NON-LINEAR RESPONSE ANALYSIS OF MULTI-STORY STRUCTURES INCLUDING ROCKING AND SWAYING VIBRATION SUBJECTED TO EARTHQUAKE GROUND MOTIONS.

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ABSTRACT

In this paper one made response analysis of M and N.K.B. buildings subjected to Kanto earthquake, 1st September, 1923. The M building *received heavy damage, while N building was sufficient for Kanto* earthquake. These structures are replaced by multi-degree of freedom systems of shearing vibration type for upper structures, and by that of swaying and rocking vibration type for under structures, and analized under the following replacing conditions in structures.

(1) The basement of structures is assumed to be fixed, and the restoring force characteristics is assumed to be linear type.
(2) The basement of structures is assumed to be fixed, and the restoring force characteristics is assumed to be non-linear type.
(3) The structures are considered as the multi-degree of freedom system with rocking and swaying vibration, and the restoring force characteristics is assumed to be non-linear type.

The response of structures is calculated under the three kinds of earthquake ground motions, where the maximum acceration is 326 cm/sec^2

By the analysis under the condition (1), we can conclude only qualitatively which story receives damage, but can not get the degree of damage quantitatively.

By the analysis under the condition (2), we get the exact degree of damage suffered by Kanto earthquake quantitatively from the distribution of the calculated ductility factor.

In order to explain the condition of damage in structures in more detail by the response analysis, it is necessary to be analized under the condition (3). But it is very difficult to evaluate the coefficients of the ground condition, because these coefficients are concerning with the frequency of the earthquake ground motion. Therefore, these coefficients are discussed and decided.

According to the analysis under the condition (3), the results of response analysis agree very well with the real damage in the Kanto earthquake.

In addition to the proceeding problem the eigen value problem and response of the structures for the some kinds of soil conditions are discussed.

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INTRODUCTION

It is very important in earthquake engineering field to compare the results of response analysis with the condition of damages for strong motions, in order to verify the sufficiency of response analysis. In order to simplify the mathematical analysis, one usually replace the structures by multi-degree of freedom systems of shearing vibration type for upper structures under the fixed condition for the structure basement. The above idealized equivalent structures show the same vibrational behavior with the actual structures to certain degree. However, in order to make more detailed analysis, it is hopeful to consider the more realistic conditions for the structure basement.

Many authors(1) have found that the actual vibration of structures also include rocking and swaying type vibration. We should notice that these vibrational behaviors of structures are influenced by the property of the ground and the condition of foundation and under-structures. It is also well known that the damping effects of the vibration of structures by the energy dissipation into ground are larger than that by the internal and external viscous damping of upper structures.

The damping by the energy dissipation into the ground is usually studied by replacing to the viscous damping in upper structures in the conventional analysis of structures. However, as we noticed above, the characters of viscous damping of upper structures are different from the damping of energy dissipation into the ground, and in order to consider rocking and swaying type vibration we should consider the correct properties of the ground and foundation separating from the condition for the viscous damping in upper structures.

In the first part, the non-linear response analysis of multi-story structures including rocking and swaying vibration is developed, parametorizing the ground property by spring constant, virtual mass and viscous damping coefficient. It is very difficult to evaluate the coefficients of the ground condition, because these coefficients are concerning

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with the frequency of the earthquake ground motions. Therefore, these coefficients are discussed and decided in the latter section.

In the second part, one makes response analysis of M and N.K.B. buildings subjected to Kanto earthquake, 1st September, 1923, and compares the results of response analysis with the conditions of a damage for strong motion. From this consideration, one gets the exact degree of damage suffered by Kanto earthquake quantitatively from the distribution of the calculated ductility factor. The digital computer used in this paper is Tosbac-3400.

VIBRATIONAL EQUATIONS OF MULTI-STORY STRUCTURES INCLUDING ROCKING AND SWAYING VIBRATION.

Actual multi-story structures considering rocking and swaying subjected to earthquake ground motions are idealized as multi-degree of freedom system of shearing vibration type and are analized under the following assumptions.

(1) The entire structure is considering as a multi-degree of freedom system in which the total mass of each story, i.e., the mass of floor, structural members, walls etc. are located at it's center of mass.

(2) The restoring force characteristics of structures are bi-linear, and elasto-plastic and linear type are regarded as limiting cases of the bi-linear type.

(3) The system has viscous damping coefficient which is proportional to relative velocity for the upper structures, and the damping coefficient by the energy dissipation into the ground is considered as a function of propagation velocity of secondary wave in the ground (hereafter abbreviate velocity of S-wave) and frequency of strong motions.

(4) The mass of the lowest story is replaced by the virtual mass.

(5) The restoring force characteristics of ground including under structure are idealized by the swaying and rotating spring constant in the ground which is studied by many investigators, and are also idealized by bi-linear.

(6) For simplicity, vibration of structures is approximated by one dimensional vibration in horizontal direction.

Under the above assumptions, the vibrational equations of structure with swaying and rocking vibration for non-linear multi-degree of freedom system are written as follows;

$$m_r \ddot{x}_r - C_{r+1}\{\dot{x}_{r+1} - \dot{x}_r - \dot{\theta}(H_{r+1} - H_r)\} + C_r\{\dot{x}_r - \dot{x}_{r-1} - \dot{\theta}(H_r - H_{r-1})\}$$
$$- f_{r+1}\{x_{r+1} - x_r - \theta(H_{r+1} - H_r)\} + f_r\{x_r - x_{r-1} - \theta(H_r - H_{r-1})\} = -m_r \ddot{Z} \quad , \quad (1)$$
$$(r = 0, 1, 2, \dots\dots, r, \dots\dots, n)$$
$$I_\theta \ddot{\theta} + C_\theta \dot{\theta} + f_\theta(\theta) + \sum_{r=1}^{n} m_r H_r(\ddot{x}_r - \ddot{\theta} H_r) = -\sum_{r=1}^{n} m_r H_r \ddot{Z} \quad ,$$

where m_r is mass of r-th story, C_r is the viscous damping coefficient of the r-th story, $f_r\{x_r - x_{r-1} - \theta(H_r - H_{r-1})\}$ is the restoring force of r-th story, $\ddot{Z}$ is acceleration of earthquake ground motions, x_r is relative displacement. I_θ is inertia of rotation and $I_\theta = \sum_{r=1}^{n} (I_{o,r} + m_r H_r^2) = I_G + \sum_{r=1}^{n} m_r H_r^2$, C_θ is the viscous damping coefficient for rotation in the ground, $f_\theta(\theta)$ is the restoring force for rotation, θ is the rotation angle, m_o is the virtual mass of the soil, C_0 is the viscous damping coefficient for swaying in the ground, $f_0(x_0)$ is the restoring force for swaying, and I_G is the summation of inertia of rotation in the foundation and each story. In the case of r=0, Eq. (1) becomes the vibrational equation for only swaying vibration.

Rewritting the first one of Eq. (1), and substituting this equation into the second one in Eq. (1), the second one of Eq. (1) becomes,

$$
\begin{aligned}
I_G\ddot{\theta} + C_\theta\dot{\theta} + f_\theta(\theta) &+ \sum_{r=1}^{n} H_r C_{r+1}\{\dot{x}_{r+1} - \dot{x}_r - \dot{\theta}(H_{r+1} - H_r)\} \\
&- \sum_{r=1}^{n} H_r C_r\{\dot{x}_r - \dot{x}_{r-1} - \dot{\theta}(H_r - H_{r-1})\} + \sum_{r=1}^{n} H_r f_{r+1}\{x_{r+1} - x_r - \theta(H_{r+1} - H_r)\} \\
&- \sum_{r=1}^{n} H_r f_r\{x_r - x_{r-1} - \theta(H_r - H_{r-1})\} = 0 \quad . \qquad (2)
\end{aligned}
$$

The restoring force for upper structure is expressed by Eq. (3) or (4).

$$
f_r(Y_r) = \frac{n_r k_r}{2}(|Y_r - Y_{r,o} + 2Y_{r,y}| - |Y_r - Y_{r,o}|) + (1 - n_r)k_r Y_r \ , \qquad (3)
$$

or

$$
\begin{aligned}
f_r\{x_r - x_{r-1} - \theta(H_r - H_{r-1})\} &= \frac{n_r k_r}{2}\{|x_r - x_{r-1} - \theta(H_r - H_{r-1}) - Y_{r,o} + 2Y_{r,y}| \\
&- |x_r - x_{r-1} - \theta(H_r - H_{r-1}) - Y_{r,o}|\} + (1 - n_r)k_r\{x_r - x_{r-1} - \theta(H_r - H_{r-1})\} \ , \qquad (4) \\
&(r = 0, 1, 2, 3, \cdots\cdots, r, \cdots\cdots, n)
\end{aligned}
$$

in which, k_e is spring constant in elastic range, n_r is spring constant ratio of plastic to elastic range and $n_r = k_p/k_e$ (kp denotes spring constant of plastic range) $0 \le n_r \le 1.0$, $Y_{r,y}$ is yield displacement, and $Y_{r,o}$ is plastic displacement when the velocity becomes zero (see Fig. 2).

The restoring force for rotation in the ground is also assumed to be bi-linear type as shown in Fig. 3, and given by

$$
f_\theta(\theta) = \frac{n_\theta k_\theta}{2}\{|\theta - \theta_o - 2\theta_y| - |\theta - \theta_o|\} + (1 - n_\theta)k_\theta\,\theta \ , \qquad (5)
$$

in which k_θ is spring constant of rotation in ground, n_θ is spring constant of rotation ratio of plastic to elastic range in the ground, θ_y is rotational yield displacement in soil and θ_o is rotational plastic displacement in soil when the velocity becomes zero. Thus, substitating the function of restoring force as given by Eq. (3) or (4) and Eq. (5) into the first one of Eq. (1) and Eq. (2), one gets the following equations.

$$
\begin{aligned}
&m_r\ddot{x}_r - C_{r+1}\{\dot{x}_{r+1} - \dot{x}_r - \dot{\theta}(H_{r+1} - H_r)\} + C_r\{\dot{x}_r - \dot{x}_{r-1} - \dot{\theta}(H_r - H_{r-1})\} - \frac{n_{r+1}k_{r+1}}{2} \\
&\{|x_{r+1} - x_r - \theta(H_{r+1} - H_r) - Y_{r+1,o} + 2Y_{r+1,y}| - |x_{r+1} - x_r - \theta(H_{r+1} - H_r) - Y_{r+1,o}|\} \\
&- (1 - n_{r+1})k_{r+1}\{x_{r+1} - x_r - \theta(H_{r+1} - H_r)\} + \frac{n_r k_r}{2}\{|x_r - x_{r-1} - \theta(H_r - H_{r-1}) - Y_{r,o} + 2Y_{r,y}| \\
&- |x_r - x_{r-1} - \theta(H_r - H_{r-1}) - Y_{r,o}|\} + (1 - n_r)k_r\{x_r - x_{r-1} - \theta(H_r - H_{r-1})\} = -m_r\ddot{Z} \ ,
\end{aligned}
$$

$$
\begin{aligned}
&I_G\ddot{\theta}+C_\theta\dot{\theta}+\frac{n_\theta k_\theta}{2}\{|\theta-\theta_0-2\theta_y|-|\theta-\theta_0|\}+(1-n_\theta)k_\theta\theta+\sum_{r=1}^{n}H_rC_{r+1}\{\dot{x}_{r+1}\\
&-\dot{x}_r-\dot{\theta}(H_{r+1}-H_r)\}-\sum_{r=1}^{n}H_rC_r\{\dot{x}_r-\dot{x}_{r-1}-\dot{\theta}(H_r-H_{r-1})\}+\sum_{r=1}^{n}H_r\Big[\frac{n_{r+1}k_{r+1}}{2}\{|x_{r+1}-x_r\\
&-\theta(H_{r+1}-H_r)-Y_{r+1,0}+2Y_{r+1,y}|-|x_{r+1}-x_r-\theta(H_{r+1}-H_r)-Y_{r+1,0}|\}+(1-n_{r+1})k_{r+1}\\
&\{x_{r+1}-x_r-\theta(H_{r+1}-H_r)\}\Big]-\sum_{r=1}^{n}H_r\Big[\frac{n_rk_r}{2}\{|x_r-x_{r-1}-\theta(H_r-H_{r-1})-Y_{r,0}+2Y_{r,y}|-\\
&|x_r-x_{r-1}-\theta(H_r-H_{r-1})-Y_{r,0}|\}+(1-n_r)k_r\{x_r-x_{r-1}-\theta(H_r-H_{r-1})\}\Big]=0 \qquad (6)\\
&\qquad\qquad (r=0,1,2,\cdots\cdots,r,\cdots\cdots,n)
\end{aligned}
$$

These equations are the vibrational equation of multi-story structures including rocking and sqaying vibration.

NUMERICAL CALCULATION AND PROGRAMING

Numerical analysis of Eq. (6) was carried out by means of Runge Kutta's method. The accuracy of numerical computation of nonlinear vibration depends on the way of judging the condition of transition between elastic and plastic range. In this paper, one uses different way from the previous paper(2) to find the boundary point where the transition from elastic to plastic range, that is, the displacement in time interval between t_{i-1} and t_i is assumed by the following cubic algebric equation

$$Y_r = f\left(\frac{\Delta t_i'}{\Delta t_i}\right) \qquad (7)$$

In this boundary point, the velocity $\dot{Y}_r$ becomes zero. Namely, when the displacement, velocity and acceleration for $t=t_{i-1}$ and t_i are known, the constants of cubic algebric equation are difined by four algebic equations, Δt_i for the boundary point is given by

$$\dot{Y}_r = \dot{f}\left(\frac{\Delta t_i'}{\Delta t_i}\right) = 0 \qquad (8)$$

The displacement corresponding to $\Delta t_i'$ is obtained by substiting which is given by Eq. (8) into Eq. (7).

This method is more convenient than that of the previous paper, because one can make numerical calculation more speedy and the results derived by this method agree very well with that by the preceding paper.

The accuracy of our numerical computation of non-linear vibration including rocking and swaying vibration is considered by comparing with the strict solution. The one-mass system with only rocking vibration subjected to sinusoidal wave is selected, and is analized by modal analysis. The solution in steady state condition of this equation is adopted as the strict solution. The results of analysis by this method is compared with the strict solution of two cases for this problem, and the errors for strict solution is 0.3 - 0.4 percent and 2.5 - 0.3 percent for displacement and rotation angle, respectively. Therefore, it is sufficient for practical problems.

DETERMINATION OF CONSTANTS IN CONNECTED WITH GROUND PROPERTY

One have to solve many difficult problems for determining the constants describing the charactristics of the ground in the preceding procedure in this paper. The replacing the under structure including ground to equivallent mass system means that one make virtual mass and spring constant of ground to be constant. But it is developing that the virtual mass and spring constant of ground change by changing the frequency of earthquake motion every moment.

Now, when the foundation plate having the mass m_f vibrate by excited for $Pe^{i\omega t}$, the generalized vibrational equation is given by

$$m_a' \ddot{u} + C\dot{u} + Ku = Pe^{i\omega t}, \qquad (9)$$

in which m_a' is virtual mass (see Fig. 4). The solution of this equation is of the form,

$$u = a e^{i\omega t}. \qquad (10)$$

The solution becomes

$$u = \frac{Pe^{i\omega t}}{K - m_a'\omega^2 + i\omega C}. \qquad (11)$$

The generalized solution of Eq. (11) is

$$u = \frac{Pe^{i\omega t}}{aG}(f_1 + if_2), \qquad (12)$$

in which f_1 and f_2 are constants.

Letting the Eq. (11) equal with Eq. (12), one have

$$f_1 + if_2 = \frac{1}{K_e + ia_0C_e}, \qquad (13)$$

where

$$K_e = \frac{f_1}{f_1^2 + f_2^2}, \quad C_e = \frac{-f_2}{a_0(f_1^2 + f_2^2)}. \qquad (14)$$

Using the Eqs. (11), (12) and (13), one obtained,

$$a_0 G K_e = K - m_a'\omega^2, \qquad (15)$$

$$a G a_0 C_e = \omega C, \qquad (16)$$

in which, a is radius of circular foundation, and in the case of rectangular foundation a is effective radius and $a = 0.55\sqrt{A}$. (A is area of foundation), G is related with density of soil ρ and velocity of S-wave V_s by

$$G = \rho V_s^2, \qquad (17)$$

a_0 is frequency ratio and is expressed by

$$a_0 = \frac{\omega}{V_s/a}. \qquad (18)$$

Thus, the virtual mass is given by

$$m_a' = \frac{1}{\omega^2}\left\{K - \frac{f_1}{f_1^2+f_2^2} a G\right\} \quad . \tag{19}$$

That is, virtual mass depends on circular frequency of earthquake ground motions, H. Tajimi(3) approximated Eq. (19) by

$$m_a' = 0.27 \rho a^3 , \tag{20}$$

When Poisson's ratio $\nu = 1/2$ in swaying vibration and in the case of the uniform distribution of soil reaction.

The spring constants in swaying and rocking vibration are also function of the circular frequency of earthquake ground motions, and the statical spring constants are used for the uniform destribution of soil reaction in this paper. Thus,

$$k_0 = \frac{2\pi a G}{2-\nu} , \tag{21}$$

$$k_\theta = \frac{\pi a^3 G}{2(1-\nu)} . \tag{22}$$

The damping coefficients are also calculated by H. Tajimi's approximational equation(3), and the following values are given,

$$C_0 = 0.58 \rho V_s \pi a^2 , \tag{23}$$

$$C_\theta = \frac{1.325^3}{12} \frac{a^3 \omega^2}{V_s^3} . \tag{24}$$

Circular frequency of earthquake ground motions in Eq. (24) is to be frequency in the case of resonance when the circular frequency of input wave coincide with the natural circular frequency of the structure.

CONPARISION WITH THE RESULTS OF RESPONSE ANALYSIS AND THE CONSIDERATION OF A DAMAGE FOR EARTHQUAKE GROUND MOTIONS.

The abstruct of the buildings which is analized; The comparision with the results of response analysis and the condition of damages for Kanto earthquake September 1, 1923 are discussed for two actual buildings. These actual buildings are called M. and N.K.B. building. M. building had completed on August, 1923. This building was strengthened making the columns to be a steel framed reinforced concrete and the external walls to be a reinforced concrete walls after repair the damage of earthquake, April 26, 1922. M. building has 8 stories in upper ground and one story in under ground.

N.K.B. building is a steel framed reinforced concrete structure with shear walls in the suitable position and completed on March 1923. This building has 7 stories in upper ground and one story in under ground. As the foundation 15 - 16 m length piles of pine are used in the both buildings. M. building received special damage but N.K.B. building was sufficient by Kanto earthquake.

Professors Sezawa and Kanai explained this reason by energy dissipation into the ground, (4) the condition of damage are explained based on the results of response analysis in this paper.

The mass, spring constants, yield shearing force and yield displacement which are used for response analysis are estimated by same method with previous papers(5), and are shown in Table 1. The constants characterizing ground are estimated by the method described in the preceding section. The spring constant, damping coefficient in the ground are shown in Fig. 5 in concerning with the velocity of S-wave.

Eigen value and the results of response analysis: The response of these buildings are analized under the following replacing conditions in structures.

Condition (1) ; The structural basement is assumed to be fixed, and restoring force characteristics is assumed to be linear type.

Condition (2) ; The structural basement is assumed to be fixed, and restoring force characteristics is assumed to be non-linear type.

Condition (3) ; The structures are considered as the multi-degree of freedom system with rocking and swaying vibration, and the restoring force characteristics of upper and under ground assumed to be non-linear and linear type, respectively.

The fraction of viscous damping and the spring constant ratio of plastic to elastic range in upper structures are assumed to be $h_1 = 0.02$ and $k_p/k_e = 0.25$, respectively (see Table 1).

The following three kinds of earthquake ground motions are used; El-Centro may 18, 1940, NS, Taft California, July 21, 1952, EW and Tokyo 101, Feb. 14, 1956, EW. In which, one revises the accelerations of earthquake ground motions to make them 0.33 g, and leave the time axis.. In these earthquake motions, the digitalized records in U.S.A. were presented by G.V. Berg and the ones in Japan were made by K. Muto(6).

The natural period and vibrational mode of structures calculated by Holtzer's method and matrix method. The only natural period is shown in Table 2. The natural period in the fixed condition of structural basement by theoretical method is shorter than observational value (7), and it is found that these buildings vibrate by swaying and rocking vibration from Table 2 and vibration tests. Then the velocity of S-wave is assumed to be Vs = 200 - 300 m/sec by Kanai's study (8), and is calculated. In order to make the 1st. natural period coincide with the observational value, one use Vs = 270 and 180 m/sec in M. and N.K.B. building, respectively. The results is shown in Table 2. The observational value agrees with the case of Vs = 270 m/sec for M. building and Vs = 180 m/sec for N.K.B. building. To make the 1st. natural period coincide with the observational value, the different value of Vs have to be adopted in spite of the same kinds of ground. This reason may be the effect of scale in building and error of measurement in vibration test.

The results of response analysis under the condition (1) is shown in Fig. 6. And the comparision of the relative displacement and condition of damage in Kanto earthquake is shown in Table 3. From Table 3, one findsthe following results. M. building suffered real damage from the earthquake mainly in the 3rd, 4th and 5th story and the calculated relative displacement is also large in the same story. N.K.B. building was sufficient, but the distribution of damage for story in the damage of shade of lamp agree very well with the one of relative displacement. From this consideration, one can conclade only qualitatively which story receives damage, but can not get the degree of damage quantitatively.

The results of response analysis in the condition (2) is shown in Fig. 7. In order to explain the condition of damage in earthquake, the distribution of ductility factor of M. and N.K.B. building are compared, and the ductility factor of M. building is larger than one at 2nd - 5th story and maximum value is about 3.5 at 4th story. But the ductility factor of N.K.B. building is larger than one at 7th story and its value is only 1.2. The distribution of shearing force coefficient for both buildings is similar, but that value of N.K.B. building is about twice as large as the one of M. building. Namely, N.K.B. building is constructed very rigid in comparing with M. building. It is able to confirm that M. building received special damage at 2nd - 4th story and N.K.B. building was sufficient by earthquake from these facts. It is also found that the results of response analysis subjected to three kinds of earthquake ground motions has the same tendency as we see in Fig. 7

The distribution of ductility factor of each story is shown in Fig. 8 for the various value of velocity of S-wave in the case of M. and N.K.B. buildings under the condition (3). Comparing the results under the condition (2), one seesthat both results under the condition (2) and (3) show almost for the real value of V_S ($V_S = 200$ - 300 m/sec for M. and N.K.B. building).
It is apparant from Fig. 12 and 13 that the displacement by rocking vibration for total displacement in N.K.B. building is about twice the one in M. building, and that the ratio of overturning moment to the statistically calculated one in N.K.B. building is nearly one, and the one in M. building is smaller than 0.6. Then one can deduce that N.K.B. building was sufficient in Kanto earthquake, since the effect of displacement by rocking in total displacement is larger in N.K.B. building than in M. building.

RELATIONSHIP BETWEEN THE RESPONSE AND VELOCITY OF S-WAVE

Eigen value; The natural period depends on the effect of rocking and swaying vibration. The relationship between the natural period and velocity of S-wave is shown in Fig. 9 for M. and N.K.B. building. It is obvious from this figure that the natural period becomes short with increasing of velocity of S-wave, and the effect of swaying and rocking vibration becomes small with increasing the order of natural period.

Results of response analysis; The relationship between response and velocity of S-wave are shown in Fig. 10 and 11. The relative displacement, ductility factor and shearing force coefficient in M. building increase with increasing the velocity of S-wave, especially increase very rapidly in $V_s = 50 - 200$ m/sec region and become slightly small at fixed condition ($V_s = \infty$). These values in N.K.B. building show same tendency with one of M. building for El-Centro May 18, 1940, NS and Tokyo 101, Feb. 14, 1956, EW, but these values increase smoothly with increasing velocity of S-wave for Taft Calif. July 21, 1952, EW. This reason may be by the condition of frequency property of Taft Calif. and natural frequency of N.K.B. building. This means that these values have the maximum value at some velocity of S-wave, that is these values have the worst value at some points.

The relationship between the ratio of each displacements by shearing swaying and rocking vibration to total displacement and velocity of S-wave are shown in Fig. 12. The displacement by rocking and swaying vibration becomes small with increasing of the velocity of S-wave, and the one by shearing vibration increases with increasing the velocity of S-wave as is generally known. And the percentage of displacement by shearing vibration and one by swaying and rocking vibration have same value in $V_s = 100 - 200$ m/sec. The ratio of dynamic over turning moment to static overturning moment is shown in Fig. 13 versus the velocity of S-wave.

RELATIONSHIP BETWEEN THE LINEAR RESPONSE AND THE VELOCITY OF S-WAVE

The linear response of structures are studied in concerning with the velocity of S-wave. M. building is used in this analysis and the earthquake ground motion is only subjected to El-Centro May 18, 1940. (max. acceleration is 0.33 g).

The relationship between the linear response and the velocity of S-wave is shown in Fig. 14. It is apparant from Fig. 14 that the displacement by shearing vibration, the shearing force and the shearing force coefficient of each story increase with increasing velocity of S-wave, and in case of Vs = 500 m/sec, these value are almost equivalent with one in case of fixed condition of basement ($V_s = \infty$). The swaying displacement and rotation angle decrease with increasing of the velocity of S-wave.

CONCLUSION

In this paper the non-linear response analysis of multi-story structures including rocking and swaying vibration subjected to earthquake ground motions is developed and the results of response analysis for two model buildings which suffered earthquake are discussed.

(1) By the analysis under the fixed condition of structures and linear type of restoring force characteristics in upper structures, one can conclude only quatitatively which story receives damage in earthquake, but can not get the degree of damage quantitatively.

(2) By the analysis under the non-linear type of restoring force characteristics and fixed condition or the condition considering the ground property, one gets the exact degree of damage suffered by Kanto earthquake quantitatively from the distribution of the calculated ductility factor, specially, in case of considering the ground property.

(3) According to the relationship between the results of response analysis and velocity of S-wave, one can find the several results. That is, it may be estimated that the responses has the worst value which make the response to have maximum value at finite value of the velocity of S-wave, and the each displacement percentage by shearing, rocking and swaying vibration is found as a function of velocity of S-wave etc..

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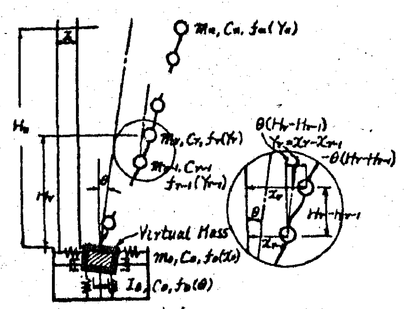
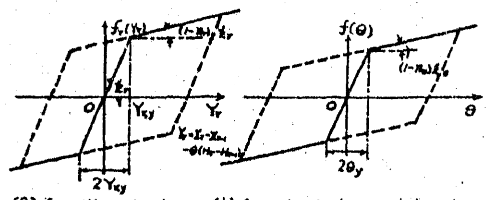


FIG. 1 Model of structure



(a) for upper structures (b) for under structures including the soil

FIG. 2 Restoring characteristics

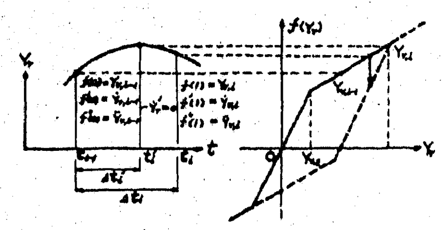


FIG. 3 Transition from elastic range to plastic range

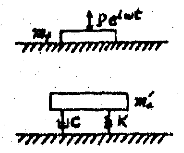


FIG. 4 Vibration of foundation plate

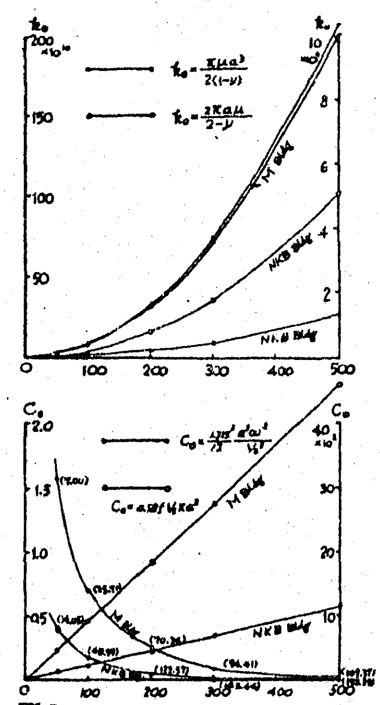


FIG. 5 Vibrational constants describing the characteristics of the ground

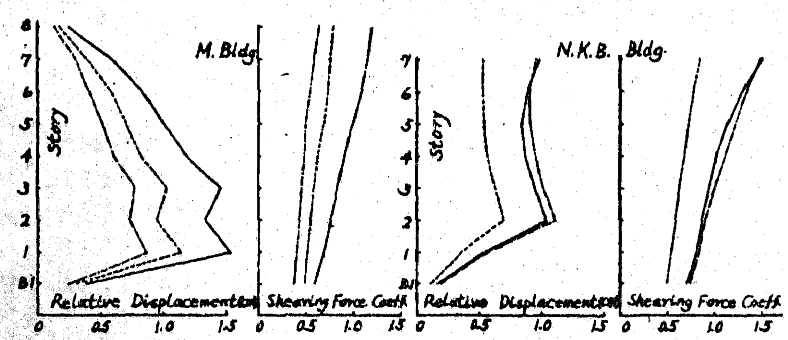


FIG. 6 Results of Linear Response Analysis in Case of Fixed Conditions of Basement

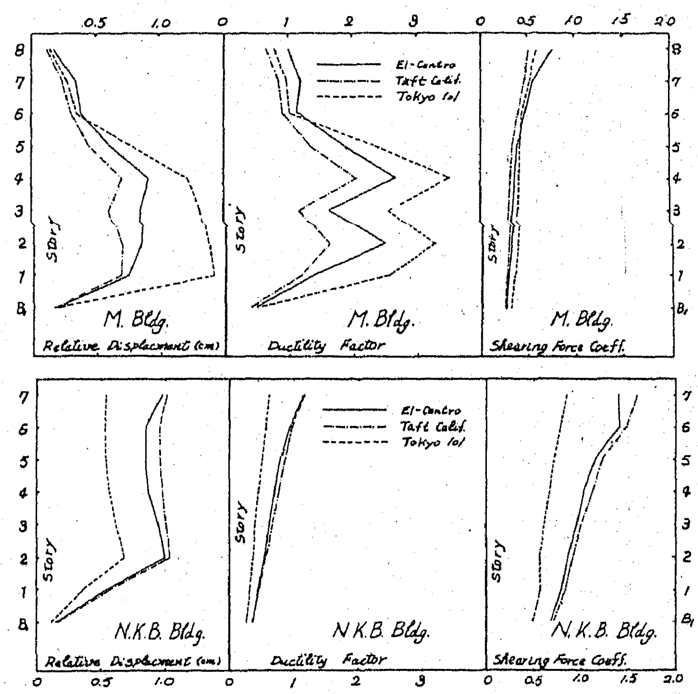


FIG. 7 Results of the non-linear response analysis in case of fixed condition of structure basement (non-linear, $h = 0.02$)

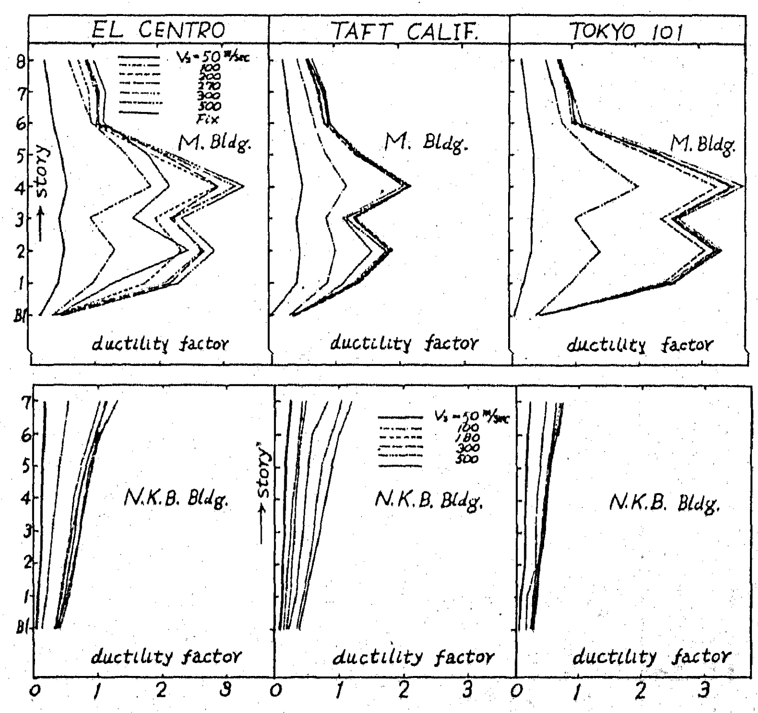


FIG. 8 Results of response analysis with rocking and swaying vibrations in case of non-linear type

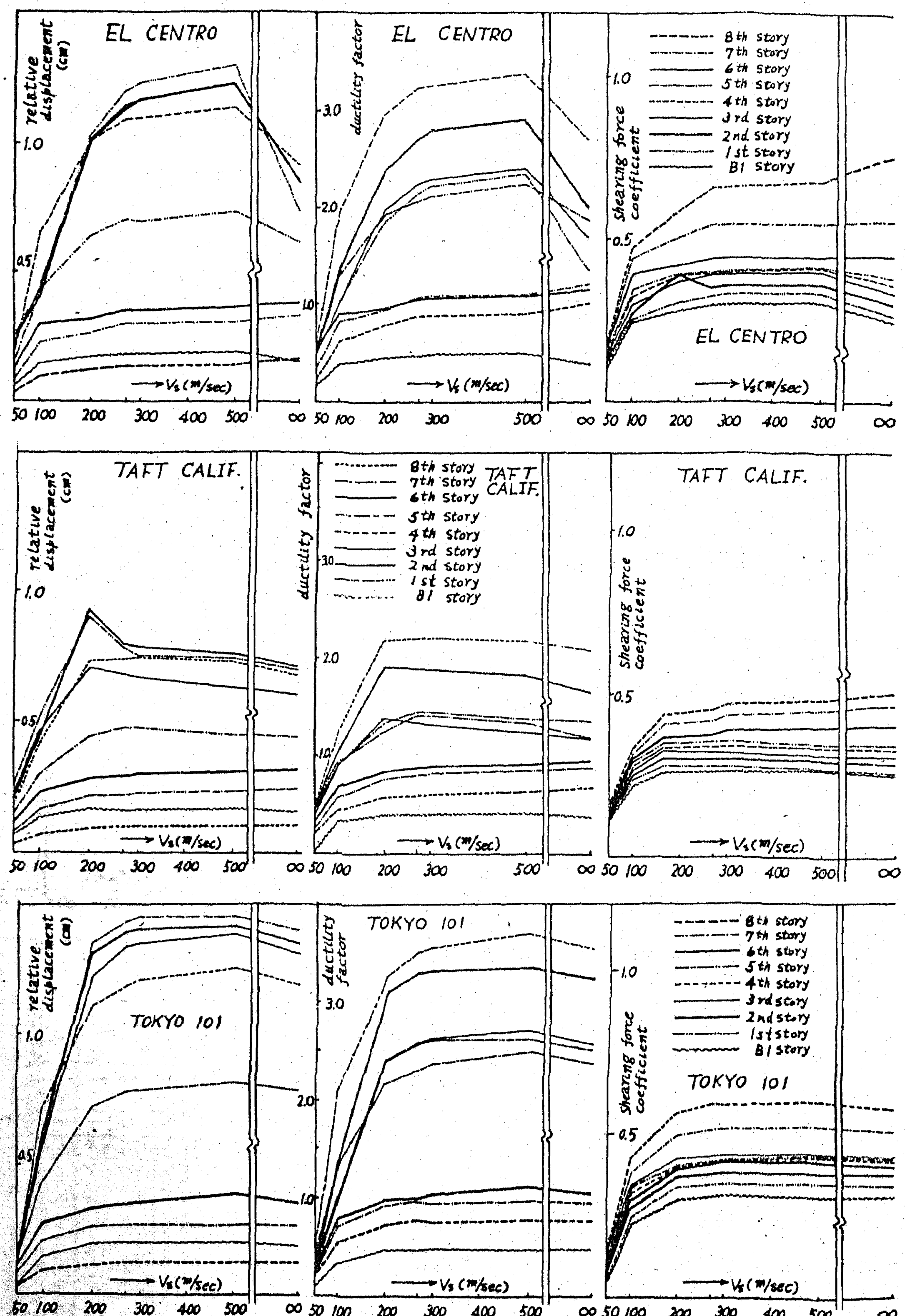


FIG. 10 Relationship between the relative displacement, ductility factor and shearing force coefficient and velocity of S-wave in M. Building

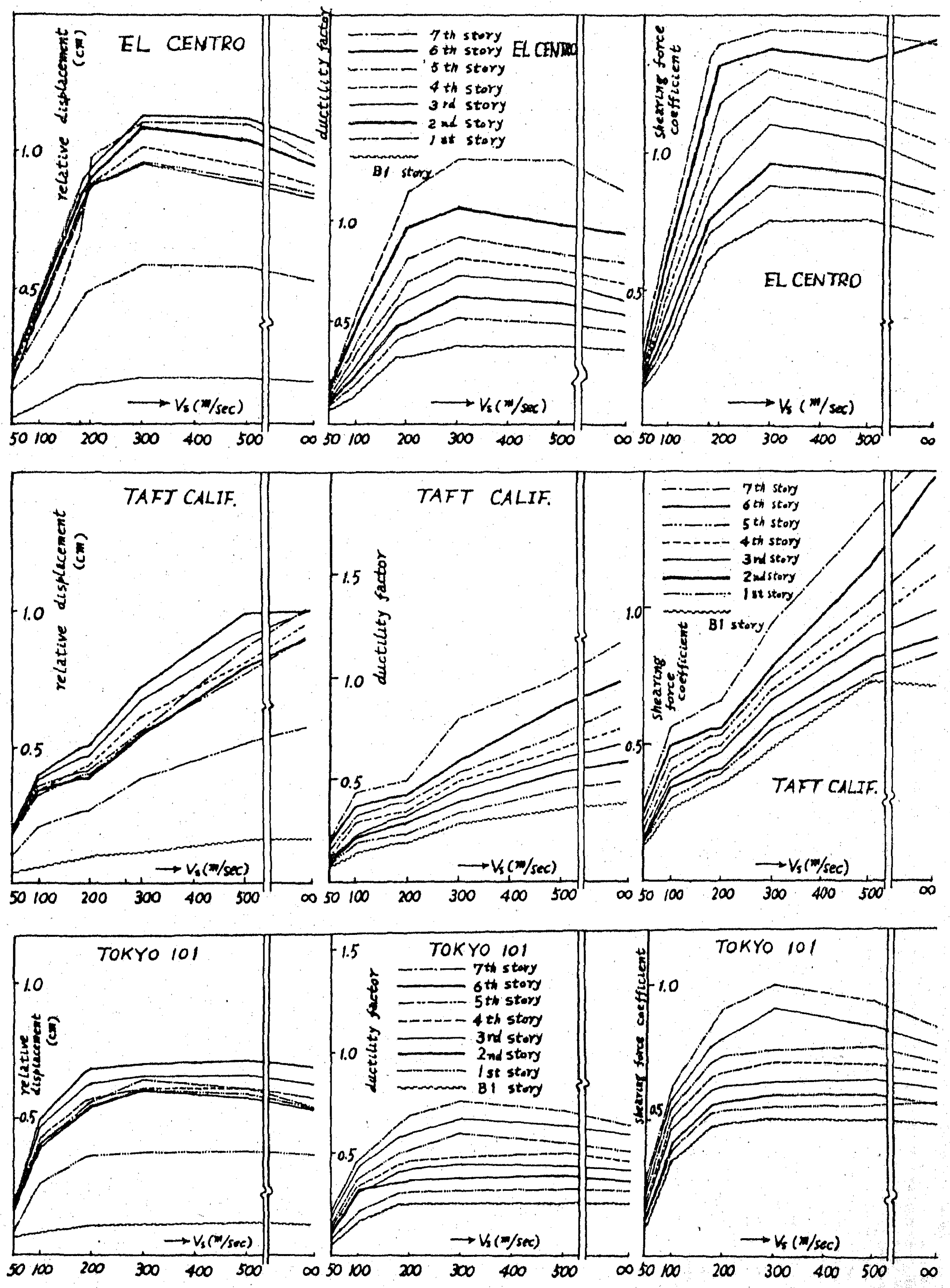


FIG. 1.1 Relationship between the relative displacement, ductility factor and shearing force coefficient and velocity of S-wave in N.K.B. Building

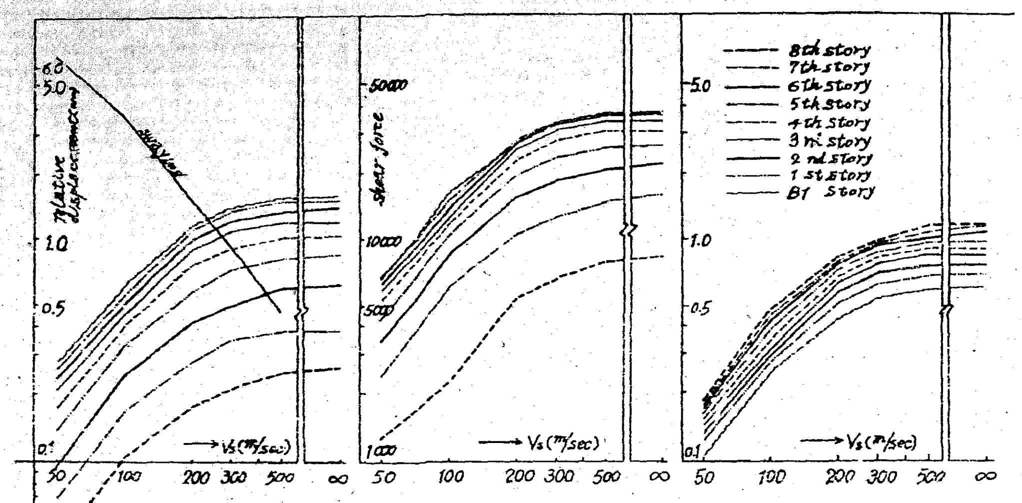


FIG. 14 Relationship between the relative displacement shearing vibration and velocity of S-wave

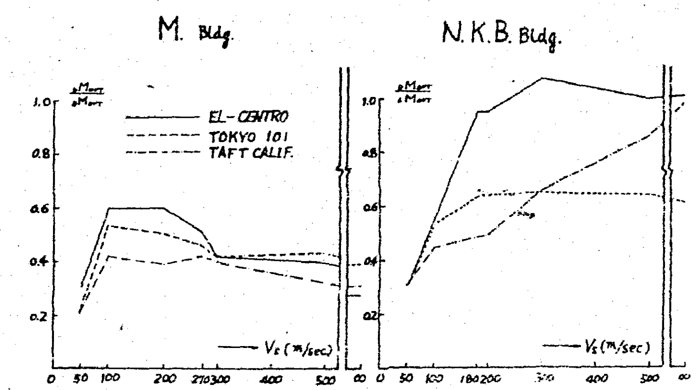


FIG. 13 Relationship between the ratio of overturning moment and propagation velocity of S-wave

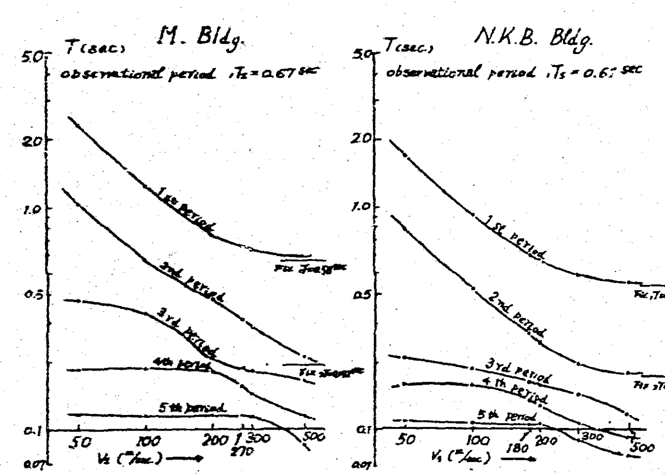


FIG. 5 Relationship between the natural and the propagation velocity of S-wave. (Vs)

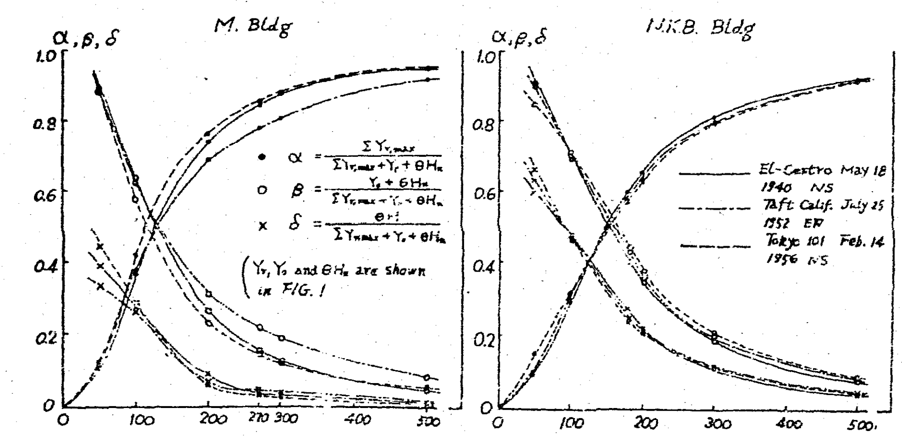


FIG. 12 Relationship between the relative displacement, swaying and rocking displacement and propagation velocity of S-wave

Table 1 Vibrational constants in M. Bldg and N.K.B. Bldg

	story	m_r ($\frac{t \cdot sec^2}{cm}$)	C_r ($\frac{t \cdot sec}{cm}$)	k_r ($\frac{t}{cm}$)	$Q_{r,y}$ (t)	$Y_{r,y}=\frac{Q_{r,y}}{k_r}$	$\frac{k_p}{k_e}$	H (cm)
M. Bldg.	8	7.00	115.915	31,500	5,166	0.164	0.25	3,280
	7	6.80	94.204	25,600	7,163	0.280	0.25	2,930
	6	6.82	95.308	25,900	8,702	0.336	0.25	2,580
	5	6.82	94.940	25,800	8,514	0.330	0.25	2,230
	4	6.82	94.572	25,700	8,687	0.338	0.25	1,880
	3	6.82	85.373	23,200	11,855	0.511	0.25	1,530
	2	6.94	100.460	27,300	11,493	0.421	0.25	1,180
	1	7.00	89.788	24,400	13,518	0.554	0.25	830
	B1	7.70	356.946	97,000	39,382	0.406	0.25	330
	Swaying	62.72			—	—	—	0
	Rocking	$I_e = 6.7077 \times 10^9$ ($t \cdot cm \cdot sec^2/rad.$)						
N.K.B. Bldg.	7	2.050	8,591	3,140	2,688	0.856	0.25	3,160
	6	2.238	17,073	6,240	5,678	0.910	0.25	2,670
	5	2.090	23,092	8,440	8,946	1.060	0.25	2,290
	4	2.049	26,594	9,720	12,247	1.260	0.25	1,930
	3	2.100	28,126	10,280	15,369	1.495	0.25	1,570
	2	2.375	28,728	10,500	19,425	1.850	0.25	1,170
	1	1.400	56,744	20,740	23,644	1.140	0.25	740
	B1	2.276	196,443	71,800	31,017	0.432	0.25	310
	Swaying	16.578			—	—	—	0
	Rocking	$I_e = 8.6160 \times 10^7$ ($t \cdot cm \cdot sec^2/rad.$)						

TABLE 2, 1st Natural Period of M. and N.K.B. Building.

	Theoretical Value (SEC)					Observational Value (SEC)
	$V_s=\infty$ (cm/sec)	300 (cm/sec)	270	200	180	
M.-BLDG. EW	0.578	0.647	0.665	0.749	—	0.670
N.K.B.-BLDG. NS	0.429	0.491	—	0.568	0.598	0.610

TABLE 3, Comparison of relative displacement and condition of damage.

		$Y_{r,max}$ (cm)			story height H_r (cm)	$Y_{r,max}/H_r$			Condition of damage.*
		El-Centro	Taft Calif.	Tokyo 101		El-Centro	Taft Calif.	Tokyo 101	
M. BLDG. (EW)	8	0.261	0.139	0.172	350	1/1341	1/2518	1/2035	
	7	0.609	0.318	0.402	350	1/575	1/1101	1/871	
	6	0.841	0.432	0.567	350	1/416	1/810	1/617	
	5	1.021	0.524	0.714	350	1/343	1/668	1/490	light
	4	1.178	0.624	0.835	350	1/297	1/559	1/419	heavy
	3	1.460	0.783	1.030	350	1/240	1/447	1/340	very heavy
	2	1.326	0.733	0.947	350	1/264	1/477	1/370	heavy
	1	1.530	0.877	1.132	500	1/327	1/570	1/442	light
	B1	0.382	0.231	0.308	330	1/864	1/1429	1/1071	
N.K.B. BLDG. (N.S)	7	0.992	0.972	0.547	490	1/494	1/504	1/896	43.5 (%)
	6	0.887	0.913	0.533	380	1/428	1/416	1/713	58
	5	0.856	0.918	0.544	360	1/421	1/392	1/662	50
	4	0.874	0.956	0.578	360	1/412	1/377	1/623	44
	3	0.952	1.017	0.628	400	1/420	1/393	1/637	23
	2	1.034	1.090	0.687	430	1/416	1/394	1/626	44
	1	0.538	0.565	0.364	430	1/799	1/761	1/1181	27
	B1	0.160	0.166	0.112	310	1/1938	1/1867	1/2768	—

* Failure of shear walls and bracing member for M. Bldg, and failure of shade of lamps for N.K.B. Bldg.