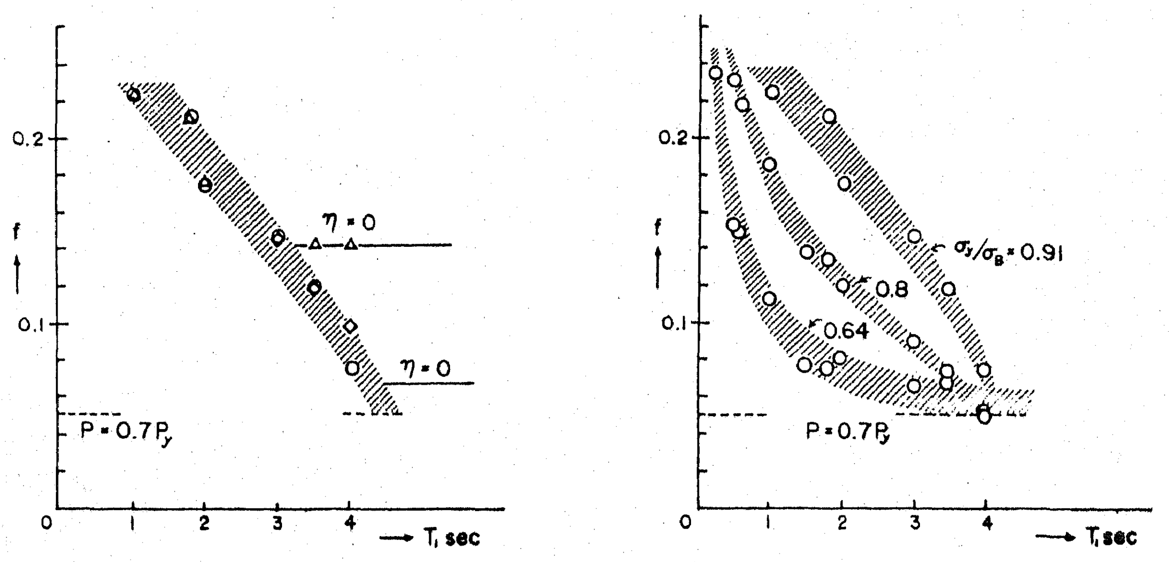


Fig.21 (b) Y = σ_y/σ_B = 0.80



(c) $Y = \sigma_y/\sigma_B = 0.91$ (d) $\lambda = 20$, $Y = 0.64, 0.80, 0.91$

Fig. 21 Fig. 21 f–T_1 RELATION AT ULTIMATE STATE

κ	T_1, sec	f_1	η
5.0	1.0, 1.5	0.05 ~ 0.25	0

Table 1

κ	T_1, sec	n	f	η
5.0	2.0	20	0.05~0.25	0.025~0.025

Table 2

n	5	10	20	30	40
T_1, sec	0.25 0.5	0.6 1.4	1.5 2.0	1.8 3.0	3.5 4.0

Table 3