

THE DISTRIBUTED-ELEMENT CONCEPT OF HYSTERETIC MODELING AND ITS APPLICATION TO TRANSIENT RESPONSE PROBLEMS

by

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ABSTRACT

This paper deals with a class of models which may be used to study the dynamic behavior of yielding or hysteretic structures. The models are based on the idea of representing the structure as a collection of linear and slip elements. Any continuous, monotonic and finite initial loading relation with negative curvature may be reproduced by the models and transient loading behavior is dictated solely by the initial loading curve and the physics of the model. The response of a rather general yielding system to eight artificial earthquakes is examined and the effects of hysteresis loop shape are discussed.

INTRODUCTION

Perhaps the most widely used model for hysteresis is the so-called bilinear hysteretic model in which the force-deflection relation is thought of as consisting of straight line segments with two possible slopes. This model provides a relatively simple framework within which both periodic and transient phenomenon can be studied. However, it has at least one drawback in that most real systems do not behave in such an ideal manner. In an attempt to overcome this difficulty, several mathematical models giving curved hysteresis loops have been proposed. Such models generally seek to relate the force-deflection relations of all subsequent loading paths to the initial loading behavior by means of mathematical relations of a given form. These models are quite acceptable when used in periodic response problems, but their application is not so clear cut in transient problems where it is necessary to make certain additional mathematical assumptions about the nature of succeeding noncyclic loading paths. In addition, it is sometimes difficult to match the assumed mathematical form to a given set of force-deflection data.

The present paper deals with a class of models which are based on the idea of representing the structure as a continuously distributed collection of linear elastic and ideal slip elements. As such, the models are capable of reproducing any arbitrary initial loading relation with negative curvature. Furthermore, the behavior of the models for an arbitrary loading history is completely specified by the physics of the models without making any additional assumptions. This makes the models particularly well suited to transient response studies. The relation of the distributed element models to other curved hysteresis models such as those of Jennings and Pisarenko is discussed. Also discussed is the manner in which actual data on initial loading behavior or energy loss per cycle may

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be used to specify the model parameters.

As an example of the application of the present class of models to transient response calculations, the response of a rather general yielding system to eight artificial earthquakes is studied. The effects of the degree of curvature of the hysteresis loop and the ratio of excitation level to yield level on the overall response spectrum of the system are discussed. These results are compared with those of a statistical electric analog study and another investigation using a different hysteretic model.

THE MODELS

The class of models which will be considered here consists of a collection of perfectly elastic and rigid plastic or slip elements arranged so that they give force-displacement relations which are hysteretic in nature. Two particular configurations will be discussed; a parallel-series configuration in which the slip elements are in series with the elastic elements and a series-parallel configuration in which the slip elements are in parallel with the elastic elements. It is assumed that the number of elastic and slip elements in a given configuration is large and therefore that the overall properties of the model depend only upon the distribution of the properties of the elements and not upon the individual elements themselves. In a complex structure, this is equivalent to stating that the gross force-displacement behavior is the result of the combined action of a great many individual components, some of which act elastically and some of which act plastically at any given point in the loading history. The applicability of the present class of models will obviously depend on the accuracy of this description for the structure in question.

Parallel-Series Configuration. The parallel-series model consists of linear elastic and slip elements arranged as indicated in Fig. 1. The modulus of the i -th linear elastic element is k_i , the critical slipping force of the i -th slip element is f_i^* , and the number of elastic-slip element combination is N . It will be assumed that the system is initially in an unstressed state. The details of obtaining the initial loading force-displacement relation have been worked out elsewhere (1) so only a summary will be given here. If the model undergoes an extension x , the restoring force will clearly result from two groups of elements; those behaving elastically, and those which have been stretched beyond their slip limit. Hence, the initial loading characteristic may be represented as

$$f = \sum_{i=1}^n k_i x / N + \sum_{i=n+1}^N f_i^* / N \quad (1)$$

where the summation from 1 to n includes all of those elements which remain elastic and the summation from $n+1$ to N includes those elements which have slipped or yielded. In general, either the k_i or the f_i^* or both could be taken as distributed parameters. However, for brevity, we will consider only the special case where the f_i^* are distributed and $k_i = k$, a constant. For N large let us define a distribution function

$\varphi(f^*)$ so that $\varphi(f^*)df^*$ is the fraction of the total number of slip elements having a slip force between f^* and $f^* + df^*$. Then, equation (1) becomes

$$f = kx \int_{kx}^{\infty} \varphi(f^*) df^* + \int_0^{kx} f^* \varphi(f^*) df^* ; \quad \dot{x} > 0 \quad (2)$$

Note that the slope of the force-displacement relation approaches k as $x \rightarrow 0$. Furthermore, if the function $\varphi(f^*)$ is sufficiently smooth as $f^* \rightarrow \infty$, it is seen that the model will have an "ultimate yield" force given by

$$f_y = \int_0^{\infty} f^* \varphi(f^*) df^* \quad (3)$$

Hence, this configuration leads to a restoring force which is bounded for all x . This provides a meaningful reference parameter for use in comparing the behavior of related systems.

Series-Parallel Configuration. Another possible configuration for the distributed-element model is shown in Fig. 2. In this case, there is one elastic element with modulus k and a series of elastic-slip element combinations having elastic moduli kN/β and slip forces f_i^* where β is an arbitrary constant. If the system is initially stress free and then loaded by an impressed force f , the displacement of the i -th elastic-slip element combination will be

$$x_i = 0 ; \quad 0 \leq f \leq f^* \\ = \frac{\beta(f - f^*)}{kN} ; \quad f \geq f^* \quad (4)$$

Hence, the total displacement on initial loading will be

$$x = f/k + \sum_{i=1}^N x_i \quad (5)$$

Let the total number of elements become large, and define a new distribution function $\gamma(f^*)$ such that $\gamma(f^*)df^*$ is the fraction of slip elements having a yield force between f^* and $f^* + df^*$. Then, from equations (4) and (5)

$$x = \frac{1}{k} \left[f + \beta \int_0^f (f - f^*) \gamma(f^*) df^* \right] ; \quad \dot{x} > 0 \quad (6)$$

For $x \rightarrow 0$, the slope of the force-displacement relation is again k as in the case of the parallel-series configuration. However, unlike the former configuration, the present representation is unbounded as $x \rightarrow \infty$. The limiting slope of the initial loading curve as $x \rightarrow \infty$ can be shown to be $k/(1+\beta)$ provided $\gamma(f^*)$ is sufficiently smooth.

STEADY-STATE CYCLIC LOADING

If the models discussed above are subjected to a steady-state cyclic loading, the resulting force-displacement diagrams will be hysteresis loops as indicated in Fig. 3. The equations for the loop curves may be obtained in a straightforward manner by noting the reversed loading behavior of the various categories of initially loaded elements. Consider, for example, the case of the parallel-series configuration. Referring to Fig. 3, let the system be initially loaded along some curve ab to a displacement x_m and then reverse loaded along a path bcd . The restoring force along the reversed loading path will be due to three different groups of elements; those elements which were in a positive yield state after initial loading and have changed to a negative yield state under reverse loading, those elements which were in a positive yield state after initial loading and have not changed to a negative yield state, and those elements which were not yielded on initial loading. The force-displacement relation along the path bcd may therefore be written as

$$f = - \int_0^{k(x_m - x)/2} f^* \varphi(f^*) df^* + \int_{k(x_m - x)/2}^{kx_m} (kx - kx_m + f^*) \varphi(f^*) df^* + kx \int_{kx_m}^{\infty} \varphi(f^*) df^* ; \quad \dot{x} < 0 \quad (7)$$

The series-parallel configuration can be treated in exactly the same way and the equation for the lower-most curve of the hysteresis loop becomes

$$x = \frac{f}{k} + \frac{\beta}{k} \int_0^{(f_m - f)/2} (f + f^*) \gamma(f^*) df^* + \frac{\beta}{k} \int_{(f_m - f)/2}^{f_m} (f_m - f^*) \gamma(f^*) df^* ; \quad \dot{x} < 0 \quad (8)$$

It should be emphasized that the loop equations follow directly from the basic assumptions used to obtain the initial loading behavior. No new assumptions were introduced. This is not the case for most of the purely empirical hysteresis formulations where it is necessary to make certain additional assumptions to get from the initial loading behavior to the hysteresis loop behavior. The usual assumption made is that the system obeys Massing's hypothesis which states that the loop curves are of the same form as the initial loading curve except for an enlargement by a factor of two (2). This hypothesis is rather well supported by tests of real systems having the general type of hysteretic behavior indicated in Fig. 3. It is therefore desirable that any new models at least approximate this behavior. It may easily be shown from equations (2) and (6)-(8) that the present class of models actually satisfies this hypothesis exactly.

TRANSIENT CYCLIC LOADING

One of the greatest advantages of the present distributed-element formulation is its ready adaptability to transient loading problems. As in the case of steady-state loading, no additional assumptions need be made. The physics of the model itself dictates the history dependence of the loading behavior. Consider the case shown in Fig. 4. The system is initially loaded along ab, reverse loaded to point c, again reversed to point d, etc. The equation for the force-displacement relation along paths ab and bc have already been given for the case $|x_c| \leq |x_b|$. The relation for the path cd may be found in a manner similar to that used to obtain equations (7) and (8). The fraction of elements in each yield state is known at c so one need only determine what happens to each group when the direction of loading is reversed. For the parallel-series configuration this will give the loading curve cd as

$$f = \int_0^{k(x-x_c)/2} f^* \varphi(f^*) df^* + \int_{k(x-x_c)/2}^{k(x_b-x_c)/2} (kx - kx_c - f^*) \varphi(f^*) df^* + \int_{k(x_b-x_c)/2}^{kx_b} (kx - kx_b + f^*) \varphi(f^*) df^* + kx \int_{kx_b}^{\infty} \varphi(f^*) df^* \quad (9)$$

If $x > x_b$ the system will again follow the initial loading curve equation (2). For the series-parallel configuration the corresponding relation will be

$$x = \frac{f}{k} + \frac{f}{k} \left[\int_{(f_b-f_c)/2}^{f_b} (f_b - f^*) \gamma(f^*) df^* + \int_{(f-f_c)/2}^{(f_b-f_c)/2} (f_c + f^*) \gamma(f^*) df^* + \int_0^{(f-f_c)/2} (f - f^*) \gamma(f^*) df^* \right] \quad (10)$$

It is apparent that this procedure could be carried on indefinitely merely by keeping track of the fraction of elements in each yielding category after each reversal in the direction of loading. In practice, the problem is actually simplified by the fact that a portion of the loading history can be ignored or "erased" every time the displacement exceeds a previous extremum which was established for loading in the same direction. Hence, referring to Fig. 4, if the system moves along the curve dec and passes to the left of point c, the actual path around the loop cdec no longer enters into the determination of the load or displacement. The force-displacement relation will be given by equation (7) or (8) and depends only on the extremum $x_b = x_m$.

PARAMETER SELECTION

One of the essential features of any useful model is the ability to select the parameters of the model in a manner that is both straightforward and meaningful. This selection is usually based on a least square error or similar fitting of the initial loading curves for the model and the system under study. Such a fitting procedure may or may not determine the model parameters uniquely depending on the nature of the model. For the present class of models, parameter selection is particularly well defined. Both of the configurations considered are capable of reproducing exactly any arbitrary, monotonic, finite, and continuous initial loading curve with negative curvature. In addition, the two major parameters may be determined uniquely. Consider the parallel-series configuration. The parameter k may be determined directly from the small amplitude slope asymptote and it may be shown from equation (2) that $\varphi(kx)$ is given by the formula

$$\varphi(kx) = -\frac{1}{k^2} \frac{d^2f}{dx^2} \quad (11)$$

Hence, the distribution parameter is directly proportional to the curvature of the initial loading curve. For the series-parallel configuration k may again be determined from the small amplitude slope and the parameter $\beta\gamma$ (note that β and γ do not appear separately) may be determined from the initial loading curve as

$$\beta\gamma(f) = k \frac{d^2x}{df^2} \quad (12)$$

The parameters β and γ could be separated by requiring that the integral of the distribution function over the range of f be unity but this is not necessary.

In some instances, it may be easier to obtain experimental data on the cyclic energy loss of a system rather than on its initial loading behavior. The distribution functions may be determined directly from such energy loss data provided the initial stiffness k is known or can be separately determined. This approach might be used for example where forced vibration test results were available. For the parallel-series configuration it may be shown from equation (7) that

$$\varphi(kx_m) = \frac{1}{4x_m k^2} \frac{d^2W(x_m)}{dx_m^2} \quad (13)$$

where $W(x_m)$ is the energy loss per cycle expressed as a function of the displacement amplitude. For the series-parallel configuration equation (8) gives

$$\beta\gamma(f_m) = \frac{k}{4f_m} \frac{d^2\bar{W}(f_m)}{df_m^2} \quad (14)$$

where $\overline{W}(f_m)$ is the energy loss per cycle expressed as a function of force amplitude.

One interesting consequence of the fact that the distribution function is related separately to the initial loading curve and the energy loss per cycle is the possibility of expressing these latter features directly in terms of each other. Hence, combining equations (11) and (13) it may be shown that the initial loading curve for the parallel-series configuration may be expressed as

$$f = kx - \frac{x}{4} \int_0^x \frac{1}{\xi^2} \left[\frac{dW(\xi)}{d\xi} - \frac{dW(\zeta)}{d\zeta} \right]_{\zeta=0} d\xi \quad (15)$$

In this way, it would be possible to deduce the initial loading curve from steady-state dynamic response data if the motion is reasonably harmonic and if the phase angle can be assumed to be equal to 90° at resonance.

Although a least square fit of initial loading or energy loss data is probably most desirable, the present class of models lends itself to another even simpler method of parameter selection based on equations (11) and (12). This method has been discussed in detail elsewhere (1) in relation to a steady-state response problem. Basically, it consists of attempting to approximate data on the slope of the force-displacement diagram of a given system by three straight line segments. For the parallel-series configuration, this implies that the distribution function $\phi(f^*)$ is a flat, band-limited function as indicated in Fig. 5a. The shape of the initial loading curve for such a system will appear as in Fig. 5b where

$$\beta = \Delta f / 2f_y \quad (16)$$

Note that this special case includes a wide range of yielding behavior from the sharp elasto-plastic characteristic to a continuously yielding system, $\beta = 1$. For all $\beta > 0$ there will be a definite proportional limit denoted by $x_{p,l}$. Whether or not this simplifying assumption about the nature of $\phi(f^*)$ is valid will of course depend on the actual system being studied. An example of data from an actual single-story structure is shown in Fig. 6 along with a possible three-segment approximation. The data was taken from hysteresis loop measurements made by Hanson (3). The system parameters resulting from the indicated fit are: $k = 5810$ lb/in, $f_y = 3310$ lb, $\beta = 0.80$. As an indication of the applicability of the simplified model, the results of actual energy loss measurements for this structure are shown along with predicted results based on the model in Fig. 7. A similar comparison based on steady-state frequency response results was presented in (1). On the basis of these results, it appears that the simplified model may be quite adequate for a reasonable range of structures.

RELATION TO OTHER METHODS

It has been seen that the two different model configurations considered lead to two entirely different formulations of the force-displacement behavior; one giving the force as a function of displacement and the other giving displacement as a function of force. In general, the different formulations cannot be inverted, so a direct comparison is not possible. However, it should be emphasized that the same set of initial loading data could be used to generate either of the two formulations. The choice is up to the investigator. For the most part, the parallel-series formulation will be more desirable in dynamic analyses since it gives the force directly. On the other hand, there may be situations where the parallel-series formulation is preferable. This latter formulation has the advantage of possessing a three-dimensional analog(4).

Because of the inversion problem, any direct comparison of the present models with other models must be limited to models which lead to the same type representation (i. e., force or displacement). Different type representation can only be meaningfully compared through some intermediate quantity such as energy dissipation (4). One of the more popular force-type representations is that used by Pisarenko (5) and reportedly due to Davidenkov. In this formulation, the lower hysteresis loop curve is given by

$$f = k \left\{ x + \frac{\nu}{n} \left[(x_m - x)^n - 2^{n-1} x_m^n \right] \right\} \quad (17)$$

where ν is a parameter to be determined from experimental data and n is usually taken to be 2 or 3. This corresponds to a parallel-series model with a distribution function $\phi(f^*)$ which is a constant for $n = 2$ (similar to the case $\beta = 1$) and which is proportional to f^* for $n = 3$. Obviously, the correspondence only holds over a finite range since the integral of the distribution function must be unity. Indeed, equation (17) itself gives questionable results for x_m large.

In structural response problems, the most widely used curved hysteresis representations are those based on the Ramberg-Osgood model particularly as formulated by Jennings (6). This is a displacement representation and the initial loading relation may be expressed in notation similar to Jennings as

$$x/x_y = f/f_y + \alpha(f/f_y)^r \quad (18)$$

where α and r are parameters to be selected and r is assumed to be an odd integer >1 . This corresponds to a series-parallel model with $k=f_y/x_y$ and a distribution function $\beta\gamma(f^*) = \alpha r(r-1)(f^*/f_y)^{r-2}/f_y$. Since $r \geq 3$, this distribution function and its integral become unbounded as $f^* \rightarrow \infty$. Hence, correspondence of the two formulations is valid only if restricted to finite loads and displacements.

APPLICATION TO A TRANSIENT RESPONSE PROBLEM

As stated above, one of the most attractive features of the present class of models is that they provide a consistent framework for the study of transient loading behavior. The complete history of loading may be determined once the initial loading behavior is specified without the need for any further assumptions. In addition, the simplified representation indicated in Fig. 5 has the desirable property that the parameters f_y and β , specifying the yield level and shape, are physically distinct. This makes it possible to study the effects of yield level and shape independently in a transient response analysis.

Let us consider the response of a class of single-degree-of freedom oscillators having the hysteretic characteristic implied by the simplified representation. Let the mass of the oscillator be m ; let the displacement of the mass with respect to the base be x and let the base have an impressed acceleration $a(t)$. Furthermore, let there be no viscous damping (we desire to study the hysteretic effects only). Then, define,

$$\begin{aligned}\omega_0 &= \sqrt{k/m} ; T = 2\pi/\omega_0 \\ \eta &= m a_{rms} / f_y \\ x_y &= f_y / k\end{aligned}\tag{19}$$

where ω_0 is the small amplitude natural frequency of the system, a_{rms} is the rms-value of the excitation $a(t)$, x_y is the yield displacement, and η has the nature of a dimensionless excitation parameter. For a given excitation it may be shown that the dimensionless response parameter x/x_y is a function only of η and ω_0 or T and not of k , m , or f_y individually. For an excitation, let us use the series of eight 30-second duration artificial earthquakes generated by Housner and Jennings (7). These artificial earthquakes were constructed so as to give an average response spectrum very close to that for known large earthquakes as determined by Housner (8). By using eight different earthquakes, it is possible to get a reasonable estimate of both the average value of the response parameters and the deviation of these parameters from the average.

The dimensionless maximum response of the present system was obtained by numerical integration of the equation of motion and the average results are shown in Fig. 8. The parameter $|x/x_y|_{max}$ could be referred to as the response ratio, the ductility factor, or the yield ratio. In what follows we will use the term response ratio. At least three major observations may be made on the basis of the results. First, it may be noted that the response ratio increases as T decreases for constant excitation ratio η . This result has been pointed out before for a similar system (6) and is to be expected. It can be made reasonable by recognizing that x_y must decrease like T^2 if m , a_{rms} and f_y are held constant.

The second important observation is that generally speaking, the elasto-plastic model ($\beta = 0$) gives a larger response ratio than the curved loops for $|x/x_y|_{\max} < 2-2.5$ and a smaller response ratio for $|x/x_y|_{\max} > 2-2.5$. In other words, the widely used elasto-plastic model gives conservative estimates of the response only for moderately low level response. For other cases, it may underestimate the response. The same conclusion was arrived at in steady-state response studies of this system (1). This observation could have important implications in design but it must be moderated by the third general observation; namely, that the relative effect of β on the response ratio decreases as the response itself increases. For $|x/x_y|_{\max} < 2$, the variation in response ratio due to changing β is as high as 60-70% while for $|x/x_y|_{\max} > 2$, the variation is rarely as large as 20%. This indicates that variations in response due to loop shape alone can possibly be neglected if the system has reached a fully yielded state and if the limiting behavior of the system is unchanged by loop shape variations.

Figure 9 shows the range of the standard deviation of the response ratio results from the mean values for $\beta = 0.5$. It can be seen that the system is fairly sensitive to variations in the individual artificial earthquakes and this sensitivity should be kept in mind when interpreting the results of Fig. 8.

In recent years, there has been a great deal of discussion related to the definition of equivalent linear systems for certain hysteretic systems. Equivalent linear parameters have been defined in a number of ways using a variety of excitations and the results must be interpreted with great caution. Some of the difficulties involved can be seen from an extension of the present investigation. Let us consider all of the cases in Fig. 8 for which $2 \leq |x/x_y|_{\max} \leq 4$. For each of these cases, there will exist an average actual maximum velocity $|\dot{x}|_{\max}$ determined from the numerical integration, and an average pseudo-maximum velocity $\omega_0 |x|_{\max}$ which can be deduced from Fig. 6 and a knowledge of the system parameters. These two maximum velocities have been plotted along with the average linear velocity response spectrum of the artificial earthquakes in Fig. 10. The maximum velocity points shown represent a wide range of systems and the fact that the scatter of similar points is so small is somewhat surprising. However, the most significant point has to do with the correspondence between the linear spectrum and the maximum velocity points. It is seen that the actual velocity points consistently fall well below the pseudo-velocity points. Indeed, the actual velocity points correspond to a linear system with approximately 10% of critical damping while the pseudo-velocity points correspond to a linear system with 3-6% damping. Hudson (9) has presented a similar analysis based on pseudo-velocity and the resulting linear system damping was 4%. A study by Iwan and Lutes (10) based on the stationary random response of a nearly elasto-plastic system showed that the viscous damping necessary to give the same rms displacement response was approximately 2% while that required to give the same rms velocity response was closer to 15%. In all of the cases considered, the equivalent linear system had the same initial stiffness and mass as the hysteretic system.

The results indicate that there is a significant difference between the actual and pseudo-velocities in a yielding system. This, in turn, raises an important question as to which set of results is most meaningful. It is the belief of the present author that the pseudo-velocity results are probably the most meaningful since they are based on actual maximum displacement and this is the quantity which is usually most important in design. In any case, the present discussion clearly points out the difficulties involved in trying to define equivalent linear systems for hysteretic systems.

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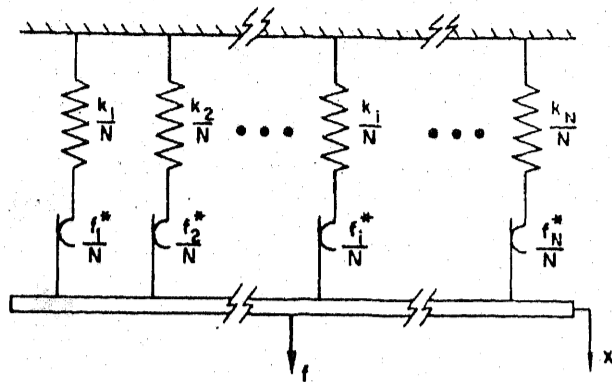


FIGURE 1 - PARALLEL-SERIES CONFIGURATION

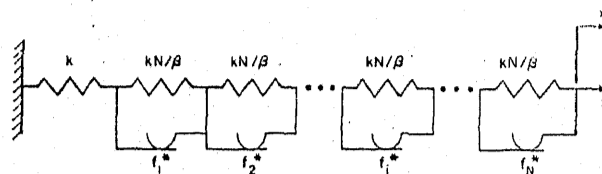


FIGURE 2 - SERIES-PARALLEL CONFIGURATION

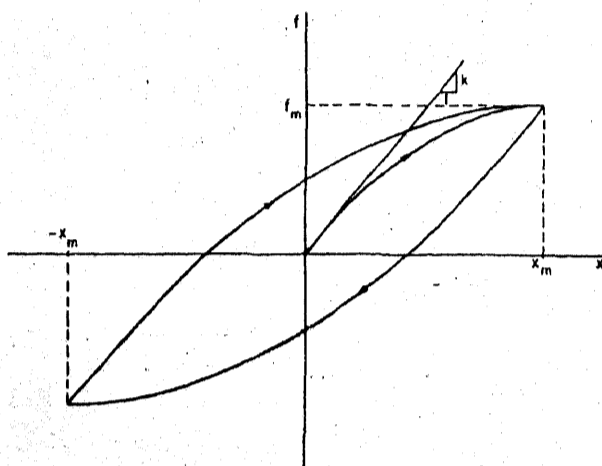


FIGURE 3 - PERIODIC LOADING CYCLE

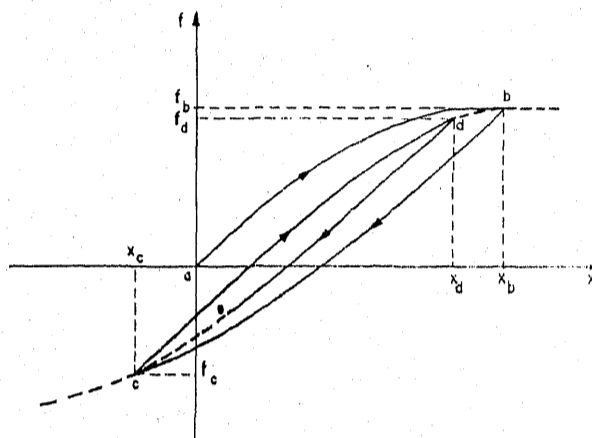


FIGURE 4 - TRANSIENT LOADING

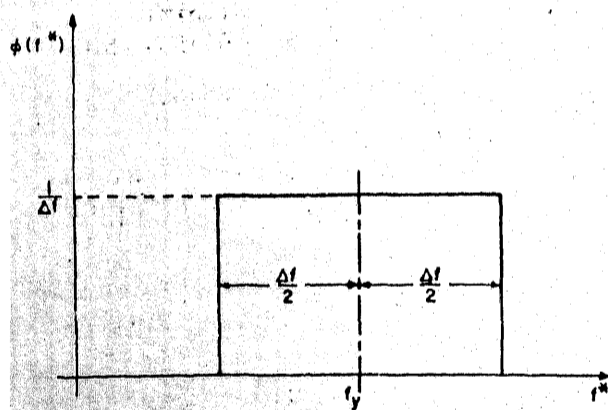


FIGURE 5a - BAND-LIMITED DISTRIBUTION FUNCTION

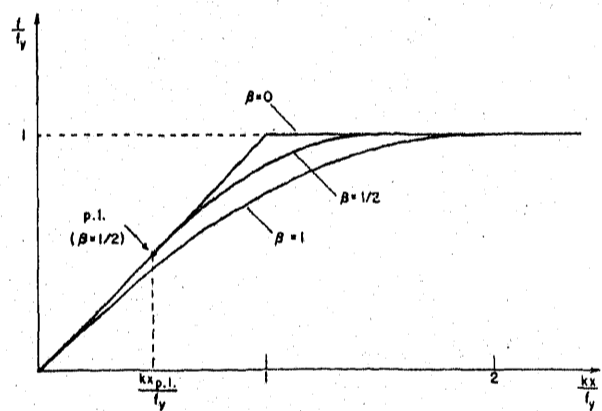


FIGURE 5b - INITIAL LOADING CURVES FOR BAND-LIMITED DISTRIBUTION FUNCTION

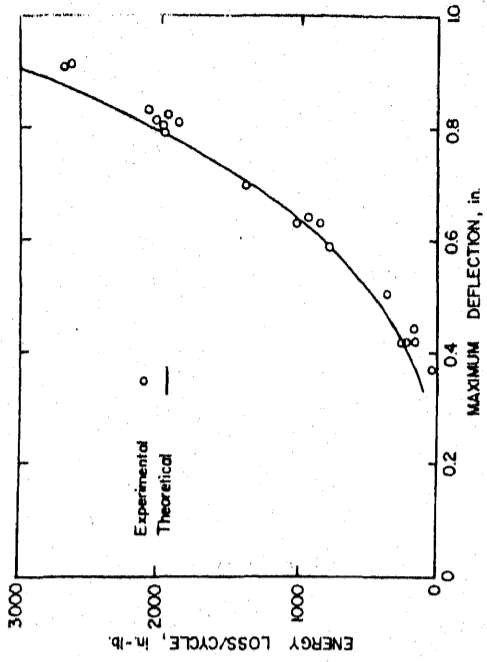


FIGURE 7 - ENERGY LOSS PER CYCLE COMPARISON

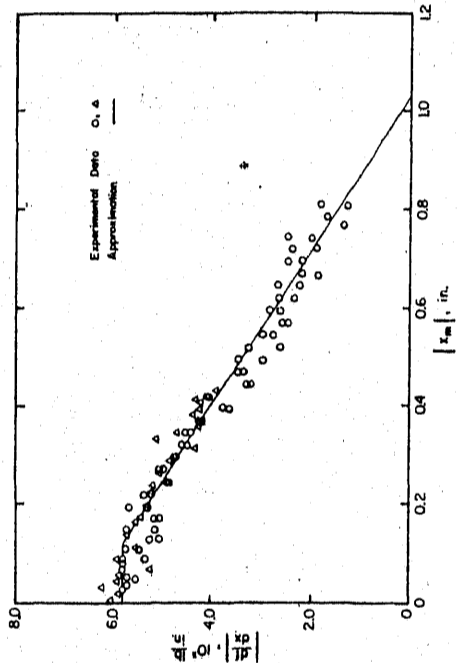


FIGURE 6 - APPROXIMATION OF HYSTERESIS DATA FROM ACTUAL STRUCTURE

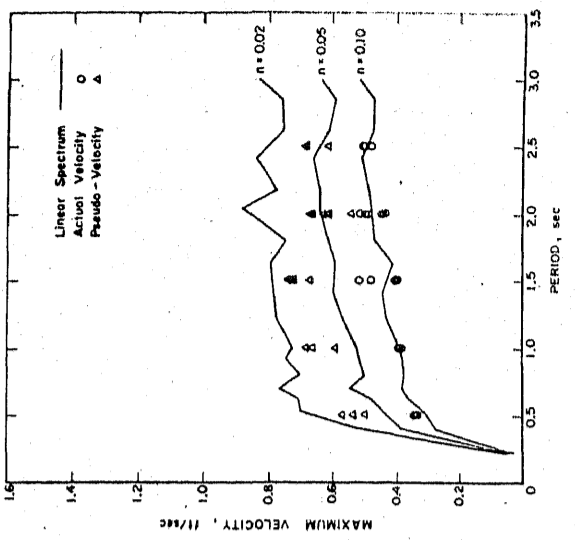


FIGURE 11 - RESPONSE SPECTRUM

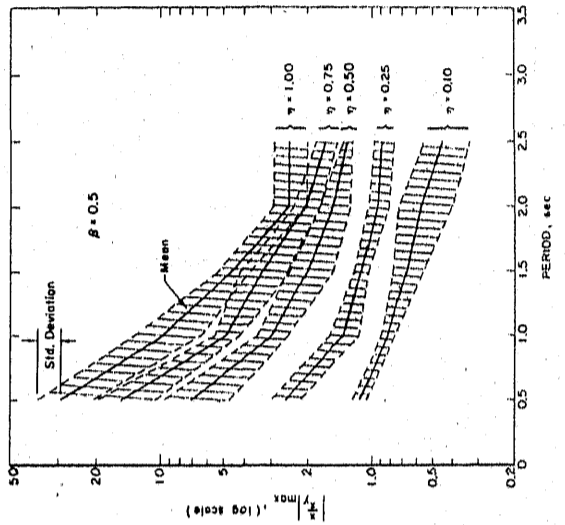


FIGURE 10 - DEVIATION OF RESPONSE RATIO FROM ACTUAL STRUCTURE

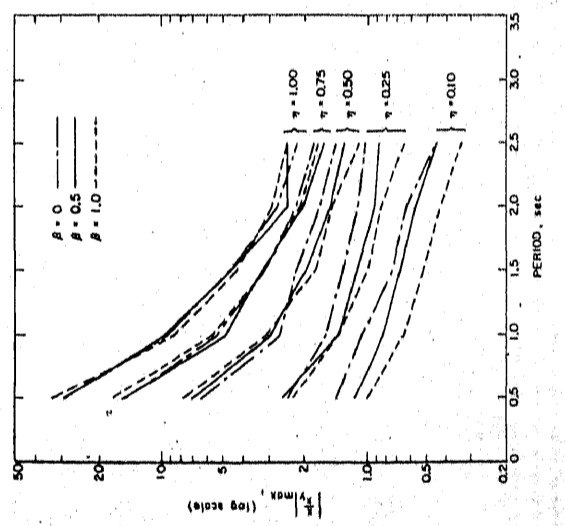


FIGURE 8 - RESPONSE RATIO FOR SYSTEMS WITH VARYING DEGREE OF CURVATURE