THE INELASTIC RESPONSE OF A STEEL FRAME

By

W.R. Walpole* & R. Shepherd**

Synopsis This paper describes a digital computer analysis of the elasto-plastic response of a proposed 14 storey flexible steel-frame New Zealand building to a large digital earthquake record. The behaviour under an intensified El Centro record is examined assuming elastic and elasto-plastic behaviour. Comparative static analyses are made using code forces and those predicted by a normal mode-response spectrum approach.

* Assistant Engineer, Beca, Carter, Hollings and Ferner, Consulting Civil and Structural Engineers, Wellington, New Zealand.

** Reader in Civil Engineering, Canterbury University, Christchurch, New Zealand

INTRODUCTION

It is generally recognized that many structures have successfully resisted the action of major earthquakes although designed to resist much smaller lateral forces than those predicted by dynamic elastic considerations. When the elastic response of a typical multi-storey building to a major earthquake is calculated it is evident that the stresses in some of the members are greater than the yield stress of the material. This is because many framed structures are designed on the basis that they will resist the more frequent ground motions without damage, but will withstand the most intense seismic shocks without total collapse occurring only by calling on the reserve strength existing beyond the yield deformation point in the individual members. Observations of buildings damaged in earthquakes support the contention that many structures possess an ability to dissipate energy due to inelastic action and thus avoid catastrophic collapse under strong motion earthquake loading.

In an attempt to allow for the inelastic behaviour and yet still make use of the normal mode-response spectrum approach to seismic design, modification of the elastic response spectra to produce a set of design curves has been practised.[1] Studies of one degree of freedom elasto-plastic systems subjected to earthquake motions[2] justified this design practice to some extent, as it was shown that the maximum displacements were reasonably independent of the yield strength of the structures. Most design methods implicitly or explicitly assign yield moments based on a load factor on the moments under code loading and are only an inadequate substitute for an authentic inelastic analysis because the magnitude of the plastic deformations which may occur are not calculated.

As an alternative to a response spectrum - normal mode approach the response of a multi-mass system to earthquake ground motion may be found by the direct integration of the equations of motion using a digital record to represent the earthquake.[3] This procedure requires a knowledge of the dynamic stiffness of the building frame, the mass to be considered lumped at each floor level, the form and magnitude of the damping present and a reliable record of the earthquake motion.

The direct integration of the equations of motion in their original form is more general then the normal mode superposition method because there is no restriction on the magnitude or form of the damping present and it may be applied to non-linear systems by modifying the linear properties at each successive step of integration.

This approach was adopted by Clough Benuska & Wilson (4) who used a digital computer program to determine the amount and distribution of plastic deformation in a typical reinforced concrete 20-storey three bay open frame building when subjected to the 1940 El Centro earthquake, N-S component. The structure was assumed to behave in a linear elastic manner during each time increment and the non-linear response was obtained as a sequence of sucessively differing systems.

The investigation described below was undertaken to establish the behaviour of a typical New Zealand structure, using methods similar to those developed by Clough, Benuska and Wilson, to give the non-linear response to strong earthquake motion and, in particular, the required member ductilites. The difficulties and danger of extending the results of the anlysis of one type of elasto-plastic system to another type made it worthwhile considering the behaviour of multi-mass structures other than the frame examined by earlier researchers.

The 1940 El Centro earthquake, N-S component, was used, intensified by a factor of 50% to bring it up to the magnitude of the largest earthquake considered possible (5) having a maximum acceleration of 0.50 g. The frame examined is modelled on a preliminary proposal for a building which is to be constructed in Wellington, New Zealand, and which is symmetrical in plan having five bays in the longidudinal direction and three bays in the transverse with a fundamental period of approximately 2.80 secs. in both directions. The structure is very flexible and the object of the anlysis was to determine the magnitude and distribution of the plastic deformations in the members of the frame and to investigate whether a major earthquake would cause a significant amount of plastic drift.

THE FRAME.

The steel frame building used as the basis of this analysis has 14 main floors of area 11,000 sq. ft. with two setback floors on the top and two basement floors which are part of massive reinforced concrete voults. An interior transverse frame was analysed and for this purpose was simplified to a regular fourteen storey three bay frame rigidly fixed at gound level, the mass of the setback floors being lumped at the fourteenth floor and the extra bays forming a podium at the lower two flocrs being of much lighter sections. The properties of the members and the floor masses are shown in Figure 1, the moments of inertia being calculated from the uncased steel sections because it was proposed to provide the fire protection with sprayed asbestos.

ELASTIC ANALYSIS.

A normal mode analysis was carried out to enable the seismic forces to be estimated from a response spectrum. The member stiffnesses were estimated from the steel sections, taking Young's Modulus as 30×10^6 lb/in^2, the lateral flexibility matrix being calculated using a digital computer program which considered bending deformations only as the effects of shear deformation, axial deformation and joint size were considered to be sufficiently small to be neglected. The seismic forces were predicted by taking the root mean square of the modal responses found using a response spectrum curve assuming 5% critical damping with an intensification factor of 1.5. The member actions caused by these predicted forces were found using a tridiagonal elimination program (6) and are shown in Figure 2(a) together with the lateral forces. The normal mode properties and the predicted response are listed in Table 1.

The elastic response to the intensified El Centro record was found by integrating the first three modal equations of motion and summing the responses at each increment using a digital computer (6). The variation of the displacements of the floors with time is shown in Figure 3, and the maximum floor displacements are plotted in Figure 2(b).

A static analysis was also carried out for comparative purposes to determine the member actions under the lateral loading required by the New Zealand code (6). Lateral loads were found using a coefficient of 0.08 on the base shear and distributed as required by the code, i.e. in approximately triangular fashion. The inter-storey deflections under code loading exceed the code requirements slightly. The lateral loads, deflections and member actions are compared with those predicted from the dynamic elastic response in Figure 2. A factor of 2.6 is needed to reduce the predicted elastic response to the code figures, comparing base shear values.

ELASTO-PLASTIC ANALYSIS.

The elasto-plastic response of the 14 storey 3 bay frame to the intensified El Centro record was found using a computer program written for this purpose (6). The ultimate moments of the members were calculated using a yield stress of 36,000 lb/in^2 and a shape factor of 1.15 for the beams and 1.00 for the columns, to allow for the effects of axial load approximately. The values assumed for the exterior beams and columns are given in Figure 2(c). The damping was assumed to be 5% of critical in the first two modes.

The variation of displacement of the floors against time is shown in figure 4. The plastic deformation reduces the maximum top storey deflection by approximately 50% and also caused the periodicity of motion to become a little longer. The displacements are similar in magnitude up to the peak at 3.3 secs, but after that the plastic action dissipates energy and prevents the build-up of elastic strain and kinetic energy which would lead to an increase of response. Figure 4 also shows that a significant amount of permanent deformation has occurred and that the top floor would be left with a permanent set of several inches beyond the base.

Figure 2 (c) shows that the beam yield moments are set much lower that the column yield moments relative to the predicted elastic response, and so it would be expected that more yielding would take place in the beams. The only yielding in the columns occurs at the base of the bottom column, most of the building's energy absorption being provided by the beams. The required ductilities of the exterior beams are compared with the moment rations in Figure 2 (b). The member ductility is defined as ratio of the maximum total end rotation to the end rotation when the member is defined as the ratio of the maximum bending moment assuming elastic behavior to the yield moment of the same section.

It can be seen that a high ductility is required near the bottom of the building where a high moment was obtained and this peak in the moment-ratio plot is accentuated in the ductility requirements. High ductilities are also required around the 9th. floor; this behaviour was also noted by Clough, Benuska and Wilson (3) and is presumably caused by the effects of modes of vibration similar to the second and third elastic modes.

The variation of bending moment with time and the growth of plastic deformation as yielding occurs is shown in Figure 5. It can be seen that the plastic deformation changes over a comparatively short time corresponding to the rotation of a plastic deformation changes over a comparatively short time corresponding to the rotation of a plastic hinge. The effect of the plastic action is to cut off the peak of the moment curve giving a distinctive flat crest to the plot. It should be remembered that the member is plastic only on the flat crest, either side the member is elastic and the plastic action does not appear to alter the shape of the moment curve either side of the crest.

CONCLUSIONS.

It has been shown that for the hypothetical frame studied the maximum displacements calculated assuming elastic behaviour and 5% damping are reduced by 50% when plastic action is taken into account.

The yielding occurs entirely in the beam members except for some at the base of the bottom column, and the ductility requirements are quite modest with a maximum of 2.74, however a significant amount of plastic drift occurs and the top floor would be left with a permanent set of several inches relative to the base.

Although these large elastic and plastic deformations are not in themselves dangerous consideration of damage to finishes, partitions and cladding and also of probable panic by occupants indicates that it would not be desirable to relax the code limitations on inter-storey deflection or to reduce the design seismic coefficients for long period structures.

ACKNOWLEDGEMENTS.

The authors record with gratitude the interest shown in this investigation by Mr. D. Bruce-Smith, Consulting Engineer of Wellington, New Zealand who made available details of preliminary design which was used in selecting the properties of the frame analysed. The final design of the building in fact differs in many significant aspects from the preliminary proposals.

The staff of the University of Canterbury Computer Centre assisted the project by providing card punching and other services. The punched card accelerograms of the 1940 El Centro Earthquake were provided by Professor G.V. Berg of the University of Michigan.

The work described in this paper was carried out by the first author as part of his studies for his Ph.D. degree, under the supervision of the second author.

The first author was supported throughout his graduate studies by a scholarship awarded by the New Zealand University Grants Committee.

REFERENCES.

(1) - - - - - N.Z.S.S. 1900, chapter 8, Basic Design Loads, New Zealand Standards Institute, Wellington, 1965.

(2) Veletsos, A.S. and Newmark, N.M. Effect of Inelastic Behaviour on the Response of Simple Systems to Earthquake Motion; Proc. 2nd. W.C.E.E., Tokyo, 1960, p. 895.

(3) Wilson E.L. & Clough R.W. Dynamic Response by Step-by-Step Matrix Analysis; Simposium on the Use of Computers in Civil Engineering, Lisbon, Portugal, October 1962.

(4) Clough, R.W., Benuska K.L. & Wilson, E.L. — Inelastic Earthquake Response of Tall Buildings; Proc. 3rd W.C.E.E., New Zealand, Vol. II, 1965, p. 68.

(5) Housner, G.W. — Intensity of Earthquake Ground Shaking near the Causative Fault. Proc. 3rd W.C.E.E., New Zealand, Vol. III, 1965, p. 94.

(6) Walpole W.R. — The Response of Structures to Earthquake Loading. Unpublished Ph.D. Thesis, University of Canterbury, Christchurch, New Zealand. 1968.

TABLE 1. continued

Mode 3.

Frequency = 1.75 c.p.s. Period = 0.572 secs.

Amplification Factor = 1.13

Floor	Displacement Ratios	1g Displacements inches	1g Shears Kips
14	1.000	.734	175.4
13	.312	.229	204.3
12	-.497	-.365	158.3
11	-1.117	-.820	54.9
10	-1.335	-.980	-68.7
9	-1.114	-.817	-171.7
8	-.574	-.421	-224.9
7	.059	.043	-219.4
6	.614	.450	-162.6
5	1.007	.739	-69.4
4	1.169	.858	38.8
3	1.096	.804	140.2
2	.772	.567	211.7
1	.353	.259	244.3

PREDICTED ELASTIC RESPONSE

Floor	Displacement inches	Shears Kips	Force Kips
14	15.95	265	264.6
13	15.34	366	101.7
12	14.44	430	63.1
11	13.36	472	42.6
10	12.18	515	42.4
9	10.95	561	46.2
8	9.65	602	41.7
7	8.39	636	33.6
6	7.19	666	30.2
5	5.96	699	33.3
4	4.80	739	39.7
3	3.67	781	42.3
2	2.23	814	32.1
1	.93	829	15.2

TABLE 1.

NORMAL MODE PROPERTIES.

Mode 1.

Frequency = 0.362 c.p.s. Period = 2.765 secs.

Amplification Factor = 0.23

Floor	Displacement Ratios	1g Displacements inches	1g Shears Kips
14	1.000	99.39	1016
13	.965	95.93	1534
12	.912	90.65	2023
11	.844	83.89	2476
10	.768	76.29	2888
9	.686	68.20	3256
8	.599	59.56	3577
7	.515	51.18	3853
6	.435	43.22	4087
5	.355	35.28	4277
4	.281	27.93	4428
3	.212	21.09	4542
2	.127	12.65	4610
1	.053	5.26	4638

Mode 2.

Frequency = 0.998 c.p.s. Period = 1.001 secs.

Amplification Factor = 0.64

Floor	Displacement Ratios	1g Displacements inches	1g Shears Kips
14	1.000	-4.68	-365
13	.749	-3.50	-509
12	.395	-1.85	-585
11	-.002	.01	-584
10	-.371	1.73	-513
9	-.666	3.11	-385
8	-.878	4.11	-216
7	-.989	4.62	-258
6	-1.003	4.69	167
5	-.938	4.39	347
4	-.818	3.83	505
3	-.657	3.07	631
2	-.413	1.93	711
1	-.177	.83	745

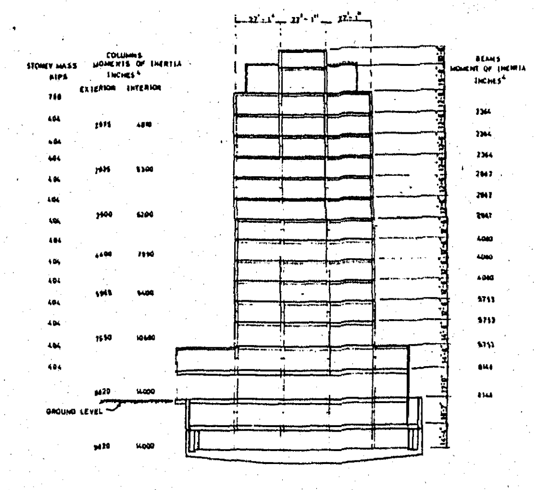

FIGURE 1

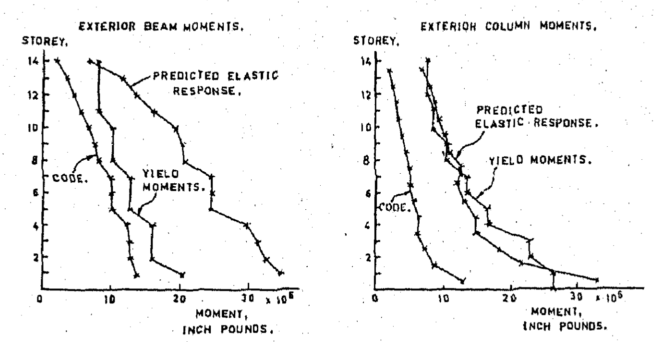

FIGURE 2(c).

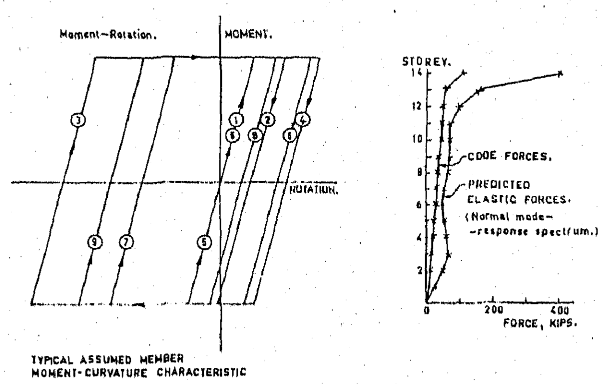

FIGURE 2(a).

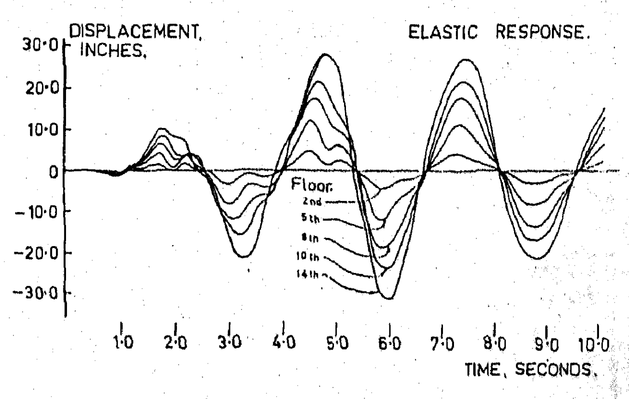

FIGURE 3.

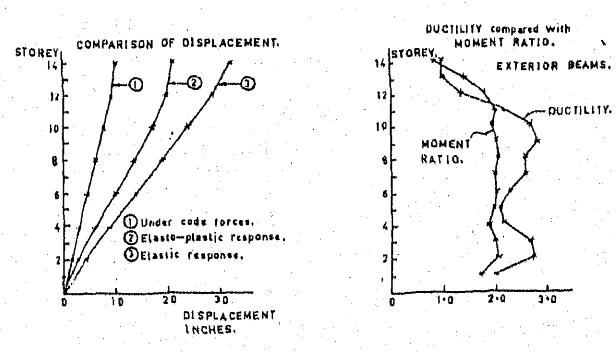

FIGURE 2(b).

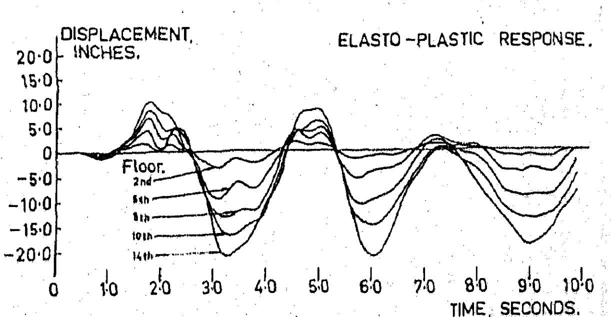

FIGURE 4.

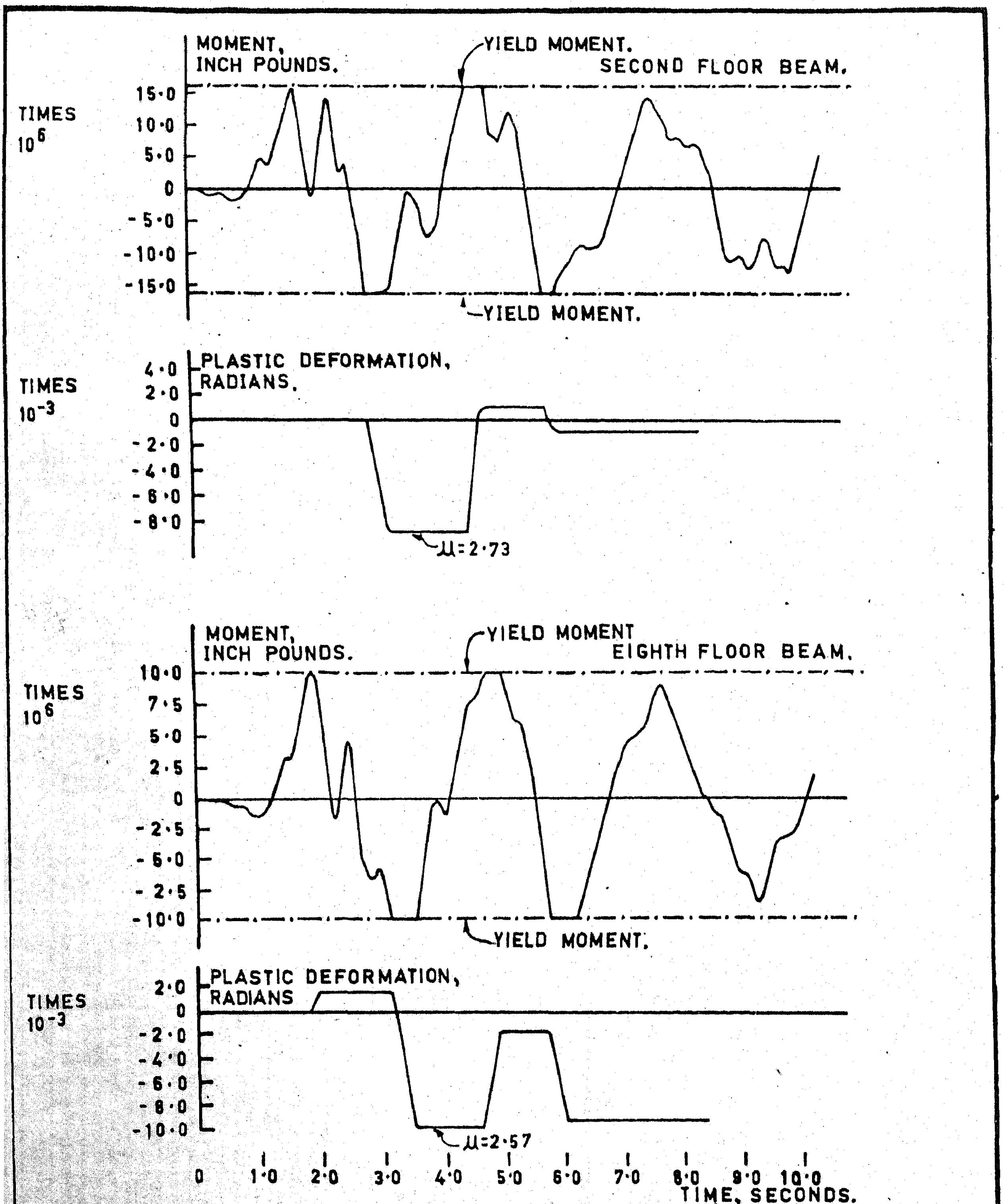
MOMENT, INCH POUNDS.
YIELD MOMENT.
SECOND FLOOR BEAM.
TIMES 10^6
15·0
10·0
5·0
0
-5·0
-10·0
-15·0
YIELD MOMENT.
PLASTIC DEFORMATION, RADIANS.
TIMES 10^{-3}
4·0
2·0
0
-2·0
-4·0
-6·0
-8·0
μ=2·73
MOMENT, INCH POUNDS.
YIELD MOMENT
EIGHTH FLOOR BEAM.
TIMES 10^6
10·0
7·5
5·0
2·5
0
-2·5
-5·0
-2·5
-10·0
YIELD MOMENT.
PLASTIC DEFORMATION, RADIANS
TIMES 10^{-3}
2·0
0
-2·0
-4·0
-6·0
-8·0
-10·0
μ=2·57
0 1·0 2·0 3·0 4·0 5·0 6·0 7·0 8·0 9·0 10·0
TIME, SECONDS.
W.M.

FIGURE 5.