MAXIMUM DEFORMATIONS OF CERTAIN NONLINEAR SYSTEMS

by

A.S. Veletsos[(I)]

ABSTRACT

This paper summarizes the results of a digital computer investigation into the response to ground motions of several types of nonlinear systems having a single degree of freedom, and of elastoplastic systems of the shear-beam type having two and three degrees of freedom. The single-degree-of-freedom (SDF) systems considered include (a) conventional elastoplastic systems, (b) bilinear hysteretic systems of the softening type with a positive second slope, (c) elastoplastic systems of the slip type which approximate the behavior of simple braced frames, (d) bilinear elastic systems, and (e) elastoplastic-elastic systems for which the behavior is linear in one direction of deformation and elastoplastic in the other. Both pulse-type excitations and an earthquake record are used as input motions.

A discussion is given of the effects and relative importance of the various factors which influence the response of these systems, and, for certain cases, simple approximate rules are presented for relating the absolute maximum deformation of a nonlinear system to the corresponding deformation of an associated linear system. The following general conclusions are deduced.

1. There is a simple and direct relationship between the response of systems subjected to earthquakes and to simpler pulse-type excitations.

2. For SDF elastoplastic systems of both the conventional and slip types, the yield level required to limit the maximum deformation of an inelastic system to a prescribed ductility factor may be related simply to the maximum deformation of an associated linear system. For design purposes, this relationship may be considered to be a function of only the natural frequency of the system.

3. Within the low-frequency region of the spectrum (defined in the paper), the maximum deformation for each of the nonlinear systems investigated, with the exception of systems of type (e), may be considered to be equal to the maximum deformation produced in the associated linear system by the same excitation.

4. Within the medium-and high-frequency spectral regions, on the other hand, the maximum deformations for several of the nonlinear systems studied may be significantly different from the corresponding deformation of the associated linear system.

5. For elastoplastic systems of type (e) which can yield in only one direction of deformation, the maximum deformations are generally significantly greater than those for the associated linear systems and the conventional elastoplastic systems over the entire frequency range of the spectrum, including the low-frequency range.

6. For the class of two-and three-degree-of-freedom systems investigated, the relationship between the absolute maximum deformations for an inelastic system and for the associated linear system is for all practical purposes the same as that for a SDF system having the same frequency value and subjected to the same excitation.

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SYNOPSIS

A summary is presented of the results of a digital computer investigation into the response to ground motions of five different types of single-degree-of-freedom nonlinear systems, both hysteretic and elastic, and of elastoplastic systems of the shear-beam type having two and three degrees of freedom. The input motions considered include both pulse-like excitations and an earthquake record. The effects and relative importance of the various factors which influence the response of these systems are discussed, and, for certain cases, simple approximate rules are presented for relating the absolute maximum deformation of a nonlinear system to the corresponding deformation of an associated linear system.

INTRODUCTION

The study reported herein is part of a broader investigation into the dynamic response of nonlinear systems, the objectives of which are: (a) to provide improved understanding of the response of such systems to transient ground excitations, and (b) to develop information and simple concepts which may be used to estimate the significant aspects of the response of a nonlinear system from the known response of an associated linear system. The approach used consists in starting with the simplest systems and simplest input motions, and then proceeding gradually to more involved systems and excitations. Both pulse-like excitations and earthquake records are considered. An earlier phase of this investigation was reported in Refs. (1)-(3).

The present paper, based mainly on Refs. (4)-(7), summarizes some of the results obtained for several hysteretic and nonlinear elastic single-degree-of-freedom systems, and for elastoplastic systems having two and three degrees of freedom. The absolute maximum deformation for each of these systems is evaluated for prescribed ground motions for a range of natural frequencies and degrees of nonlinear action, and the results are compared with the corresponding deformations obtained for associated linear systems when subjected to the same input motions. All data were obtained on a digital computer by numerical integration of the equations of motion.

SYSTEMS AND GROUND MOTIONS

The systems considered in this study are the conventional single-degree-of freedom(SDF), mass-spring-dashpot oscillator, and systems of the shear-beam type having two and three degrees of freedom. For the systems with more than a single degree of freedom, the resistance-deformation relationship for each spring is assumed to be of the conventional elastoplastic type, as shown in part (a) of Fig. 1, whereas for the SDF systems four additional relationships are used. These are the conventional bilinear hysteretic relationship of the softening type with a non-zero positive second slope, the slip-type elastoplastic relationship shown in part

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(b) of the figure, the bilinear elastic relationship shown in part (c), and the elastoplastic-elastic relationship shown in part (d) for which the behavior is linear in one direction of deformation and elastoplastic in the other. A possible cycle of deformation for each diagram is shown by arrows or by consecutively numbered points on the figure.

Diagram (b), which appears to have been first considered in Ref. (8), approximates the behavior of simple frames braced with elastoplastic diagonals, whereas diagram (d) represents a limiting form of an elastoplastic system having unequal yield levels in the two directions of deformation. For exciting motions which coincide with the direction of gravity, the inequality in yield levels may be due to the effect of the dead weight of the system.

The yield levels of the systems in diagrams (a) and (b), and the coordinates of the points in diagram (c) where the slope is discontinuous, are assumed to be the same in the two directions of deformation. Unloading and reloading from regions of inelastic deformation for diagrams (a) and (d) are assumed to take place along paths parallel to the initial elastic portion of the diagram. The same is also assumed to be true of the initial portions of the unloading and reloading paths for diagram (b).

The ground motions considered include a half-cycle velocity pulse, a half-cycle displacement pulse, and the first 6.29 secs. of the N-S component of the El Centro California earthquake record of May 18, 1940. Only limited data are presented for the half-cycle velocity input. The displacement, velocity, and acceleration diagrams of these motions are shown in Figs. 2 and 3, starting from top to bottom. The absolute maximum values of these functions, without regard for signs, are denoted by y_o, $\dot{y}_o$, and $\ddot{y}_o$, respectively.

Associated Linear System and Definition of Terms. For each of the systems and ground motions referred to above, it is desired to evaluate the absolute maximum deformation of the system, and to compare this deformation to the corresponding deformation produced by the same excitation in an associated linear system. The latter system is considered to have the same stiffness as the initial stiffness of the nonlinear system.

The resistance diagrams of the two systems are compared in Fig. 1e, in which the symbols u_m and Q_m denote the numerical values of absolute maximum deformation and absolute maximum spring force of the nonlinear system, and u_o and Q_o denote the corresponding quantities for the associated linear system. For systems having more than one degree of freedom, these symbols define the absolute maximum values of the quantities for any of the springs. Where no confusion can arise, the symbol U will be used to denote u_m and u_o collectively. The yield point deformation for the hysteretic systems is denoted by u_y, and the corresponding force by Q_y. The same symbols also define the coordinates of the point in Fig. 1c where the slope of the diagram changes. For the first three diagrams in Fig. 1, $Q_m = Q_y$, whereas for the bilinear hysteretic relationship with a nonzero second slope and for the diagram in Fig. 1d, Q_m may be greater than Q_y.

In what follows, the "yield" level of a nonlinear system will be expressed in terms of the dimensionless yield factor, c, defined as

$$c = Q_y/Q_o = u_y/u_o.$$

For a linear system, c is unity, since in this case u_y and Q_y may be considered to be equal to u_o and Q_o, respectively. A value of, say, c = 0.25 represents a yield level which is one-fourth of that required for linear behavior, or a ground motion which is four times as intense as that which would cause the system be at the verge of yielding.

SINGLE-DEGREE-OF-FREEDOM SYSTEMS

Conventional Elastoplastic Systems. In Fig. 4 are given spectra of absolute maximum deformation for linear and conventional elastoplastic systems without damping subjected to the half-cycle displacement pulse. The results are presented in a dimensionless form for fixed values of the yield factor, c. The symbol f on the abscissa represents the natural frequency of vibration of the system, computed from the stiffness of the initial elastic portion of the resistance diagram. Inasmuch as the values of this frequency for an inelastic system and for the associated linear system are the same, the ratio of the ordinates of the curves for inelastic and linear systems for a given frequency value represents the desired ratio of maximum deformations u_m/u_o. Similar spectra are presented in Fig. 5 for the El Centro record for systems with 2 percent critical damping, $\beta = 0.02$. The latter value is based on the small-amplitude natural frequency of the system, f.

For a given input motion, the relationship between u_m and u_o depends on the frequency parameter of the system and on the yield factor c. In the left-hand region of the spectrum, corresponding to systems with low natural frequencies, the two deformations are for all practical purposes equal. This result, which is independent of the characteristics of the input motion and the yield level of the system, is to be expected. By virtue of its low stiffness, the system in this case acts as a displacement meter, and irrespective of its yield level, it experiences a maximum deformation equal to the maximum ground displacement. Moving from left to right, this region is followed by a range within which u_m is in general smaller than u_o, and a range where u_m is again approximately equal to u_o. The effect of the yield factor, c, while not entirely negligible in these ranges, is generally small, and u_m is affected irregularly by variations in c. Finally, in the right-hand, high-frequency spectral region, u_m is greater than u_o, and the relationship between u_m and u_o is extremely sensitive to variations in both the frequency parameter and the yield factor, particularly at the higher frequency values. For systems in this region, even a slight lowering of the yield level of the system below that required for elastic response may produce large inelastic components of deformation. For reasons given later, the ratio u_m/u_o is particularly large in Fig. 4.

A qualitative explanation for these general trends may be provided by noting that an inelastic system differs from the associated linear system in two respects: (a) it dissipates energy by hysteretic action, and (b) it is a softer system, with a lower apparent "natural frequency" of vibration. The first factor tends to decrease the resulting value of maximum deformation, whereas the second may either decrease it or increase it, depending on the value of the frequency parameter. The relative importance of these factors is discussed in the following.

It is sometimes thought that the overall effect of inelastic action may be expressed by the addition of viscous damping to the associated linear system. When one recalls that the addition of viscous damping tends to decrease the maximum deformation of a linear system for the entire range of natural frequencies, one concludes that while this approach is capable of explaining that the maximum deformation for a hysteretic system is equal to or less than that for the associated linear system in the low-frequency and the medium-frequency spectral regions, it cannot by itself account for the fact that the inverse is true in the high-frequency region. The application of this concept to high-frequency systems leads to large negative values of equivalent viscous damping which cannot, of course, be justified physically. The boundaries of the low-, medium-, and high-frequency spectral regions are defined later.

The major trends in Figs. 4 and 5 can be attributed mainly to the effect of reduced apparent frequency of vibration for the yielding system. Examination of the spectra for elastic systems in these figures reveals that a reduction in the natural vibrational frequency of a system would have practically no effect on the response of systems in the left-hand region of the spectrum, would tend to reduce the response in the middle region where the elastic spectrum attains its larger values, and would tend to increase the response in the right-hand region. Furthermore, the fact that in the left-hand and middle regions the elastic spectra are relatively insensitive to changes in frequency suggests that the relationship between u_m and u_o in these regions would also be insensitive to changes in the yield factor c. On the other hand, in the right-hand region of the figure where the elastic spectra are sensitive to frequency changes, the relationship between u_m and u_o should likewise be sensitive to variations in c.

On the same basis, one would expect that as the yield factor of a system in the high-frequency region is decreased below one, the value of u_m would increase towards the peak of the elastic spectrum (about $1.6 y_o$ for the half-cycle displacement input) and, as c tends to zero, it will approach the limiting value of $u_m = y_o$. The peak value of the elastic spectrum may not be attained, however, because of the effect of hysteretic energy dissipation which tends to limit the maximum deformation to a value smaller than that applicable to an undamped linear system. These deductions are in agreement with the observed trends. It is of interest to note, in particular, that the level of the spectrum for c = 0.3 in the high frequency region of Fig. 4 is higher than that for c = 0.1. Similar results have also been obtained for the earthquake record for smaller values of the yield factor than those used in Fig. 5.

Within corresponding regions of the spectra, the relationship between u_m and u_o for both the El Centro input and the simple input have been seen to be similar. The boundaries of these regions may be determined by plotting the spectra for the linear systems on a log-log scale, as shown in Fig. 6, with the abscissa again representing the frequency parameter, and the ordinate representing the pseudo-velocity of the system $V = pu_o$, where p is the circular natural frequency. The 45^o diagonal scales on the left and right represent constant values of u_o and of the pseudo-acceleration $A = p^2 u_o$. In this particular figure, the scales for u_o, V and A have been normalized with respect to y_o, $\dot{y}_o$ and $\ddot{y}_o$, respectively.

When plotted in this form, the central portion of the spectrum may be approximated by a horizontal line and two 45^o diagonal lines as shown by the dashed lines. The region to the left of the intersection point b is defined as the "low-frequency" region of the spectrum, the region between points b and c is defined as the "medium-frequency" region, and the portion to the right of point c is defined as the "high-frequency" region. It is sometimes desirable to subdivide the low-frequency region into an "extremely low-frequency" subregion for which u_o is equal to or less than y_o, and a "moderately low-frequency" subregion where u_o is greater than y_o. The high-frequency region is similarly subdivided into a "moderately high-frequency" subregion for which the pseudo-acceleration A is greater than $\ddot{y}_o$, and an "extremely high-frequency" subregion for which A is for all practical purposes equal to $\ddot{y}_o$. The boundaries of these subregions are identified in Fig. 6 by the points a and d. The frequency values for these points and for points b and c are summarized in the following for both the half-cycle displacement pulse and the El Centro input:

	a	b	c	d
Values of ft_1 for simple pulse	0.20	0.55	0.76	5
Values of f, in cps, for El Centro record	0.10	0.38	2.1	25

It should be noted that the medium-frequency region for the El Centro input is considerably broader than for the simple input, and that Fig. 5 does not go as far into the high-frequency region as does Fig. 4. It is mainly for the latter reason that the ratio u_m/u_o in the extreme right-hand region of Fig. 4 is substantially greater than in the corresponding region of Fig. 5.

The relationship between u_m and u_o for each of these frequency regions may be considered to be independent of the characteristics of the input motion, and may be summarized approximately as follows. In the extremely low-frequency region of the spectrum, $u_m = u_o = y_o$; in the moderately low-frequency region, u_m is less than u_o; and in the medium-frequency region, u_m is generally less than or approximately equal to u_o. Note that these relationships are considered to be independent of the yield factor. For design purposes, they may further be simplified by taking $u_m = u_o$ throughout the low-and medium-frequency regions.

In the high-frequency region, u_m cannot be related simply to u_o, as the ratio of these quantities is sensitive to variations in both the yield factor and the frequency parameter of the system. For design purposes, however, instead of the value of u_m corresponding to a given yield resistance, one normally needs to know the yield resistance required to limit u_m to a prescribed value. The latter quantity, which is effectively the inverse of the former, is insensitive to changes in the amount of inelastic deformation that may be tolerated, and can be related simply to the maximum response of the associated linear system.

To this end, it is desirable to display the results on a logarithmic plot of the type shown in Fig. 7, in which the diagonal scale on the left represents the yield deformation of the system, u_y, instead of its maximum deformation, and each curve refers to a fixed value of the ductility factor, $\mu = u_m/u_y$. For linear systems, $\mu = 1$. The quantities V_y and A_y in this plot are alternative measures of yield deformation, defined as

$$V_y = p u_y \quad \text{and} \quad A_y = p^2 u_y$$

where p for a nonlinear system refers to its small-amplitude circular natural frequency. The resulting spectra then give the value of the yield deformation or yield resistance which, for the given input, is required to limit the maximum deformation of the system to a specified multiple of its yield deformation. The particular data given in Fig. 7 are for the El Centro input.

When plotted in this form, the spectra for inelastic and elastic systems may be related as follows. In the low-frequency and medium-frequency regions, the spectral ordinates for the inelastic systems may be taken as $1/\mu$ times those for linear systems. This is equivalent to taking $u_m = u_o$. In the extremely high-frequency region (i. e. for frequency values greater than that for point d in the tabulation given previously), the curves for the inelastic and elastic systems may be considered to coincide, and the spectra may then be completed by drawing smooth transition curves between the limiting curve on the right and the curves established for the medium-frequency region. These approximate design rules are a simplified version of those proposed in Ref.(3).

Conventional Bilinear Hysteretic Systems. From the discussion of the data presented in the preceding section, one would expect that the maximum deformation of a bilinear system with a nonzero positive second slope can differ appreciably from the maximum deformations of the associated elastoplastic and linear systems only in the high-frequency region of the spectrum. That this is indeed so has been verified by numerical computations.

Representative results are given in Fig. 8 for undamped systems with c = 0.3 subjected to the half-cycle displacement excitation. It can generally be stated that, within the low-frequency and medium-frequency spectral regions, the relationship between u_m and u_o is for all practical purposes independent of the ratio of stiffnesses of the resistance diagram, k_2/k_1. In the high-frequency region, on the other hand, even a slight increase in k_2/k_1 above zero may cause a significant decrease in the resulting value of u_m.

As the yield factor of the bilinear hysteretic system is lowered toward zero, the maximum deformation of the system tends to the value obtained for a linear system having a stiffness k_2. The corresponding natural frequency, f_2, is equal to $\sqrt{k_2/k_1}$ times the small-amplitude natural frequency of the hysteretic system. Even for the value of c = 0.3 considered in Fig. 8, it is noteworthy that the high-frequency portion of the spectrum for the systems with $k_2 = 0.1\, k_1$ is separated from the linear spectrum by an amount which can be accounted for largely by a change in frequency from f to $f_2 = \sqrt{0.1}\, f$.

Slip-Type Elastoplastic Systems. In Figs. 9 and 10 some of the spectra for conventional elastoplastic systems presented in Figs. 4 and 5 are compared with the corresponding spectra for slip-type systems. It can be seen from these plots that the maximum deformation for the slip model is generally greater than or equal to that for the conventional model, and that the relationship between the two deformations is a function of both the frequency parameter and the yield factor, c. For the limiting values of c = 1 and c = 0, the spectra for the two models coincide and, as would be expected, the larger differences in response are obtained for the intermediate values of c and for the larger values of the frequency parameter.

These differences are due mainly to the fact that the slip model can dissipate less energy by hysteretic action than the conventional model. Examination of the time histories of deformation reveals that there is also a slight reduction in the apparent frequency of vibration of the slip model as compared to that of the conventional model. Representative plots are given in Fig. 11 for systems with f = 0.5 cps and β = 0.02 subjected to the El Centro input. The yield factor for the hysteretic systems is taken as c = 0.5. The bar diagrams in the lower part of this figure define the time intervals during which the deformations of the hysteretic systems are in the inelastic range, i.e. the range within which the Q-u relationship is governed by the horizontal segments of the resistance diagram.

The effect of reduced hysteretic energy dissipation is most pronounced in the high-frequency region of the spectrum, because an inelastic system in this region undergoes a greater number of oscillations before attaining its peak response than does such a system in the lower frequency regions. In fact, in the extremely low-frequency region, the absolute maximum deformation for each of the two models considered corresponds to the first maximum, and no difference in response can be expected in this case. One accustomed to thinking of the effect of hysteretic energy dissipation in terms of an equivalent amount of viscous damping may find this explanation to be incompatible with the known fact that viscous damping has a negligible effect on the response of high-frequency linear systems. (It is assumed that the input acceleration has no discontinuities. See for example Refs. 2 and 3). This apparent inconsistency is eliminated, however, when one realizes that the hysteretic models are "equivalent" to lower-frequency linear systems for which the effect of viscous damping is generally substantial.

Another manifestation of the decreased importance of hysteretic energy dissipation for the slip model is provided in Ref. (7), in which spectra of maximum deformation for the half-cycle displacement input are given for several additional values of c to those considered in Fig. 9. These data show that the peak level of deformation for systems in the high-frequency region is of the order of about $1.5 y_o$, a value which is only slightly lower than the value of $1.6 y_o$ obtained from the peak of the spectrum for linear systems.

The relationship between the maximum deformations for the slip and conventional elastoplastic models and for the associated linear model may be summarized approximately as follows. In the low-frequency spectral region, u_m for all three models is for all practical purposes equal to the maximum ground displacement, y_o.

In the medium-frequency region, the ratio of u_m for the slip model to that for the conventional model is found to vary irregularly with variations in either frequency or yield factor, but there is a definite trend for the ratio to increase with increasing frequency. Pending the results of additional studies, it is recommended that this ratio be considered to increase from one in the lower frequency limit of the region (point b in Fig. 6) to a maximum value of 2 in the upper frequency limit (point c in Fig. 6). The increase is considered to be linear on a logarithmic plot. This relationship, which is stated independently of the yield factor of the system, c, is intended to approximate the maximum values of the ratio. It should be remembered, however, that as c tends to either zero or one, the ratio of maximum deformations tends to unity.

In the high-frequency region of the spectrum, the values of u_m for the two hysteretic models cannot be related simply. On the other hand, the yield resistance required for a given ductility ratio may be estimated by a procedure analogous to that described earlier for conventional elastoplastic systems.

Bilinear Elastic Systems. The maximum deformations of the two types of elastoplastic systems considered in the preceding sections are compared in Figs. 12 and 13 with the results obtained for a bilinear elastic system having the same resistance-deformation relationship as the skeleton curve of the elastoplastic relationship. The data are for undamped systems with a value of c = 0.5.

It is important to note that, except for a few isolated cases which are of no practical consequence, the deformations for the elastic systems are greater than for any of the other systems, with the spectra for the slip systems falling between those for the remaining systems. The relative position of these curves is understandable when one recalls that the conventional elastoplastic model has the highest hysteretic energy dissipating capability whereas the elastic system has none. It should further be noted that the differences start to become of practical consequence in the medium-frequency spectral region, attaining their maximum values in the high-frequency region. For the half-cycle velocity input considered in Fig. 12a, the medium-frequency region extends from 0.48 to 1.1. The general trend of these results could have been anticipated from the data and the discussion presented in the preceding sections.

Further appreciation of the relationship between the absolute maximum deformation for a conventional elastoplastic system, U_h, and the corresponding deformation for the associated bilinear elastic system, U_e, may be gained from Fig. 14, in which the ratio U_e/U_h is plotted against the frequency parameter for fixed values of the "yield factor" c. These data are for undamped systems subjected to the half-cycle displacement input. The circles in this figure define the frequency values beyond which the ratio u_m/u_y for the bilinear elastic system, U_e/u_y, is greater than 10.

Elastoplastic-Elastic Systems. The deformation spectra for this type of system (Fig. 1d) have been found to differ significantly from the spectra for the hysteretic systems and the bilinear elastic systems considered in the preceding sections.

Representative data are given in Figs. 15 and 16 for systems with c = 0.2 subjected to the half-cycle displacement input and the El Centro earthquake input, respectively. The results for the systems yielding in one direction only are compared in each case with those for linear systems and for conventional elastoplastic systems. Two separate curves are presented for the elastoplastic-elastic systems, depending on whether the initial deformation of the system is along the elastic or the inelastic branch of the resistance diagram.

For the critical direction of initial deformation, it can be seen that the maximum deformation for a system that can yield in one direction only is significantly greater than the deformations for the associated linear system and for the conventional elastoplastic system. This is true over the entire frequency range of the spectrum, including the low-frequency region for which u_m is approximately equal to u_o for all the systems considered in the preceding sections. It can further be shown that the difference in response increases with decreasing value of the yield factor. This result should be contrasted with the fact that, within the low-and medium-frequency spectral regions, the maximum deformation for conventional elastoplastic systems is for all practical purposes independent of the yield factor.

Reference to parts(a) and (d) of Fig. 1 reveals that, whereas for an elastoplastic-elastic system, the instantaneous permanent set of the system (i.e. the value of the spring deformation associated with zero spring force) increases with each successive cycle of inelastic deformation, for a system with equal positive and negative yield levels, it may either increase or decrease depending on the relative magnitudes of the peak inelastic excursions along the increasing and decreasing directions of deformation. This difference in the Q-u relationship of the two systems is one of the important reasons for which the maximum deformation of the elastoplastic-elastic system is generally greater than that of the associated conventional elastoplastic system. For a further discussion of the factors responsible for the difference in the behavior of these two types of systems, the reader is referred to Ref. (4).

SYSTEMS WITH MORE THAN ONE DEGREE OF FREEDOM

The following discussion refers to conventional elastoplastic systems to the shear-beam type having two and three degrees of freedom. For each system, the masses and the yield deformations of the springs are considered to be equal, and the following assumptions are made concerning the initial stiffnesses of the springs: (a) all the stiffnesses are equal, and (b) the distribution of the stiffnesses is such that, under elastic conditions of response, all springs experience the same maximum value of deformation for the particular excitation considered. The first class of systems is referred to as "uniform" and the second as "elastically balanced".

The relative spring stiffnesses for an elastically balanced system are functions both of the characteristics of the system, as reflected in the value of its fundamental frequency of vibration, and of the characteristics of the input motion. These stiffnesses were determined by trial and error.

To provide a basis for the interpretation of the inelastic response of these systems, it is desirable to investigate first their linear response. In Fig. 17 are given spectra of maximum deformation for the individual springs of three-degree-of-freedom, uniform, linear systems without damping subjected to the half-cycle displacement input. These spectra are plotted on a four-way logarithmic plot in a manner analogous to that used previously in Fig. 6, with the ordinate representing the pseudo-velocity of the ith spring, V_i. The latter quantity is the product of the maximum deformation for the ith spring, $(u_i)_o$, multiplied by the fundamental circular natural frequency of the system, $p_1 = 2\pi f_1$, and $A_i = p_1 V_i$. For the base spring, i=1.

It is important to note in this figure that, except for a narrow range in the lower part of the moderately low-frequency region of the spectrum for which the maximum deformations of the three springs are about equal, the maximum deformation for the base spring is significantly greater than for any of the other springs.

The corresponding spectra for the inelastic systems are presented in Fig. 18, plotted in a form analogous to that used in Fig. 7 for SDF systems. The ductility factor in this case is defined as the ratio of u_m, the absolute maximum deformation for <u>any</u> of the three springs, divided by the yield deformation, u_y. The latter quantity is assumed to be the same for all the springs.

Several important trends are worthy of note in Fig. 18. First, in those regions of the spectrum where the maximum deformation for the base spring of an elastic system is substantially greater than for the remaining springs, the largest deformation for the associated elastoplastic system also occurs in the base spring. Of course, this result is not surprising, since as the yield level of the system is lowered and the base spring yields, the maximum value of the force that can be transmitted from the ground through the base spring to the upper part of the structure is decreased from Q_o, the value developed under elastic conditions of response, to $Q_y = cQ_o$. The response of the upper part of the structure is therefore reduced, and the base spring continues to be the controlling spring.

What was unexpected in Fig. 18 is that, in the region of the spectrum where the maximum deformations of the individual springs are of comparable magnitude for linear response, the largest deformation for the inelastic systems occurs in a spring other than the base spring for the lower values of ductility ratio. From an examination of the time histories of deformation, it has been found that, under elastic conditions of response, the absolute maximum deformation for one of the upper springs in these cases occurs prior to that for the base spring and that it is, in general, appreciably greater in magnitude than the earlier peak deformation(s) of the base spring. Consequently, yielding begins in an upper spring and, as might be expected, the same spring controls the response in the inelastic range of behavior as well. Only when the yield resistance of the system is lowered to a sufficiently low level so that yielding initiates in, and is maintained for some time in, the base spring does the effect of decreased force transmissibility discussed earlier begin to take over, and the base spring becomes the controlling spring.

The final and most important feature of the data displayed in Fig. 18 concerns the relationship between the spectra for the inelastic and the elastic systems (top curve). This is found to be very similar to the corresponding relationship for SDF systems subjected to the same input.

Similar results have also been obtained for elastically balanced systems, except that in this case a spring other than the base spring was found to be the controlling element for a wider range of the frequency parameter than for the uniform systems considered previously. In Fig. 19 are given the results obtained for two-degree-of-freedom, elastically balanced systems subjected to the El Centro input. In these solutions, a value of "interfloor" damping of 2 percent critical is assumed, based on the fundamental mode of vibration, i.e. $\beta_1 = 0.02$. The relationship between the spectra for the inelastic and linear systems in this figure is for all practical purposes the same as that for the SDF systems considered in Fig. 7.

On the basis of these data and additional data given in Ref. (5), it has been concluded that, for the class of multi-degree-of-freedom systems investigated, the absolute maximum deformations for an elastoplastic system and for the associate linear system bear approximately the same relationship as do the corresponding deformations for SDF systems having the same frequency value and subjected to the same excitation. For preliminary design purposes, therefore, the approximate rules formulated for SDF systems may also be used in this case.

It should be noted, however, that the relationship between u_m and u_o is not, strictly speaking, independent of the number of degrees of freedom of the system. This is illustrated in Fig. 20, which shows the ratio u_m/u_o for uniform systems with one, two, and three degrees of freedom subjected to the El Centro record. The damping factor β_1 in these solutions was taken as 0.02. It is clear from this figure that the number of degrees of freedom of the system has a small but distinct influence on the relationship between u_m and u_o, and that the approximate design rules which have been presented tend to lead to non-conservative results as the number of degrees of freedom increases. This is particulary true of the moderately low-and medium-frequency regions of the spectrum for which $u_m = u_o$ for SDF systems. Accordingly, care should be exercised in applying these relationships to systems having more than three degrees of freedom. Some data for such systems are available in Ref. (6).

CONCLUSIONS

1. There is a simple and direct relationship between the response of systems subjected to earthquakes and to simpler pulse-type excitations.

2. For single-degree-of-freedom(SDF) elastoplastic systems of both the conventional and slip types, the yield level required to limit the maximum deformation of an inelastic system to a prescribed ductility factor may be related simply to the maximum deformation of an associated linear system. For design purposes, this relationship may be considered to be a function of only the natural frequency of the system.

3. Within the low-frequency region of the spectrum, the maximum deformation for each of the nonlinear systems investigated, except for the elastoplastic-elastic systems, may be considered to be equal to the maximum deformation produced in the associated linear system by the same excitation.

4. Within the medium-and high-frequency spectral regions, on the other hand, the maximum deformations for most of the nonlinear systems studied are significantly different from the corresponding deformation of the associated linear system.

5. For elastoplastic-elastic systems, the maximum deformations are generally significantly greater than those for the associated linear systems and the conventional elastoplastic systems over the entire frequency range of the spectrum, including the low-frequency range.

6. For the class of two-and three-degree-of-freedom systems investigated, the relationship between the absolute maximum deformations for an inelastic system and for the associated linear system is for all practical purposes the same as that for a SDF system having the same frequency value and subjected to the same excitation.

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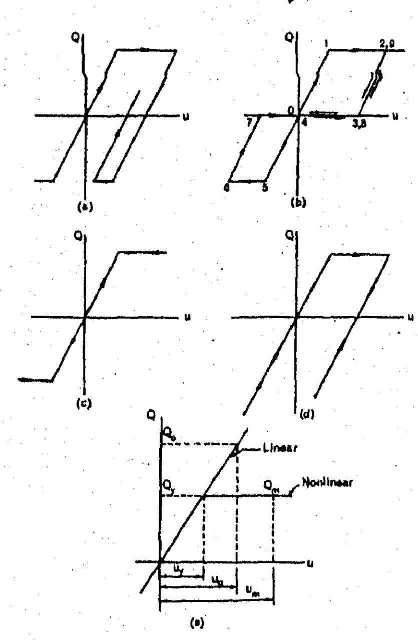

FIG.1 RESISTANCE-DEFORMATION DIAGRAMS

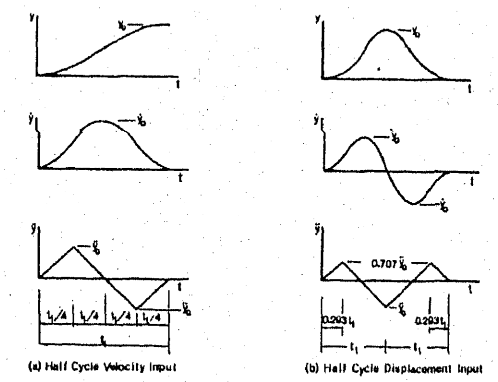

FIG.2 SIMPLE INPUT MOTIONS CONSIDERED

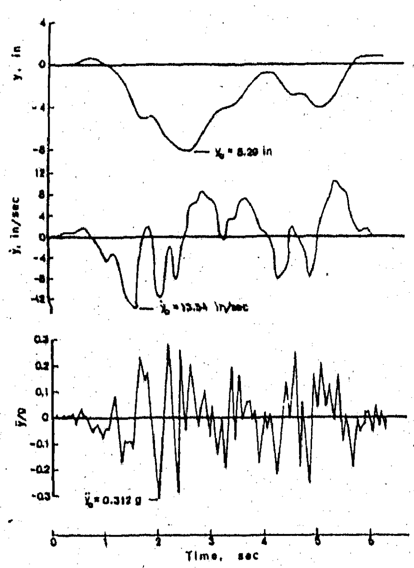

FIG. 3 EARTHQUAKE MOTION CONSIDERED

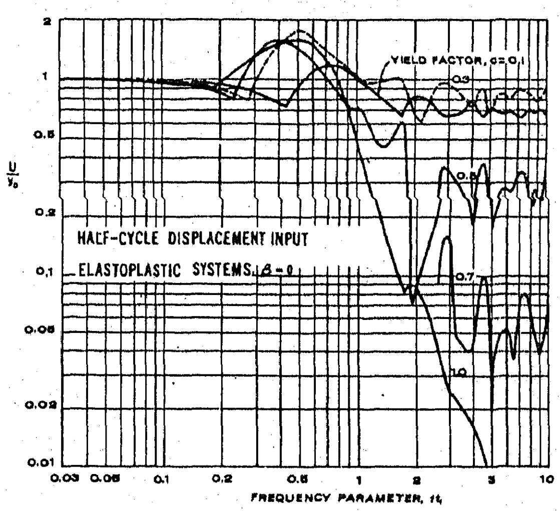

FIG.4 SPECTRA FOR CONVENTIONAL ELASTOPLASTIC SYSTEMS

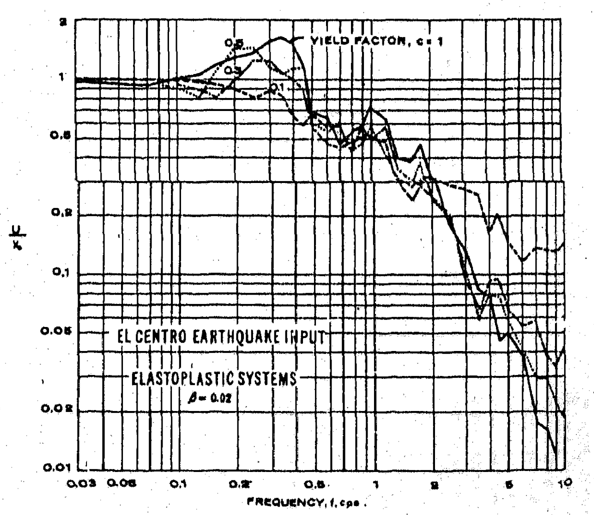

FIG.5 SPECTRA FOR CONVENTIONAL ELASTOPLASTIC SYSTEMS

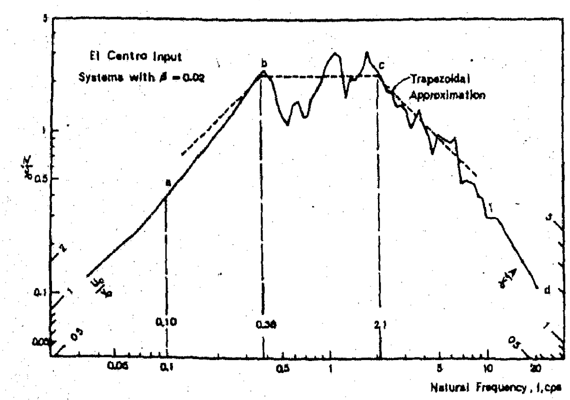

FIG.6 DEFINITION OF SPECTRAL FREQUENCY REGIONS

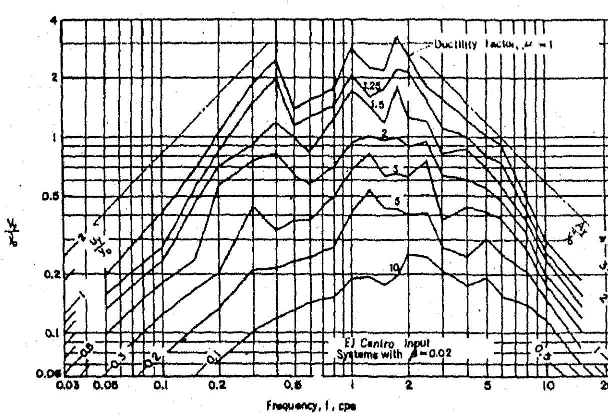

FIG. 7 SPECTRA FOR CONVENTIONAL ELASTOPLASTIC SYSTEMS

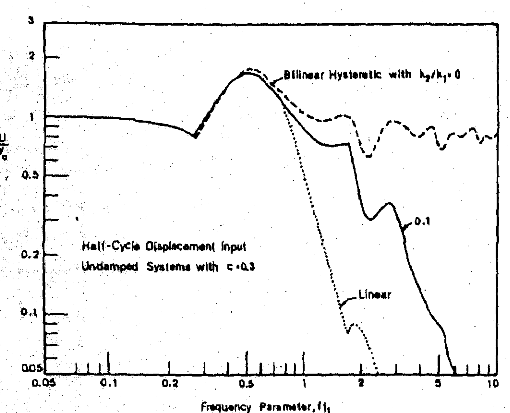

FIG. 8 SPECTRA FOR BILINEAR HYSTERETIC SYSTEMS

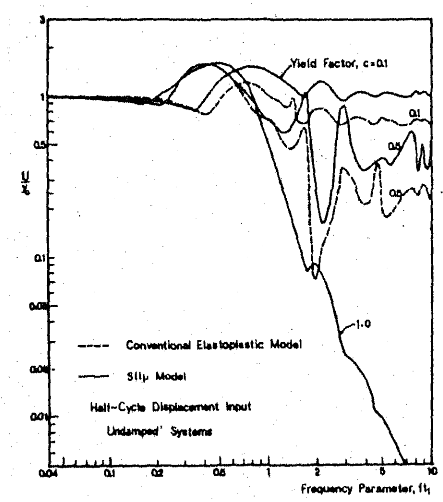

FIG.9 SPECTRA FOR SLIP AND CONVENTIONAL ELASTOPLASTIC SYSTEMS

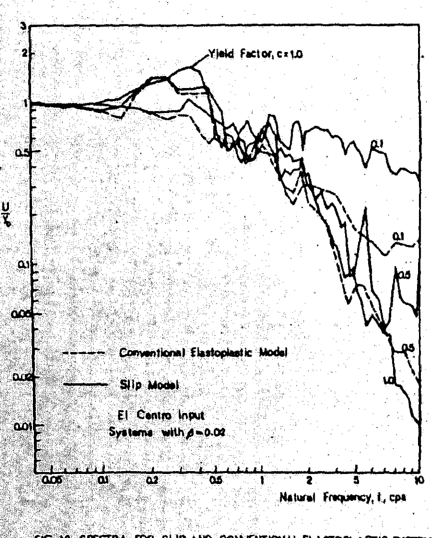

FIG. 10 SPECTRA FOR SLIP AND CONVENTIONAL ELASTOPLASTIC SYSTEMS

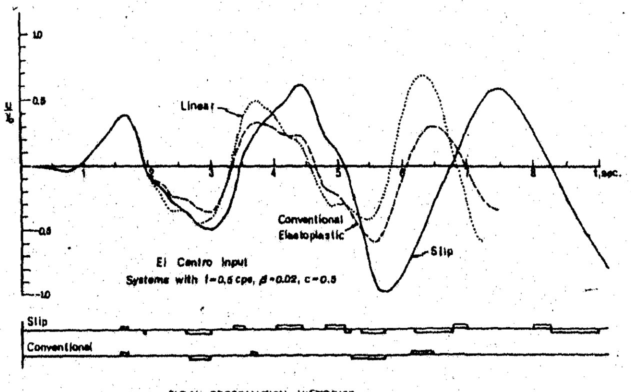

FIG.11 DEFORMATION HISTORIES

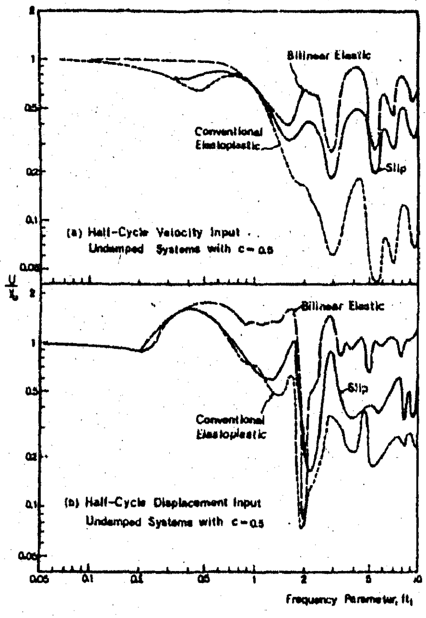

FIG.12 SPECTRA FOR BILINEAR ELASTIC AND HYSTERETIC SYSTEMS

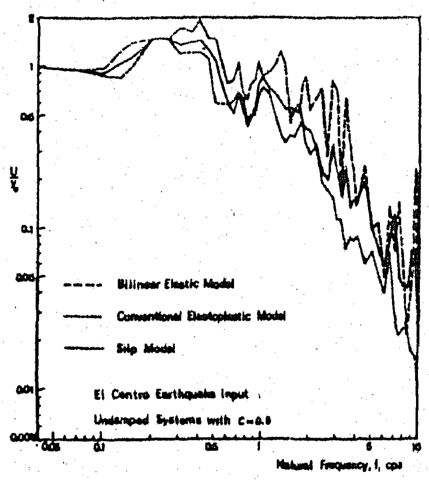

FIG. 13 SPECTRA FOR BILINEAR ELASTIC AND HYSTERETIC SYSTEMS

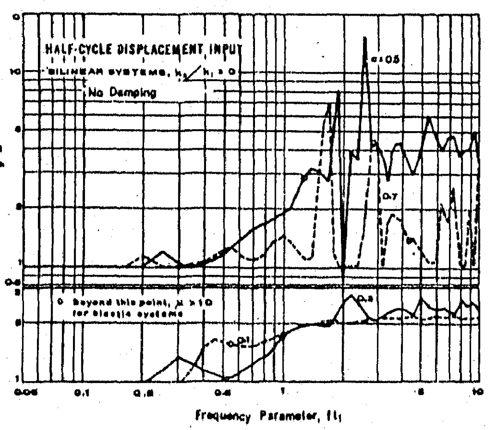

FIG.14 SPECTRA FOR BILINEAR ELASTIC AND HYSTERETIC SYSTEMS

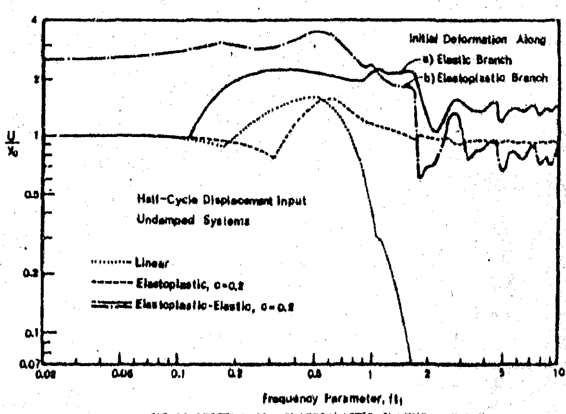

FIG. 15 SPECTRA FOR ELASTOPLASTIC-ELASTIC SYSTEMS

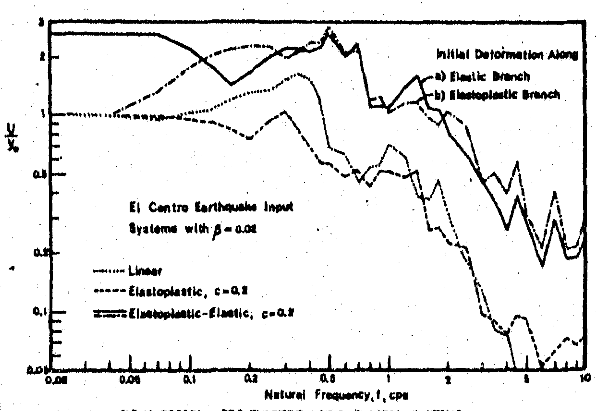

FIG. 16 SPECTRA FOR ELASTOPLASTIC-ELASTIC SYSTEMS

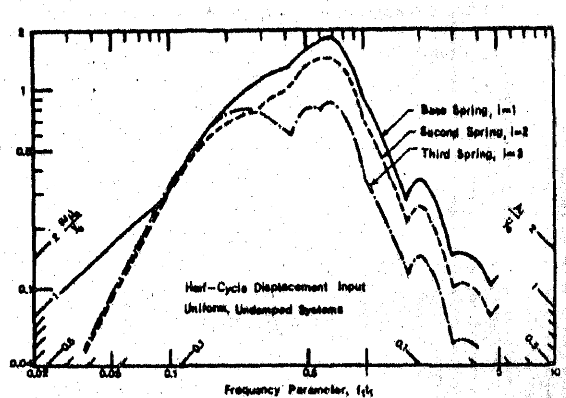

FIG. 17 SPECTRA FOR 3DF ELASTIC SYSTEMS

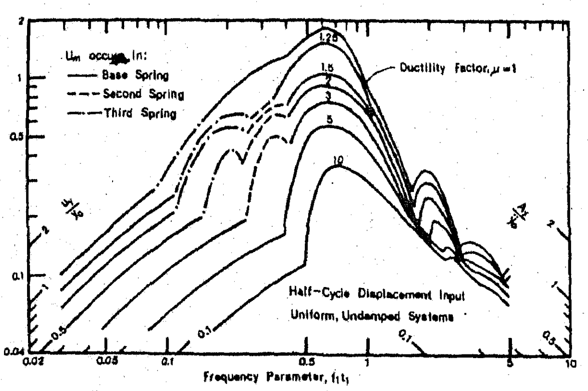

FIG. 18 SPECTRA FOR 3DF ELASTOPLASTIC SYSTEMS

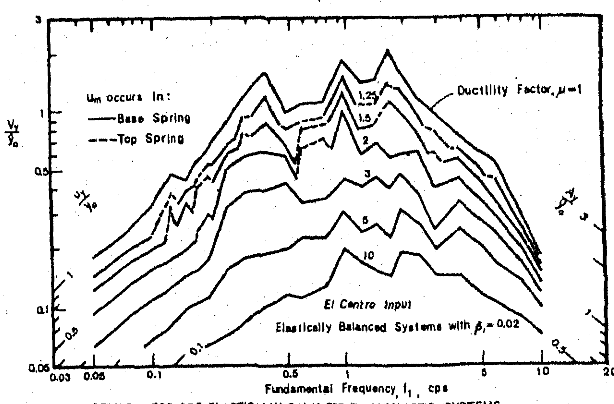

FIG. 19 SPECTRA FOR 2DF, ELASTICALLY BALANCED, ELASTOPLASTIC SYSTEMS

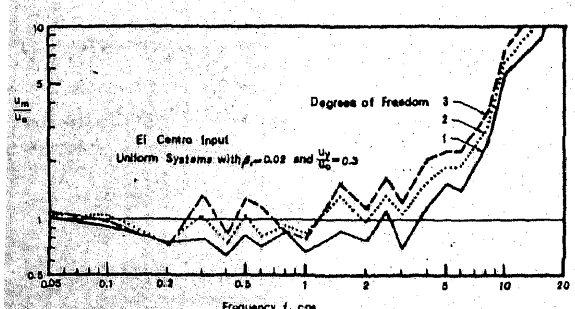

FIG. 20 SPECTRA FOR UNIFORM ELASTOPLASTIC SYSTEMS