THE EFFECT OF GRAVITY ON THE COLLAPSE OF YIELDING STRUCTURES WITH EARTHQUAKE EXCITATION

by

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SYNOPSIS

The effect of gravity on the collapse of simple yielding structures with earthquake excitation is studied. The results show that yielding structures capable to resist earthquakes when gravity is ignored may collapse when considered.

A statistical study of the time to failure for elasto-plastic and bilinear structures showed that an earthquake of short duration would have to possess significantly higher acceleration than a longer earthquake to cause failure of a given structure.

Calculations made with simultaneous vertical and horizontal excitation, and with recorded strong earthquake accelerograms, indicate that the results of this study should be applicable for strong earthquake excitation.

INTRODUCTION

The behaviour of yielding structures subjected to earthquake motions usually has been studied under the assumption that it will not suffer collapse during strong earthquakes. Usually, this hypothesis implies rather small displacements, and therefore the effects of gravity upon the response can be neglected (II) (1,2). However, for structures that may yield during strong earth motions, gravity effects may become important because when the displacements approach the failure range the weight becomes the prevailing force. Thus, since a structure designed to yield when subjected to strong earthquakes allows permanent displacements, it is of fundamental interest to find how close to collapse is the yielding structure. This makes it necessary to consider gravity when studying the response.

It is the purpose of this study to present the first stone to build a better understanding of the collapse of buildings under strong earthquake excitation.

Probably the first examination of the effects of gravity on earthquake response was that by A.C. Ruge (3) who discussed the gravity effect in the determination of earthquake stresses in elastic structures with the aid of models and made estimations of the changes in period and deflection of a simple vertical cantilever loaded at the end with a weight.

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(II) Arabic numbers indicate references given in the Bibliography.

EXAMPLES OF GRAVITY EFFECTS

In a broad sense every structure that has collapsed when subjected to an earthquake is an example of the effect of gravity, but to understand how gravity affects earthquake response, it is informative to survey structures where collapse, though near, has no yet occurred.

Near Matsushiro, Japan (4,1) several houses inclined about four degrees in the direction of the street faced by each other. The final inclination resulted from consecutive increments produced by several earthquakes. Presumably, further shaking would eventually cause collapse of the houses.

During the great Alaska earthquake of 1964 a simple structure with a heavy wooden roof and supported by steel pipe columns was severely damaged. The heads of the columns were displaced horizontally more than 10 inches. It is clear that the structure suffered large deformations in the yielding range and it appears that gravity was in this case on the border of producing collapse.

Several canopies at the Macuto Sheraton Hotel (Venezuela) presented similar damage after the destructive earthquake of July 29, 1967. The heads of the columns of one of the canopies were displaced more than 11 inches and a similar structure suffered complete collapse.

MATHEMATICAL MODEL

A model of a simple structure that considers the effect of gravity is shown in Figure 1. The girder is considered to be infinitely rigid with a total mass 2 m. The columns are also assumed to be infinitely rigid but without mass. The connections between columns and the girder and between columns and the foundation are represented by nonlinear springs that operate in torsion to generate nonlinear restoring moments. Damping, assumed to be viscous, is supplied by the dashpot. The deflection angle specifies the position of the mass.

Two yielding relations are used for the nonlinear springs in this study: The elasto-plastic and the bilinear hysteretic $F(\phi,\dot{\phi})$ defined in Figure 1 is shown in Figure 2 for the two yielding relations considered.

EQUATION OF MOTION

The equation of motion of the structure shown in Figure 1 when the foundation is subjected to horizontal acceleration $\ddot{u}(t)$ may be written as

$$\ddot{\phi} + 2n\,\omega_0\,\dot{\phi} + \frac{K}{ml^2}\,F(\phi,\dot{\phi}) - \frac{g}{l}\sin\phi = -\frac{\ddot{u}(t)\cos\phi}{l} \qquad (1)$$

where n is the fraction of critical damping of small oscillations with frequency ω_0:

$$\omega_0 = \sqrt{\frac{K}{ml^2} - \frac{g}{l}} \qquad (2)$$

Equation 1 characterizes the response of the simple yielding structure of Figure 1 and considers the effect of gravity. When small and linear vibrations are considered equation 1 becomes

$$\ddot{\phi} + 2n\,\omega_0\,\dot{\phi} + \omega_0^2\,\phi = -\frac{\ddot{u}(t)}{l} \qquad (3)$$

When the structure has no damping and gravity is ignored, its period is T_o. When gravity is considered the period of the structure, T, is such that:

$$\frac{T}{T_o} = \left\{ 1 - g \frac{T_o^2}{4\pi^2 l} \right\}^{-\frac{1}{2}} \tag{4}$$

Thus the effect of gravity in the linear range is to increase the natural period of vibrations. For most structures this change in period is small because g/l is much less than k/ml^2 since equality of these two terms implies that the structure will fail by elastic buckling.

IMPORTANT PARAMETERS

Equation 1 describes the motion of a simple yielding structure when gravity is considered. Before solving this equation numerically an exam of the parameters is made.

The yield angle, ϕ_y, is the yielding level for the nonlinear springs shown in Figure 1. The yield level, a_y, is the lateral acceleration of the foundation necessary to start yielding in the structure. It is easily verified that

$$\phi_y = \lambda \frac{a_y}{g} \tag{5}$$

where

$$\lambda = \frac{g}{l\left\{\frac{k}{ml^2}\cos\phi_y - \frac{g}{l}\frac{\tan\phi_y}{\phi_y}\right\}} \tag{6}$$

and for small yield angles, $\lambda = \frac{g}{l\omega_o^2}$

Introducing a dimensionless time τ, a scale factor of the earthquake excitation E, and designating with $\sigma(\tau)$ an accelerogram with standard strength, the equation of motion becomes

$$\frac{d^2\phi}{d\tau^2} + 2n\frac{d\phi}{d\tau} + F(\phi,\dot\phi) + \frac{g}{l\omega_o^2}\left[F(\phi,\dot\phi) - \sin\phi\right] = -\frac{E}{l\omega_o^2}\sigma(\tau)\cos\phi \tag{7}$$

where $\tau = \omega_o t$, and $\ddot{\mu}(t) = E\sigma(\tau)$

Note that F $(\phi,\dot\phi)$ is not defined unless ϕ_y, or a_y is given.

The angle of static failure ϕ_s is defined as the maximum angle for which the yielding structure is statically stable. For an elasto-plastic structure with $\phi_y \ll 1$ it is found

$$\sin|\phi_s| = \phi_y + \frac{a_y}{g} \tag{8}$$

If ϕ_s is such that $\sin|\phi_s| \approx \phi_s$, which is generally true for real structures, then equation 8 becomes

$$|\phi_s| = \phi_y + \frac{a_y}{g} \tag{9}$$

Numerical values of E for four of the strongest earthquakes recorded, the yield angle and the angle of static failure, as functions of a_y/g and λ, are given in Reference 1.

SELECTION OF PARAMETERS

It is seen from equation 7 that the parameters of the problem include the fraction of critical damping, the period of small vibrations, the yield level (implicit in F $(\phi,\dot\phi)$), the height of the structure and the intensity and form of the earthquake excitation. Values of every one of the listed parameters must be selected before obtaining a numerical solution of equation 7.

From equation 7 follows that the lengths of the columns will have an important influence on the development of collapse under the action of gravity. The values of l used in the calculations are $l = 5$, 10, 15, 20, 25 and 30 ft. This range extends well below and above the heights of typical story heights in structures.

Because of time limitations the amount of viscous damping was not varied in this study. The value selected, 2 percent of critical, was based on results from vibration tests of multi-story buildings (5).

Four values of the period for small amplitudes were selected which cover most of the range of interest in the earthquake problem. The values considered are $T = 0.5$, 1.0, 1.5 and 2.0 seconds, a range including the periods of most multi-story structures. It is noted that the present study is not aimed at real one story buildings. It represents a first step in the analysis of the effects of gravity; aprelude to the study of multi-story buildings. Although the analysis may apply to some special one-story structures (Examples of gravity effects), most modern one-story buildings are constructed in such a way that collapse during earthquakes is not a problem.

It proved convenient to used ϕ_y and a_y/g to describe the yield level of the structure. These two parameters are related by equation 5. For most of this study two values of a_y/g were selected: $a_y/g = 0.05$ and 0.10. As will be shown below, the results obtained for these values can be extrapolated to other cases of interest.

RESPONSE OF AN ELASTO-PLASTIC STRUCTURE. EXCITATION.

The influence of gravity in the response of the selected elasto-plastic structure near collapse is made clear with an example. Figure 3 shows the response of the structure to 60 seconds of pseudo-earthquake (6) for four cases as function of time. The structure used has a height of 10 ft., a yield level of 10 percent of g and a period of 0.5 seconds. The pseudo-earthquake is scaled ($E = 3.45$) to be about 20 percent stronger than the N.S. component of the El Centro 1940 record.

In case a), the structure is linear and gravity was neglected; while in b) the structure remains linear, but gravity is considered in the response. In c) the structure is elasto-plastic and gravity is disregarded; while in d) the structure remains elasto-plastic and gravity is included.

From Figure 3 it is seen that the effect of gravity upon the response in the linear case is negligible. The elasto-plastic response in c) presents an appreciable drift of the equilibrium position and at the end of the excitation a permanent set remains. A different response results when gravity is considered. It is seen that the effect of gravity is to increase significantly the development of permanent set, and in this case the displacement eventually exceeds the failure angle. Because the gravity effect increases as the deflection grows the effects are accelerative and the drift increases rapidly as the failure angle is approached.

For a statistical study of gravity effects it is convenient to use a collection of recorded earthquakes with similar statistical properties. Such a collection is not available yet and recorded earthquakes have different frequency contents, durations and strengths and depend on the epicentral distance, local geology, depth of the hypocenter, etc. Considering that the number of accelerograms recorded is quite small (7), it was thought preferable to do the basic part of this study using an ensemble of well defined pseudo-earthquakes (6).

These artificial earthquakes have ensemble average r.m.s. acceleration of 0.697 ft/sec^2 and they produce average damped velocity spectra corresponding closely to the standard spectra computed by Housner (8,6). The strength of the artificial earthquake is varied by changing the factor E. For E = 2.88 gives an ensemble of accelerograms whose strength is comparable to the El Centro 1940 earthquake.

The duration of the motion has an important role in the collapse of yielding structures. In particular, too short a duration does not make clear the relative safety of the structure as a function of time. For example, Figure 3d presents no evidence of collapse during the first 11 seconds of response, but the subsequent response shows clearly that the structure is very close to failure at t = 11 seconds. The strong phases of recorded U.S. earthquakes have not exceeded 25 sec. and the basic duration for the artificial earthquakes was chosen 60 seconds.

Eight 30 second accelerograms of artificial earthquakes are available on punched cards for use in digital computer calculations. Four 60 seconds accelerograms were formed arbitrarily by joining artificial earthquakes 1 and 2, 3 and 4,5 and 6, and 7 and 8.

RELATIVE EARTHQUAKE STRENGTH

It seems reasonable to expect that a structure with a high yield level subjected to a strong earthquake would collapse in about the same time as a lower yield level structure subjected to proportionally weaker excitation. This was proved analitically for ϕ sufficiently small, and numerical computations showed that for most cases of interest the time to collapse depends only on the ratio of the intensity of the earthquake (E) to the lateral yield level and not upon their individual values (1). The parameter $\theta = E/(a_y/g)$ is introduced to denote this ratio. For most practical cases, θ will be in the range between 20 and 60. $\theta = 20$ for the El Centro 1934 earthquake and $a_y = 0.1g$, and $\theta = 60$ for the El Centro 1940 earthquake with $a_y = 0.05g$.

Introducing $E = \theta\, a_y/g$ in equation 7 it becomes:

$$\frac{d^2\phi}{d\tau^2} + 2n\frac{d\phi}{d\tau} + F(\phi,\dot{\phi}) + \frac{g}{l\omega_0^2}\left\{F(\phi,\dot{\phi}) - \sin\phi\right\} = -\frac{\theta a_y}{g l \omega_0^2}\nabla(\tau)\cos\phi \qquad (10)$$

DIGITAL COMPUTATIONS

Equation 10 was used to evaluate the response of the yielding structures to earthquake-like excitation. The computations were done on an IBM 7094 computer using 3rd and 4th order Runge-Kutta methods (8). The integration step was made smaller than 1/20 of the period of small oscillations. This was done after noting that further reduction of the step size did not alter appreciably the response. For the elasto-plastic structure, 169 response evaluations were made. The characteristics of the structures, time to collapse t_o, average time to collapse $\bar{t}_o$, yield and failure angles, are presented in several tables in Reference 1.

EFFECT OF THE RELATIVE EARTHQUAKE STRENGTH UPON THE TIME TO FAILURE

With ℓ fixed at 10 ft., the response was calculated for T = 1.0 sec. and for six value of θ (17, 23, 34.5, 46, 69 and 90) for each of the four 60 second artificial earthquakes. The results, shown in Figure 4, suggest a nearly hyperbolic relation between $\bar{t}_o$ and θ. For each θ, Figure 4 gives $\bar{t}_o$ and the range of individual values of t_o. Figure 5 presents the dependence of $\bar{t}_o$ on θ for four values of period with ℓ = 10 ft. The effects of the natural period on $\bar{t}_o$, if any, appears comparable to the dispersion in the results.

EFFECT OF PERIOD

With ℓ = 10 ft., four values of period and four values of θ (23, 34.5, 46 and 69) for each of the four 60 second pseudo-earthquakes, the response was calculated. Figure 6 shows $t_o/\bar{t}_o$ plotted as a function of θ and no strong trend exists in the data for constant θ. The dispersion of the data increases with increasing θ, indicating more percentage variation in t_o for weak structures subjected to strong earthquakes than for strong structures subjected to weak earthquakes. This may be explained observing that relatively small times to collapse are obtained when θ is large. The r.m.s. acceleration over such short durations tends to vary considerably among the artificial earthquakes (1) and this greater variability in the strength of the excitation may be responsible for the increased dispersion for large θ.

It was concluded that the mean $\bar{t}_o$ and $t_o/\bar{t}_o$ do not depend strongly on T. It was found that $\bar{t}_o$ is strongly dependent on θ, decreasing as θ increases (1).

EFFECT OF STORY HEIGHT

To study the effect of story height values of 5, 10, 15, 20, 25 and 30 ft. for ℓ were selected, with a_y = 0.05g and E = 2.3 (θ = 46). Figure 7 shows $\bar{t}_o$ for the four artificial earthquakes and four values of T plotted against ℓ. It shows that the results have a strong linear dependence on ℓ and again, that the effects of period, if any, are masked by the dispersion of the results.

ESTIMATES FOR $\bar{t}_o$

Correlation and regresion analyses of the results (1) showed that $\bar{t}_o$ is approximately given by:

$$\bar{t}_o = \frac{C\ell}{\theta^{r}} \qquad (11)$$

where $\bar{t}_o$ is in seconds, ℓ is in feet and θ is dimensionless.

A least squares fit was made and it gave C = 2000. Equation 11 with C = 2000 is valid for ℓ ranging between 5 to 30 ft. and θ between 20 and 70.

For $\theta = 46$, equation 11 (C = 2000) gives

$$\bar{t}_0 = 0.945\ell \tag{12}$$

Equation 12 is plotted as a straight line in Figure 7 and it is seen that it gives an adequate representation of the dependence of $\bar{t}_0$ on ℓ for this value of θ .

SPECIAL EARTHQUAKES

The results presented are pertinent to the response of yielding structures to very short duration shocks such as the Port Hueneme quake of March 18, 1957 and the Parkfield earthquake of June 27, 1966 (9), both of which have a very short strong part. The latter did not produce major damage in spite of the large maximum acceleration.

To examine the effects of duration, select an elasto-plastic structure with yield level a_y and height ℓ , and let E_1 be the intensity of the acceleration. Assume that $\bar{t}_0$ is 25 seconds and using equation 11 obtain:

$$E_1 = \frac{a_y}{g}\sqrt{80\ell} \tag{13}$$

Consider next an earthquake like Parkfield with an intensity E_2 . If the same structure will collapse after the end of the strong shaking (1 second), then E_2 must satisfy the relation:

$$E_2 = 5E_1 \tag{14}$$

indicating that it would take approximately 5 times the intensity of earthquake shaking to fail the elasto-plastic structure in one second compared to the intensity causing failure in 25 sec. This shows why the capacity to produce collapse is rather low for earthquakes like Parkfield even though the maximum acceleration is very large.

BILINEAR STRUCTURES

A bilinear hysteretic model often is a more realistic representation of real yielding structures than the elasto-plastic model. For a given earthquake, the effect of a positive second slope (Figure 2b) in $F(\dot{\phi},\phi)$ will be to increase the time to collapse because the restoring moment and, hence, ϕ_s will be larger. For the bilinear model ϕ_y and a_y/g are also related by equation 5, and ϕ_y is the angle at which the stiffness changes. The angle of static failure ϕ_s , is derived from equilibrium consideration which give:

$$mg\ell \sin\phi_s - K_2\phi_s = (K - K_2)\phi_y \tag{15}$$

where K and K_2 are the stiffnesses (Figure 2b). When $\sin\phi_s \approx \phi_s$, last equation can be written:

$$\frac{\phi_s}{\phi_y} = \frac{K-K_2}{mg\ell - K_2} \tag{16}$$

For small displacements there is a singular case for which gravity does not tend to make ϕ increase beyond ϕ_s. In this case, the effect of the spring K_2 just balances the effect of gravity, that is:

$$K_{2c} = mgl \tag{17}$$

DIGITAL RESPONSE AND SPECIAL EFFECTS

In this study calculations were made using equation 10 only for two specific structures, four artificial earthquakes, and 3 real earthquakes, so that the entire range of possible bilinear structures is not covered. The computations were done for varying values of K_2. In Figure 8 the ratio of $\bar{t}_\star$ for the bilinear structure to $\bar{t}_0$ for the elasto-plastic structure with the same lower slope are plotted as a function of K/K_{2c}. The most important feature of the results is the increase in average time to collapse with increasing upper slope.

No clear influence of period is observed in Figure 8. Since no such dependence was found for the elasto-plastic case, it seems acceptable to combine the results. Under this assumption, a least square fit of the data was done and the following expression was obtained:

$$\bar{t}_0 = \frac{2000\,l}{\left(\frac{E}{a_y/g}\right)^2 \left\{1 - \left(\frac{K_2}{mgl}\right)^{0.8}\right\}} \tag{18}$$

The last equation gives an approximate relation for the average time to collapse of bilinear and elasto-plastic structures and is informative as to the influence of the properties of the structure and the strength of the earthquake.

Figure 9 shows the time history of the response of one of the structures for K_2/K = 0, 0.02, 0.05 and 0.075 when earthquake (3+4) was used. For this structure K_{2c} = 0.075 K. It is seen that for the case when $K_2 = K_{2c}$ the structure does not collapse and the response shows two dominant periods that can be associated with the first and second slopes of the bilinear hysteretic relation.

The results of Figures 8 and 9 show that the second slope can influence strongly the response of a yielding structure. In the example K_2 = 0.075 K = K_{2c} prevents collapse completely.

Calculations similar to those just presented were done using strong-motion earthquake records to see if the results are consistent with those obtained using artificial earthquakes (1). From these calculations it can be concluded that there is no major difference between the results using the strong portion of typical recorded accelerograms and the artificial earthquakes used for this study.

Several structures analyzed previously were re-examined when vertical and horizontal motions are acting simultaneously to check whether the vertical motion will appreciably change the time to failure. Based on results obtained for the particular structures and earthquakes considered, and on theoretical considerations (1), it is concluded that the vertical component of earthquake motion does not appreciably affect the lateral vibrations and collapse of structures of the type considered in this study.

CONCLUSIONS

An investigation is made of the effect of gravity upon the response and collapse of one degree of freedom yielding structures subjected to earthquake-like excitation. The results demonstrate that the effect of gravity is to increase significantly the amount of permanent set over that found when gravity is ignored. Large values of permanent set, of course, will lead to collapse of the structure.

The average time to failure does not appear to depend on the period of small vibrations for the range 0.5 to 2.0 sec. considered. The important parameters are the height of the structure, the relative earthquake strength, and the upper slope of the bilinear hysteretic yielding relation Equation 18 shows approximately how t_c depends on the parameters listed above.

It is concluded that the time to collapse of a bilinear hysteretic structure is strongly influenced by the second slope of the yielding relation. It was found that an appreciable increase in the time to collapse occurs when the second slope K_2 increases from zero towards $K_2 = mg\lambda$ and collapse does not occur if $K_2 \geq mg\lambda$.

The vertical component of the earthquake motion does not appreciably affect the lateral vibrations of simple yielding structures (elasto-plastic and bilinear hysteretic).

The results show for the structures considered that the average time to failure varies as the square root of the intensity of the earthquake. Therefore the intensity of ground motion needed for failure under short duration earthquakes is much higher than that for longer quakes. This shows why earthquakes like the Parkfield quake 1966, even though the maximum acceleration may be very large, have a rather low capability to produce collapse.

ACKNOWLEDGEMENTS

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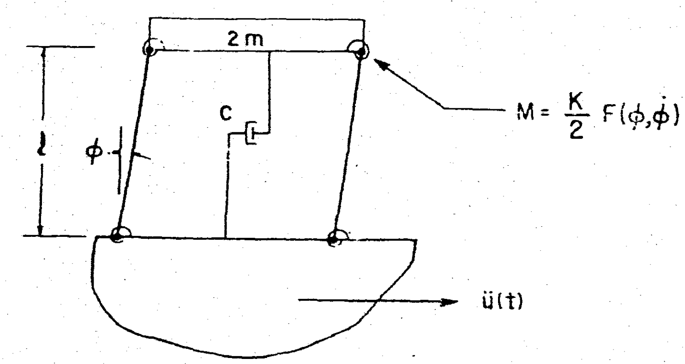

Figure 1 Simple model that considers gravity

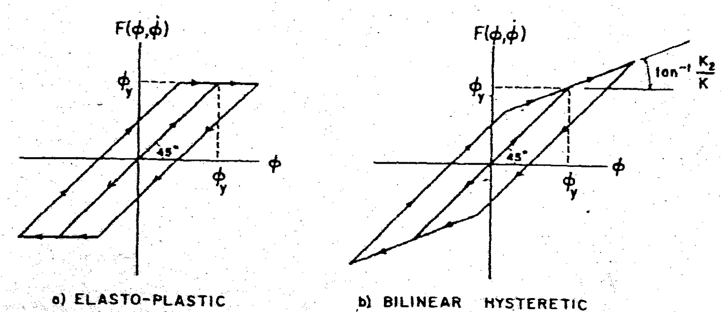

Figure 2 Yielding force-deflection relations

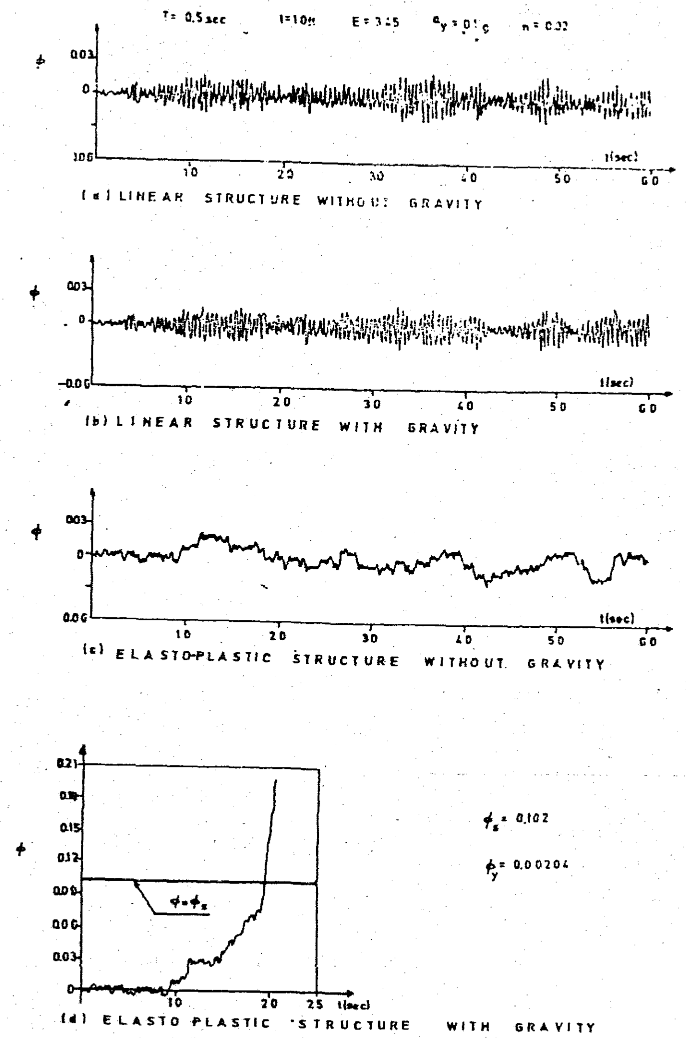

FIGURE 3 EXAMPLE OF RESPONSE TO PSEUDO EARTHQUAKE

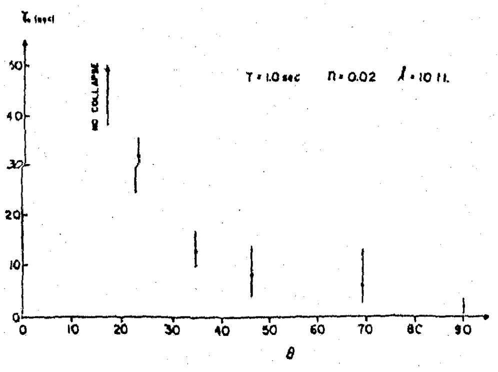

Figure 4 $\bar{T}_a$ as a function of θ for elasto-plastic structures

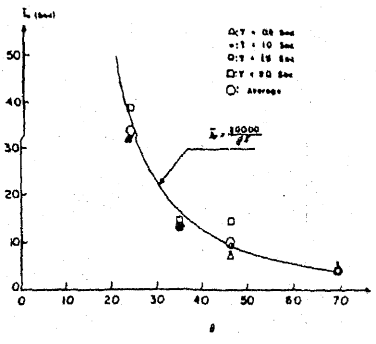

Figure 5 $\bar{T}_a$ as a function of θ

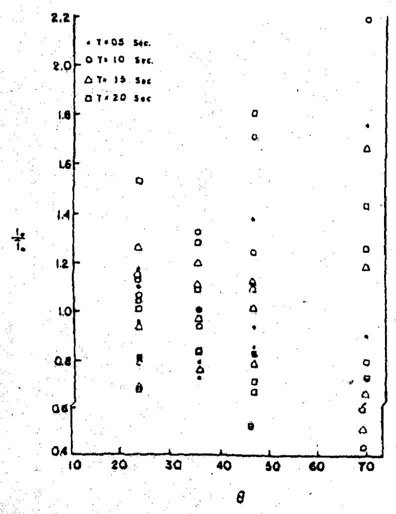

Figure 6 Normalized times to collapse

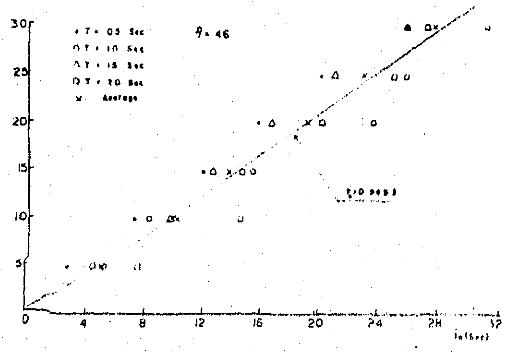

Figure 7 $\bar{T}_a$ as a function of P

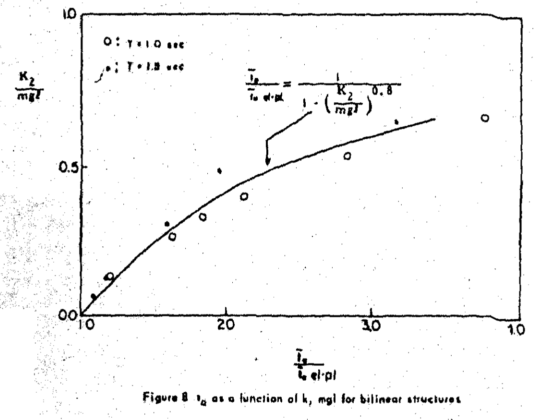

Figure 8 t_a as a function of k_2 mgl for bilinear structures

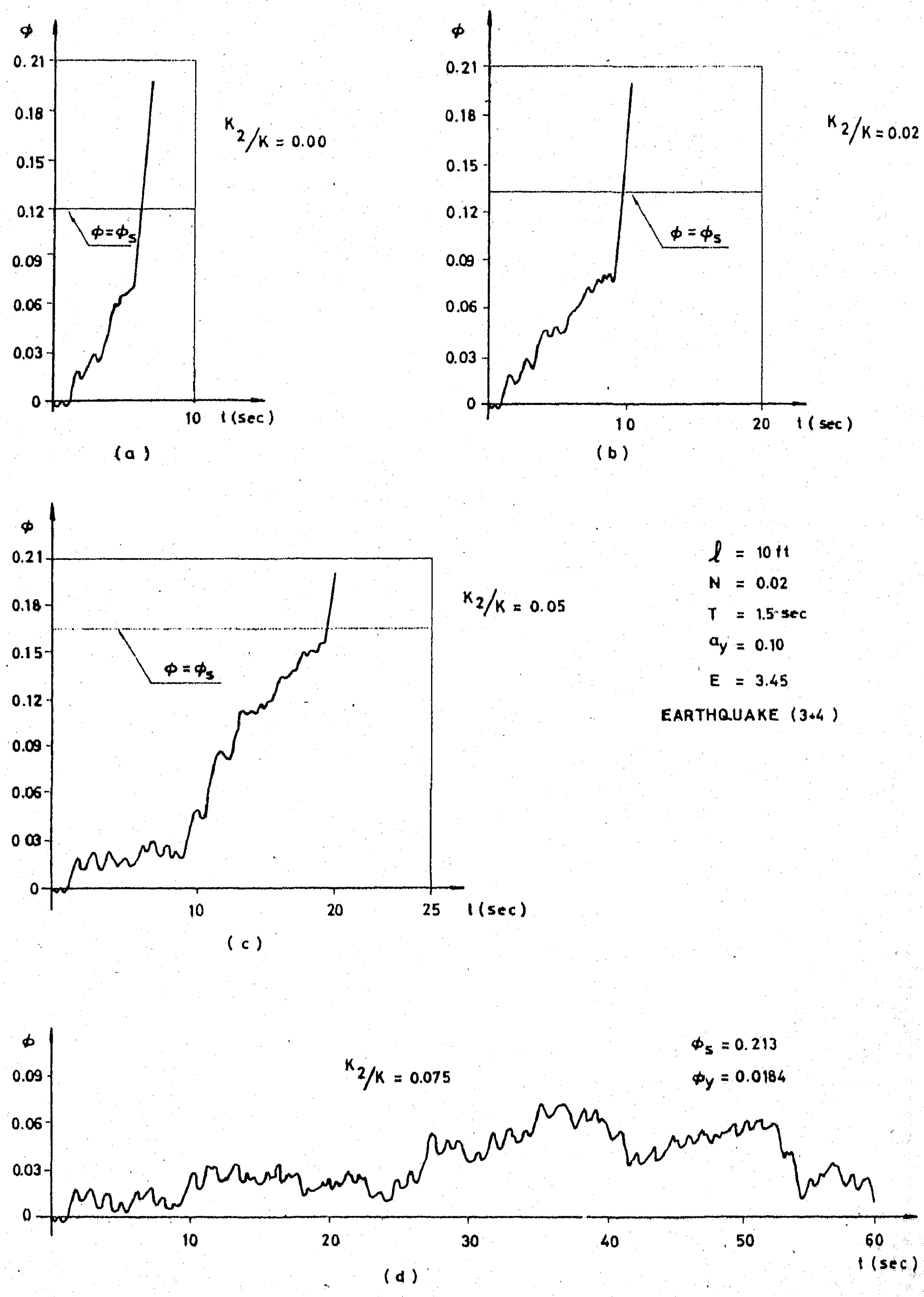

FIGURE 9 RESPONSE OF BILINEAR HISTERETIC STRUCTURES