THE EFFECT OF MINIMUM CROSS BRACING ON THE INELASTIC RESPONSE OF MULTI-STORY BUILDINGS

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SYNOPSIS

A step-wise numerical procedure is used to calculate the inelastic response of nine ten-story single-bay braced and unbraced steel frame structures. An elasto-plastic type of inelastic response was assumed for the cross braces and the girders, while the columns were assumed to remain elastic. The minimum cross braces with an L/r circa 300 were assumed to have no compression strength. Five of the structures were designed to satisfy period and parameter requirements, but the remaining four frames were conservatively designed using current code requirements. These structures were subjected to the first seven seconds of the N-S component of the El Centro 1940 earthquake in which the accelerogram ordinates were multiplied by 1.5. Together with a brief description of the method of design and response analysis, some of the results are presented which indicate the influence of the bracing placement and design procedure.

The following results appear significant:

1. The importance of including the P-Δ effect in the analysis was well documented by the results.

2. The higher modes of response are an important aspect of the apparent instability of frames with minimum tension bracing.

3. Minimum cross braces cause significant reductions in the ductility and energy absorption capacity requirements of the main frame members. However, they also impose higher strength demands upon the columns.

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INTRODUCTION

Although many investigations of the linear and nonlinear response of framed structures subjected to earthquakes have been made, relatively few studies of the response of braced frame structures have been reported. The majority of the braced frame studies have considered the elastic response (1,2,3) while a few have considered the post-elastic response (4,5). Increasing numbers of braced frame buildings are being constructed in seismically active zones throughout the world. Since most of these buildings in the United States are designed on the basis of building codes developed from experience with the earthquake response of moment resisting frames, it is felt that analytical information concerning the inelastic response of braced frames would be useful.

The concepts of structural ductility and energy absorption capacity necessary for a structure to survive a strong motion earthquake are important to adequate, economical design. This initial study attempts to make a small step in providing this information for braced frames. Minimum cross bracing normally would be used to control lateral displacements of the building under wind and weak earthquake excitation and to supply additional energy absorption capacity when subjected to a strong earthquake. The major advantage of initiating the braced frame investigation with minimum cross bracing is that a comparison of the response of a lightly braced structural frame to the response of an unbraced frame provides a basis for extrapolating our engineering intuition.

METHOD OF ANALYSIS

After listing the assumptions used to establish the mathematical model of the structures, a brief review of the numerical procedure used to determine the response will be given.

Assumptions

1. The actual structure can be represented as a single-bay multistory, rigid-jointed frame with the mass of the structure (equivalent to the dead weight of the finished building) concentrated at the floor levels.

2. The braces are tension members having an elasto-plastic type load-deformation relationship, Fig. 1, with no compression strength. The moment-end rotation relationship of the girders, Fig. 2, is also of the elasto-plastic type, but the columns behave elastically regardless of load.

3. In addition to the lateral displacement of the structure, two joint rotations and two vertical displacements are included at each floor. Thus, the number of degrees of freedom becomes five times the

number of stories[iii]. Only the lateral inertia force was considered - the rotatory and vertical inertia effects were assumed negligible.

4. One lateral deflection per floor implies that the length of the girders remains constant. The moment distribution along the girders and columns is linear and the braces are pin-ended.

5. Axial loads did not affect the stiffness of the columns and the stability of the members or structure was not considered.

6. The P-Δ effect of both the gravity loads and the dynamic axial loads in the columns were included. The gravity load P-Δ effect was included in the stiffness expression of the structure while the dynamic P-Δ effect was determined from the axial and lateral deformations of each column at each substep of the solution.

Procedure

At the beginning of the earthquake the frame is at rest, and all lateral forces equal zero. For a known value of ground acceleration and a selected time interval Δt, a set of relative displacements are calculated from the equation of motion by numerical integration. The lateral forces induced by these displacements are obtained from the force-displacement relationship of the structure. Using these lateral forces, displacements and velocities as new initial conditions, a set of new relative displacements corresponding to the next values of ground acceleration at $t + \Delta t$ can be obtained. This procedure is repeated. A fourth-order Runge-Kutta numerical procedure was used to perform this integration.

As previously mentioned the braces and the girders are assumed to have elasto-plastic force-deformation relationships. Whenever the behavior of these members changes from one phase to another, the stiffness matrix of the frame changes simultaneously. Whenever a member is loaded to its ultimate capacity, the member loses its incremental stiffness until a loading reversal occurs. Thus, the contribution of a member to the stiffness matrix is removed or restored whenever the behavior of this member changes from one phase to another. It is more convenient to work with these increments of displacement and loads than the actual quantities because this approach makes the stiffness matrix independent of the previous loading history.

The computer execution time depends on the number of stories and braces, the size of the partitioned stiffness matrix, the number of yield changes each of which requires a new stiffness matrix, and the length of the accelerogram. It was observed, Fig. 3, that over 75 percent of the total energy input to a structure occurred within the first seven seconds of the N-S component of the El Centro 1940 earthquake accelerogram and for economy this shortened accelerogram was used. The acceleration ordinates

[iii] After a number of response calculations it was observed that the dynamic joint rotations at a floor were nearly identical. Therefore, for the structures included herein the joint rotations at a floor were taken to be equal. This reduces the system to 4N degrees of freedom.

of this accelerogram were multiplied by 1.5 to provide a stronger earthquake than the ones recorded to date. The velocity response spectrum of this accelerogram, Fig. 4, is relatively independent of the period of the structure when compared with other accelerograms. This has certain advantages when interpreting dynamic response.

The accuracy of the method of analysis was initially established by static equilibrium of the deformed structure. While this check could be maintained for each step of the dynamic response computation, an energy check proved to be more versatile. The energy input to the structure at any instant can be computed by integrating the product of the base shear and the ground velocity. This must check with the sum of the recoverable strain energy and the kinetic energy present in the structure at that instant plus the total energy dissipated to that instant of time. The energy input by the dead weight of the structure was calculated separately and was excluded from both terms of the energy check and the reported energy input by the earthquake. These energy evaluations never differed by more than one percent except in certain instances which are noted herein.

PROGRAM OF INVESTIGATION

Series One. The basic program consists of comparing the response of three braced frames to the response of an unbraced frame. The pattern of the bracing layout and frame designations are given in Figs. 5 and 6. The structures of the first series have identical beam-column frames but have various distribution of brace location as shown. The minimum cross bracing system adopted in this analysis was designed so that the slenderness ratio of tension members was about 300. Thus, each brace has equal area and is treated as a tension member with zero compression strength. The floor mass of each frame is properly adjusted to maintain a fundamental period of about 1.25 seconds for each structure. This could be considered the same as having different amounts of floor area contributing its inertia forces to the one braced bay. That is, probably each bay would not be braced. The structural properties of the frames of first series are given as follows:

Effective Story Height	=	12.0 feet	=	144 inches
Bay Width	=	20.0 feet	=	240 inches
Area of Brace	=	2.88 square inches		

The additional structural properties of these frames are given in Tables I and II.

The results of subjecting series one structures to the N-S El Centro accelerogram are presented in terms of the following parameters.

(1) Maximum lateral floor displacement.

(2) Maximum relative lateral floor displacement.

(3) Girder ductility ratio, defined as the ratio of the maximum end-rotation to yield end-rotation θ_y, where $\theta_y = \frac{\sigma_y Z_x L}{6 E I_x}$.

(4) Energy dissipation per floor.

(5) Dynamic column moment ratio, defined as the ratio of the maximum end-moment caused by dynamic loading to the initial yield moment of each column.

(6) Maximum axial forces induced in columns by dynamic loading alone. $P_y = A \sigma_y$.

(7) Maximum energy input and dissipated during the first seven seconds of the accelerogram.

The first six quantities are presented in Fig. 7, the last one is given in Table III. From these results the following points should be noted.

(1) The presence of bracing tends to reduce the floor displacements and girder ductility ratios. The omission of bracing at certain floors may cause substantial relative displacements. This certainly induces large bending moments in the columns as can be observed in Fig. 7(e) where the column moment ratios of the fully braced frame are less than unity while that of the alternately braced frame exceed 1.3.

(2) Early yielding of the cross bracing results in low energy dissipation requirements for the girders. Energy dissipation per floor is more uniform in the fully braced frame than in other frames.

(3) The axial dynamic loads induced in the columns by the earthquake, especially in braced frames, are quite large. This has the effect of reducing the inelastic moment resistance capacity of the columns and may affect the stiffness of the column members.

(4) The maximum energy input to a structure by the earthquake is a function of the mass of the structure. It can be seen from Table III that the amount of energy input increases with increase in mass of the frame.

Series Two. In the second series, the floor mass for each of the frames was kept the same, the bracing systems were identical to those used in the structures of first series, and the column and beam frame systems were redesigned so that the fundamental periods of these structures were nearly equal to 1.25 seconds. The results of this analysis have been deleted so that Series Three may be discussed in some detail.

Series Three. The ten-story structures of series one and two were established to provide a fundamental period of vibration of about 1.25 seconds utilizing "reasonable" structural members and gravity loads.

However, in the present state-of-the-art the period of the structure is not a primary design factor. Primary design factors are the stresses or forces in the members and the lateral deflection of the structure under the combined dead, live, and earthquake or wind loads. The four ten-story structures of series three were designed using typical dead and live loads for multi-story office buildings and lateral earthquake forces similar to those specified by the Uniform Building Code 1967.

Two design criteria, member stress and a lateral deflection limit of 0.35 percent of the height were used. The lateral earthquake forces are based on a dead load per floor of 44 kips, C = 0.05 regardless of period, K = 1.0 regardless of the style of construction, and J = 1.0. This is an extremely crude but conservative estimate of the code lateral forces. The frames F and B were considered to carry three bays of lateral force and one bay of vertical load. The frame members were designed for 25 percent of this total lateral load while the complete system carried 100 percent of the load. Frames U and A carried one bay of lateral and vertical forces, Table IV. Column stresses were calculated using AISC Formula (7a) with $C_m = 0.85$ for combined dead, live and lateral load(iv). The live load was taken as 32 kips per floor with no reduction because of the small floor areas. Bracing areas were determined using an allowable stress of 22 ksi without utilizing the 33 percent increase. A minimum area of 2.88 in.^{2} was used.

The lateral deflections of the structures were found to be within the 0.35 percent limit without influencing the designs. The story heights and bay widths are the same as in series one while other properties and design values are given in Tables V through VIII. Even though column stresses as calculated from AISC Formula (7a) are larger than those which would be used in normal design practice, it is felt that these designs are reasonably conservative. No reduction in the live load and the use of C = 0.05, K = 1.0, and J = 1.0 all lead to an over-estimation of the design forces. For example, the code values of C determined from the calculated periods of vibration would be at least 10 percent smaller than the value used.

The response of Frames U3, F3, and A3 are presented in Fig. 8 and Table IX in the same format as the series one results. It should be remembered that these results are for the first seven seconds of 1.5 times the El Centro 1940 N-S accelerogram. The following aspects of the results should be particularly noted.

1. The addition of braces at alternate floors, A3, effectively reduces the displacements of the structure, reduces the girder ductility and energy absorption requirements, but increases the axial loads in the columns.

(iv)

AISC Formula (7a): $$\frac{f_a}{F_a} + \frac{C_m f_b}{\left(1 - \frac{f_a}{F'_e}\right) F_b} \leq 1.0$$

2. The large relative displacement between the 7th and 8th floors in F3 results in extremely large column moments in that story. Column axial loads in this frame are also large.

3. The energy absorption by F3 is more uniformly distributed along the height of the structure than in the other two cases. It should be remembered that F3 has three times more mass than Frames U3 or A3.

The absence of results for Frame B3 in the preceding discussion was probably observed. The time history response of this structure, Fig. 9, indicates an apparent instability occurring between six and seven seconds. The energies, checked by the method discussed earlier, agreed within less than 1 percent at 6.459 seconds and differed by about 11 percent at 6.991 seconds. It was noted that the energy dissipated by the structure exceeded the base shear energy input at 6.991 seconds. A similar time history for F3 is given in Fig. 10. These two structures are identical except for the first story columns and the first floor girder. The elastic periods, Table X, are also nearly the same. The effect of P-Δ upon the response is illustrated in Fig. 11 for both B3 and F3. The most striking features are that the P-Δ effect tends to reduce the amplitude of the motion except in the case of instability and that the response of these two frames neglecting P-Δ are almost identical.

Why did B3 become unstable while F3 did not? While the answer to this question is not known at the present time, a major factor leading to the instability was a higher mode contribution. This can best be illustrated by plotting the deformation shape of these structures at various times. These shapes, Fig. 12, show that the B3 instability had characteristics of a second mode. The development of the higher mode participation was delayed in F3, however, it cannot be definitely concluded whether F3 would become unstable in the remaining 23 seconds of the earthquake or not.

Since the braces and the girders have elasto-plastic characteristics, it is conceivable that the incremental lateral load at a floor would have to be carried by deformation of the elastic columns - the brace and girder at their yield values cannot contribute additional resistance. This will cause large relative floor displacements and together with the softening effect of P-Δ may have initiated the instability. But this does not explain why F3 did not behave similarly.

<u>Series Four</u>. A structure with both flexibility and strength was made identical with A3 except that the girder rotational stiffness were taken as 10 percent of the full girder stiffness. The full yield strength of the girders were retained. This structure was capable of resisting the same static ultimate lateral loads, neglecting P-Δ, as A3 but would have much larger lateral displacements. The elastic periods and maximum energy input are given in Table X for A4 as well as for the variations of A4. The two variations investigated were the elimination of the P-Δ effect on the dynamic response and, independently, total removal of all braces.

Structure A4 suffered a second mode type of instability as shown in Fig. 13. The energies agreed within 10 percent until 6.645 seconds of the