EARTHQUAKE RESPONSE OF IRREGULARLY SHAPED BUILDINGS

by

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SYNOPSIS

Presented is an approximate method of determining the peak seismic response of certain irregularly shaped buildings when subjected to base accelerations corresponding to the N-S component of the El Centro 1940 earthquake. This method is based on the forced response of two degree of freedom systems and is applied to the lateral motion of buildings having large set-backs and to the coupled lateral-torsional motion of eccentric buildings. Response spectra for a two degree of freedom system are included to facilitate the use of this method. A comparison of the results is made with an accurate solution and a more approximate solution using single degree response spectra. The conditions under which buildings demonstrate abnormal response are discussed and recommendations are made for avoiding such undesirable characteristics.

INTRODUCTION

One of the most significant contributions to the analysis of structural response during an earthquake has been the introduction of the concept of response spectra[1]. These spectra for a specific earthquake make it possible to immediately determine the maximum response of an elastic single degree of freedom system to that earthquake. Thus, earthquake response spectra provide a direct means of determining the maximum elastic response of a multi-story building in any one of its natural or normal modes of vibration.

Maximum response due to all modes of vibration is often obtained by an approximate method of taking the square root of the sum of the squares of individual modal contributions[2]. This method is reasonable in most cases provided the natural frequencies of the contributing modes are fairly well spread. However, in those cases where two of the major contributing modes have frequencies close together, this method leads to large errors, particularly when damping is small. Unfortunately, irregularly shaped buildings may have two contributing mode shapes with frequencies of nearly the same magnitude. Therefore, it is important to understand the response characteristics of such systems when subjected to earthquake excitation.

The abnormal behavior caused by the close frequencies of certain types of structures has already been reported in the literature[3,4,5]. It is the purpose of this paper to show that the mathematical models of these special structures are identical and that the method of analysis

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previously reported by J. Penzien and A. Chopra for appendage response[3] is applicable in each case.

APPROXIMATE METHODS OF ANALYSIS

A. Lateral Motion of Buildings Having Large Set-Backs

1. Two-degree of freedom analysis - The mass of the set-back portion of a building as shown in Fig. 1a can, in some cases, be small compared with the mass of the lower portion of the building which supports it. In such cases the set-back portion acts like an appendage with its fundamental mode response during an earthquake being controlled primarily by the response of the main building. This condition suggests that one could determine the seismic response of the set-back portion (or of an appendage) approximately using a separate two-degree of freedom system for each of the lower normal modes of the building (without set-back structure) as represented in Fig. 1b. The terms M_n, K_n, and C_n in this figure represent the generalized mass, generalized spring constant and generalized damping factor, respectively, for the nth lateral vibration mode of the building (without set-back structure) while the terms m_a, k_a, and c_a represent the corresponding quantities for the fundamental mode of vibration of the set-back structure. These generalized quantities are given by the relations

$$M_n = \sum_{i=1}^{N} m_i \Phi_{in}^2; \quad K_n = \omega_n^2 M_n; \quad C_n = 2M_n \omega_n \xi_n \tag{1}$$

$$m_a = \sum_{i=1}^{S} m_i \Phi_{ia}^2; \quad k_a = \omega_a^2 m_a; \quad c_a = 2m_a \omega_a \xi_a \tag{2}$$

where Φ_{in} is the dimensionless mode shape quantity at the location of lumped mass m_i ($i = 1,2,\ldots,N$) due to the nth mode of the main building itself without the set-back structure, ω_n is the corresponding nth mode frequency, ξ_n is the damping ratio for the nth mode, Φ_{ia} is the dimensionless mode shape quantity at the location of lumped mass m_i ($i = 1,2,\ldots, S$) due to the fundamental mode of the set-back structure by itself, ω_a is the corresponding frequency, and ξ_a is the damping ratio of the fundamental mode.

The two degree system of Fig. 1b is excited through its support by the motion $U_s(t)$ which is related to the ground motion $U_g(t)$ by the relation

$$U_s(t) = \left[\sum_{i=1}^{N} m_i \Phi_{in} \Big/ \sum_{i=1}^{N} m_i \Phi_{in}^2 \right] U_g(t) \equiv -\gamma_n U_g(t) \tag{3}$$

The constant quantity γ_n in the brackets of Eq. (3) is a participation factor defined so that the resulting support displacement time history $U_s(t)$, if applied to the generalized single degree system representing the nth mode of the building (with set-back structure removed) will produce a displacement time history of mass M_n identical to the nth mode displacement time history of the top of the main building, i.e. at the location of the set-back.

Using Eqs. (1), (2), and (3), and introducing a mass ratio $\beta_n \equiv m_a/M_n$, the coupled equations of motion for the two degree of freedom system shown in Fig. 1b may be expressed as

$$\ddot{X}_n + 2\omega_n\xi_n\dot{X}_n + \omega_n^2 \dot{X}_n - 2\beta_n\omega_a\xi_a(\dot{X}_a-\dot{X}_n) - \beta_n\omega_a^2(X_a-X_n) = -\gamma_n\ddot{U}_g(t)$$

$$\ddot{X}_a + 2\omega_a\xi_a(\dot{X}_a-\dot{X}_n) + \omega_a^2\,(X_a-X_n) = -\gamma_n\ddot{U}_g(t) \qquad (4)$$

The above coupled equations of motion can be solved numerically(6) for any prescribed ground acceleration $U_g(t)$ and for any set of parameters ω_n, ω_a, ξ_n, ξ_a, β_n, and γ_n to yield the desired seismic coefficient C_{an} for the set-back portion of the building which is defined as

$$C_{an}\,(\omega_n, \omega_a, \xi_n, \xi_a, \beta_n, \gamma_n) \equiv \left|(X_a-X_n)\,\omega_a^2 \Big/ g\right|_{max.} \qquad (5)$$

This seismic coefficient is simply the ratio of the maximum dynamic shear developed at the location of the set back to the static shear developed at this same location due to a 1g lateral loading.

After obtaining C_{an} for n = 1,2,...,r, the combined seismic coefficient C_a for the fundamental mode response of the set-back, including the first r modal contributions of the main building, can be obtained using the approximate relation

$$C_a = (C_{a1}^2 + C_{a2}^2 + \ldots + C_{ar}^2)^{1/2} \qquad (6)$$

Usually, only the first 2 or 3 modes of the main building need be considered in evaluating C_a; i.e., r can be set equal to 2 or 3.

2. Single-degree of freedom analysis - For easy application of the previously described two degree of freedom analysis, it is necessary that response spectra for the two degree of freedom system be available. Since response spectra are readily available for the single degree system, one might consider reducing the two degree of freedom system of Fig. 1b to two generalized single degree of freedom systems representing its first and second modes of vibration. The maximum response of the system in each of these two modes can, of course, be obtained directly using the single degree response spectrum corresponding to the prescribed ground motion. Hopefully, one might then take the square root of the sums of the squares of the two maximum set-back (or appendage) spring forces to obtain the true maximum spring force produced by the nth building mode.

Letting ω_{n1}, Φ_{n1}, Φ_{a1}, Y_{n1} and ω_{n2}, Φ_{n2}, Φ_{a2}, Y_{n2} represent the frequency, dimensionless mode shape values and the generalized coordinate for the first and second modes, respectively, the corresponding generalized masses, stiffness factors, damping factors, forcing functions, and displacements are given by the relations

$$M_{n1} = M_n \Phi_{n1}^2 + m_a \Phi_{a1}^2 \quad ; \quad M_{n2} = M_n \Phi_{n2}^2 + m_a \Phi_{a2}^2$$

$$C_{n1} = C_n \Phi_{n1}^2 + c_a (\Phi_{a1} - \Phi_{n1})^2 ; \quad C_{n2} = C_n \Phi_{n2}^2 + c_a (\Phi_{a2} - \Phi_{n2})^2$$

$$K_{n1} = \omega_{n1}^2 M_{n1} \quad ; \quad K_{n2} = \omega_{n2}^2 M_{n2} \qquad (7)$$

$$P_{n1} = -(M_n \Phi_{n1} + m_a \Phi_{a1})\ddot{U}_s(t) \quad ; \quad P_{n2} = -(M_n \Phi_{n2} + m_a \Phi_{a2})\ddot{U}_s(t)$$

$$X_n = Y_{n1} \Phi_{n1} + Y_{n2} \Phi_{n2} \quad ; \quad X_a = Y_{n1} \Phi_{a1} + Y_{n2} \Phi_{a2}$$

Making use of available single degree relative velocity response spectrum curves, $S_V(\omega)$ vs. ω, for the specified ground acceleration, $\ddot{U}_g(t)$, the absolute maximum values for the generalized coordinates Y_{n1} and Y_{n2} can be determined using the relations

$$|Y_{n1}(t)|_{max.} = \frac{\gamma_n S_v(\omega_{n1})}{\omega_{n1}} \quad \frac{(M_n \Phi_{n1} + m_a \Phi_{a1})}{(M_n \Phi_{n1}^2 + m_a \Phi_{a1}^2)}$$

$$|Y_{n2}(t)|_{max.} = \frac{\gamma_n S_v(\omega_{n2})}{\omega_{n2}} \quad \frac{(M_n \Phi_{n2} + m_a \Phi_{a2})}{(M_n \Phi_{n2}^2 + m_a \Phi_{a2}^2)} \qquad (8)$$

Accepting at this point the square root of the sums of the squares approach, the seismic coefficient C_{an} as previously defined becomes

$$C_{an} = (\omega_a^2/g) \left[|Y_{n1}(t)|_{max.}^2 \ (\Phi_{a1} - \Phi_{n1})^2 + |Y_{n2}(t)|_{max.}^2 \ (\Phi_{a2} - \Phi_{n2})^2 \right]^{1/2} \qquad (9)$$

The combined seismic coefficient for the set-back structure would then be approximated using Eq. (6).

B. Coupled Lateral-Torsional Motion of Eccentric Buildings

1. Two degree of freedom analysis - It is fairly common to design a building with centers of gravity of the floor masses being located on one vertical axis through the building and with the elastic centers (or centers of twist) being located on another vertical axis; thus, providing an eccentricity which couples torsional motion with bending. A building of this type is represented schematically in Fig. 2a. Assuming ground motion in a direction normal to the plane of above mentioned axes, the amount of torsion which couples with any one of the bending modes can be predicted using the two degree of freedom model shown in Fig. 2b where M_n and K_{nb} represent the generalized mass and generalized spring constant, respectively, for the nth lateral mode of vibration (without eccentricity) and K_{nt} represents the generalized spring constant for the nth torsional mode. This two degree of freedom model is presented again in Fig. 3 where r is the radius of gyration of mass M_n, $r\delta$ is the eccentricity, $r(1 + \Delta)$ is the radius of torsional stiffness, q_{1n} is a generalized coordinate measuring translation of the mass system, and where q_{2n} is a generalized coordinate measuring rotation of the mass system about the center of twist. If

coordinate q_{1n} is to represent the nth lateral mode displacement of the top of the building, it is once again necessary that the support excitation of the model be prescribed in accordance with Eq. (3). It can easily be shown for the model in Fig. 3 that the generalized polar mass moment of inertia J_n, the generalized torsional spring constant K_{nt}, the uncoupled ($\delta = 0$) torsional frequency ω_{nt}, and the uncoupled ($\delta = 0$) bending frequency ω_{nb} are given, respectively, by the relations

$$J_n = M_n r^2 \; ; \; K_{nt} = K_{nb} r^2 (1 + \Delta)^2 \; ; \; \omega_{nt}^2 = \frac{K_{nt}}{J_n} \; ; \; \omega_{nb}^2 = \frac{K_{nb}}{M_n} \tag{10}$$

It should be noted that (1) when $\delta = 0$ there is no eccentricity, i.e. torsion and bending are completely uncoupled, (2) when both δ and Δ equal zero the frequencies of the uncoupled torsional and bending modes are equal, (3) when $\delta = 0$ and Δ is greater than zero, the torsional frequency exceeds the bending frequency, and (4) when $\delta = 0$ and Δ is less than zero, the bending frequency exceeds the torsional frequency.

Damping is not present in the model as shown in Fig. 3, but will be introduced subsequently into the generalized equations of motion. To establish these equations of motion, consider first the undamped system for which the force and moment equations of motion in terms of coordinates q_{1n} and q_{2n} are, respectively,

$$\begin{aligned} M_n \ddot{q}_{1n} + M_n r\delta \, \ddot{q}_{2n} + K_{nb} \, q_{1n} &= -M_n \ddot{U}_g(t) \, \gamma_n \\ M_n r\delta \, \ddot{q}_{1n} + (M_n r^2 \delta^2 + M_n r^2) \, \ddot{q}_{2n} + K_{nt} q_{2n} &= -M_n r\delta \, \ddot{U}_g(t) \, \gamma_n \end{aligned} \tag{11}$$

Substituting the last two of Eqs. (10) into Eqs. (11) and dividing by M_n and $M_n r\delta$, respectively, gives the relations

$$\begin{aligned} \ddot{q}_{1n} + r\delta \, \ddot{q}_{2n} + \omega_{nb}^2 \, q_{1n} &= -\ddot{U}_g(t) \, \gamma_n \\ \ddot{q}_{1n} + (r\delta + \frac{r}{\delta}) \, \ddot{q}_{2n} + \omega_{nt}^2 \, \frac{r}{\delta} \, q_{2n} &= -\ddot{U}_g(t) \, \gamma_n \end{aligned} \tag{12}$$

Using the linear transformation

$$\begin{aligned} X_n &\equiv q_{1n} + r\delta \, q_{2n} \quad ; \quad & q_{1n} &= (1 + \delta^2) \, X_n - \delta^2 X_t \\ X_t &\equiv q_{1n} + (r\delta + \frac{r}{\delta}) \, q_{2n} \quad ; \quad & q_{2n} &= -\frac{\delta}{r} (X_n - X_t) \end{aligned} \tag{13}$$

and introducing the dimensionless parameter

$$\beta \equiv \frac{\delta^2}{(1 + \Delta)^2} \tag{14}$$

Eqs. (12) become

$$\ddot{X}_n + \omega_{nb}^2 X_n - \beta\omega_{nt}^2 (X_t - X_n) = -\ddot{U}_g(t) \gamma_n$$
$$\ddot{X}_t + \omega_{nt}^2 (X_t - X_n) = -\ddot{U}_g(t) \gamma_n \tag{15}$$

It is significant to note that Eqs. (15) are identical to Eqs. (4) when the damping ratios ξ_n and ξ_a are set equal to zero; thus, it is obvious that the two degree of freedom system shown in Fig. 1b can also be used to study the response of the two degree of freedom system shown in Fig. 3.

Appropriate damping can now be introduced into the torsion-bending model (Fig. 3) using its equivalent model shown in Fig. 1; thus, damping terms similar to those shown in Eqs. (4) are introduced into Eqs. (15) as follows:

$$\ddot{X}_n + 2\omega_{nb} \xi_b \dot{X}_n + \omega_{nb}^2 X_n - 2\beta\, \omega_{nt} \xi_t(\dot{X}_t-\dot{X}_n) - \beta\, \omega_{nt}^2(X_t-X_n) = -\ddot{U}_g(t) \gamma_n$$
$$\ddot{X}_t + 2\omega_{nt} \xi_t(\dot{X}_t-\dot{X}_n) + \omega_{nt}^2(X_t-X_n) = -\ddot{U}_g(t) \gamma_n \tag{16}$$

It should be noted that for small values of δ, i.e. small eccentricities, coordinate X_n represents primarily bending motion with only small torsional motion added while coordinate X_t represents primarily torsional motion with only small bending motion added. Therefore, damping ratios ξ_n and ξ_t, in effect, represent the bending and torsional damping, respectively.

As previously indicated, the above coupled equations of motion can be solved numerically for any prescribed ground acceleration $\ddot{U}_g(t)$ and for any set of parameters ω_{nb}, ω_{nt}, ξ_n, ξ_t, and β to yield the desired torsional seismic coefficient C_{tn} which is defined as

$$C_{tn}(\omega_{nb},\omega_{nt},\xi_n,\xi_t,\beta,\gamma_n) \equiv |(X_t-X_n)\, \omega_{nt}^2/g|_{\max.} \tag{17}$$

This seismic coefficient is simply the ratio of the maximum dynamic torsion developed at the base of the building to the static torsion developed at this same location by a 1g lateral loading. Note that Eq. (17) is identical in form to Eq. (5).

Obtaining C_{tn} for $n = 1,2,\ldots,r$, one can determine the true torsional seismic coefficient C_t resulting from the first r lateral vibration modes using the approximate relation

$$C_t = (C_{t1}^2 + C_{t2}^2 + \ldots + C_{tr}^2)^{1/2} \tag{18}$$

Again, as with Eq. (6), only the first 2 or 3 modes of vibration need be considered in evaluating C_t by Eq. (18).

2. Single-degree of freedom analysis - Since the torsion bending problem governed by Eqs. (16) is exactly the same as the set-back problem governed by Eqs. (4), the previously developed single degree

of freedom analysis can be applied directly to the torsion-bending problem; therefore, no further discussion of this method is required.

ACCURACY OF APPROXIMATE METHODS OF ANALYSIS

To determine the accuracy of the two approximate methods of analysis previously described, these methods were used to evaluate the set-back (or appendage) seismic coefficient C_a using the 6-story shear building shown in Fig. 4 on top of which a single generalized mass m_a representing the set-back structure (or appendage) was placed. The seismic coefficients obtained by these approximate methods, using a horizontal ground acceleration $\ddot{U}_g(t)$ corresponding to the N-S component of the 1940 El Centro earthquake, are shown in Figs. 5 and 6 where coefficient C_a is plotted against period T_a of the set-back structure (or appendage). Also shown in Figs. 5 and 6 are the seismic coefficients obtained by an exact multi-degree of freedom analysis. Mass m_a equals 0.001 times the total mass of the building in Fig. 5 and 0.01 times the total mass of the building in Fig. 6. The damping ratio ξ_a equals 0.02 in each case. The periods of vibration $(T_1, T_2, \ldots, T_6)$ of the 6-story building are indicated along the abscissa in each figure.

A comparison of the results in Fig. 5 shows that both approximate methods of analysis are in reasonable agreement with the exact method except in those regions where the frequency of the set-back structure (or appendage) approaches the frequency of one of the lower building modes. In these regions, the two degree of freedom approximate method agrees quite well with the exact method; however, the single degree of freedom approximat method is obviously considerably in error. Fig. 6, which represents a larger mass m_a by a factor of ten, shows reasonable agreement of all three methods over the entire period range.

SINGLE DEGREE OF FREEDOM ANALYSIS NEAR RESONANCE

For small values of the mass ratio β_n, the single degree of freedom method of analysis as previously presented is considerably in error when the frequency ω_a is near the frequency of one of the lower modes of the building. To provide additional information on the behavior in such cases, the two degree of freedom system shown in Fig. 1a was subjected to a support acceleration $U_s(t)$ corresponding to the N-S component of the El Centro 1940 earthquake; i.e., γ_n was set equal to unity, and its forced response was determined by solving the two generalized equations of motion.

The coupling of the above mentioned generalized equations of motion due to damping was neglected for this particular study; thus, yielding equations of motion for two separate single degree of freedom systems whic are characterized by the generalized quantities of Eqs. (7). The error introduced by neglecting the coupling of the generalized equations of motion due to damping was investigated and found to be small for the damping ratios considered in this particular study.

The results of the above general investigation to determine the so-called "resonance" effects are presented in Figs. 7 and 8. The seismic coefficients are plotted as ordinates in each of these figures with period T_n as the abscissa in Fig. 7 and mass ratio β_n as the abscissa in Fig. 8.

In Fig. 7, mass ratio β_n, period T_a, damping ratio ξ_n, and damping ratio ξ_a are held constant and equal to 0.001, 0.40 seconds, 0.05, and 0.02, respectively. Curve No. 1 shows the contribution of the first mode of vibration to the seismic coefficient C_a, Curve No. 2 shows the contribution of the second mode, Curve No. 3 shows the total coefficient based on the square root of the sum of the squares of the 1st and 2nd mode maxima, and Curve No. 4 is the exact coefficient. As the period T_n approaches T_a, it is clear that the square root of the sums of the squares approach is appreciably in error. This error results because the frequencies of the two degree system are very close to each other and are greatly out of phase during the critical period of the earthquake; thus, giving first and second mode contributions which are of opposite sign.

It is evident that when the earthquake ground motion retains its high intensity level over a relatively long duration, the two above mentioned mode contributions will come more into phase with each other; thus, producing very large seismic coefficients. This behavior can be clearly demonstrated by calculating the response of the two degree system of Fig. 1b to a stationary Gaussian, random support acceleration which has a constant power spectral density; i.e., is a "white" input. Crandall and Mark have thoroughly studied this problem and show, for example, that the mean square acceleration of the secondary mass m_a becomes extremely large as the frequency ratio ω_a/ω_n approaches unity(7). This increase in response at "resonance" is even more pronounced for the stationary random input than for the transient El Centro 1940 input previously discussed.

In Fig. 8 "resonance" is maintained; i.e., $T_a = T_n = 0.40$ seconds; therefore, this graph shows the effect of mass ratio β_n on the appendage seismic coefficient for this resonant condition. Damping ratios are similar and Curves 1-4 represent the same quantities as in Fig. 7. Large errors in the single degree method of analysis are observed for $\beta_n < 0.005$.

TWO DEGREE OF FREEDOM RESPONSE SPECTRA

The two degree of freedom method of analysis has been shown to yield good results even in the vicinity of the so-called "resonance" range; therefore, this method can be used reliably by the structural designer for either the "set-back" problem or the "torsion-bending" problem.

The "set-back" seismic coefficient C_{an} as defined by Eq. (5) has been determined for a wide range of the parameters ω_n, ω_a, ξ_n, ξ_a, and β_n and for $\gamma_n = 1$ by solving Eqs. (4) numerically using the N-S component of the 1940 El Centro earthquake as the prescribed excitation $\ddot{U}_g(t)$. This coefficient is plotted in Fig. 9 as a function of period T_n for 5 percent of critical damping and for various combinations of period T_a and mass ratio β_n. Similar plots for other values of damping have been published previously by J. Penzien and A. Chopra(3). The numerical values of C_{an} given in these plots are based on a unit value for the participation factor γ_n. To obtain C_{an} for $\gamma \neq 1$, simply multiply the value taken from these plots by the value of γ_n.

Due to the fact Eqs. (4) and (16) are of identical form, the ordinates of Fig. 9 also represent the "torsion-bending" seismic coefficient C_{tn} defined by Eq. (17) provided the abscissa $T_n = 2\pi/\omega_{nb}$, $\omega_a = \omega_{nt}$, $\xi_n = \xi_n$, $\xi_a = \xi_t$, and $\beta_n = \beta = \delta^2/(1 + \Delta)^2$.

INFLUENCE OF INELASTIC DEFORMATIONS

The influence of permitting inelastic deformations to be developed in the basic model (Fig. 2a) is now under investigation by J. Penzien and D. E. Ross[II]. Preliminary results of this investigation indicate that the maximum relative displacement response, $|X_a - X_n|_{max.}$, are considerably reduced for longer period systems ($T_n > 0.6$ seconds) which possess "resonant" characteristics, i.e. $T_n \doteq T_a$. These results are to be expected as the "resonant" influence is eliminated once appreciable yielding takes place. However, for short period systems ($T_n = 0.2$-0.3), the maximum relative displacement response, $|X_a - X_n|_{max.}$, are considerably increased when they possess "resonant" characteristics. These results are to be expected also on the basis of previous studies of the inelastic response of single degree of freedom systems.

CONCLUDING REMARKS

The following conclusions and recommendations are based upon the results of the investigation reported herein:

(1) The two degree of freedom method of analysis accurately predicts the set-back seismic coefficient C_{an} even when the period of the set-back structure coincides with the fundamental period of the building. This method of analysis also accurately predicts the "torsion-bending" seismic coefficient C_{tn} even when the period of the fundamental torsional mode of vibration equals the period of the fundamental lateral mode of vibration. Thus, the availability of the two degree of freedom response spectra makes this method of analysis practical in each case.

(2) The single degree of freedom method of analysis is considerably in error when the period of the set-back structure is near one of the lower periods of the main building which supports it. This method also is considerably in error when the period of the fundamental torsional mode is near the fundamental lateral vibration mode. Therefore, this method should not be used in these cases.

(3) To greatly reduce the seismic forces in a set-back structure (or appendage), it should be designed so that its fundamental period of vibration differs considerably (preferably higher) from the first lateral vibration mode of the building and also does not coincide with the periods of other lower lateral vibration modes. Likewise, to reduce the seismic forces in a building caused by an eccentricity, it should be designed to that its fundamental period in torsion differs considerably (preferably higher) from the first lateral vibration mode of the building.

(4) The seismic forces in a set-back structure (or appendage) and the seismic torsion developed in an eccentric building can be much larger than predicted by standard methods. For example, consider a multi-story

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II. Graduate Student, University of California, Berkeley building having a uniform eccentricity equal to 5 percent of the radius of gyration (δ = 0.05), damping of 5 percent in both torsion and bending ($\xi_n = \xi_t$ = 0.05), a fundamental bending period of 0.6 seconds (T_n = 0.6), and a fundamental torsional period assuming no coupling of 0.6 seconds ($T_t = 2\pi/\omega_{1t} = 0.6$). Fig. 9 gives a torsion seismic coefficient C_{1t} equal to 3.8 (β = 0.0025) which is about 6 times as large as the seismic coefficient for the fundamental lateral vibration mode. Thus, applying the static design lateral loading through the mass centers and assuming that the eccentricity present (δ = .05) will give the proper proportion of torsion to lateral loading is erroneous. In this case, torsion is underestimated by a factor of 6. If, in the above example, all parameters given remain the same except T_t is changed from 0.6 to 0.4 seconds, then Fig. 9 gives a torsion seismic coefficient C_{1t} equal to 1.8 in which case torsion is under estimated by a factor of 3. Further spreading of the periods will, of course, lower the torsional seismic coefficient even more.

(5) The seismic forces developed in a set-back structure (or appendage) and the seismic torsion forces developed in an eccentric building, assuming elastic systems, are much larger than standard code values even when designed in accordance with the recommendations of (3) above; therefore, the desirable effects of inelastic deformations must be considered as is standard practice in the design of buildings.

The quantitative results presented in this paper are based on the N-S component of ground motion recorded during the 1940 El Centro, California earthquake. Considering the fact that such results differ appreciably when using other earthquake ground motions of similar intensity, one must use the data presented herein with caution and with good engineering judgment.

REFERENCES

(1) Alford, J. L., G. W. Housner, and R. R. Martel, Spectrum Analyses of Strong-Motion Earthquakes, Earthquake Research Laboratory, California Institute of Technology, August, 1951.

(2) Clough, R. W., Earthquake Analysis by Response Spectrum Superposition, Bulletin of the Seismological Society of America, Vol. 52, No. 3, July, 1962.

(3) Penzien, J., and A. Chopra, Earthquake Response of Appendage on Multi-Story Building, Proceedings of the Third World Conference on Earthquake Engineering, New Zealand, Vol. II, 1965.

(4) Skinner, R. I., D. W. C. Skilton, and D. A. Laws, Unbalanced Building and Buildings with Light Towers Under Earthquake Forces, Proceedings of the Third World Conference on Earthquake Engineering, New Zealand, Vol. II, 1965.

(5) Shibata, H., H. Sato, and T. Shigeta, Aseismic Design of Machine Structure, Proceedings of the Third World Conference on Earthquake Engineering, New Zealand, Vol. II, 1965.

(6) Wilson, E. L., and R. W. Clough, Dynamic Response by Step by Step Matrix Analysis, Symposium on the Use of Computers in Civil Engineering, Lisbon Portugal, October, 1962.

(7) Crandall, S. H., and W. D. Mark, Random Vibration in Mechanical Systems, Academic Press, New York and London.

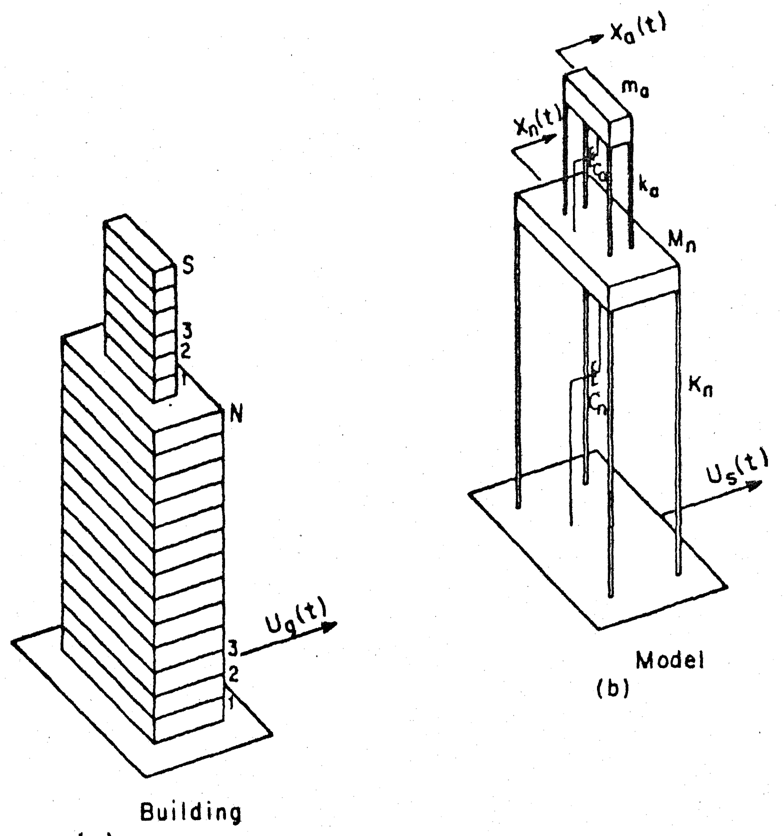


FIG. 1 SYMMETRIC BUILDING WITH SETBACK AND ITS IDEALIZED MODEL

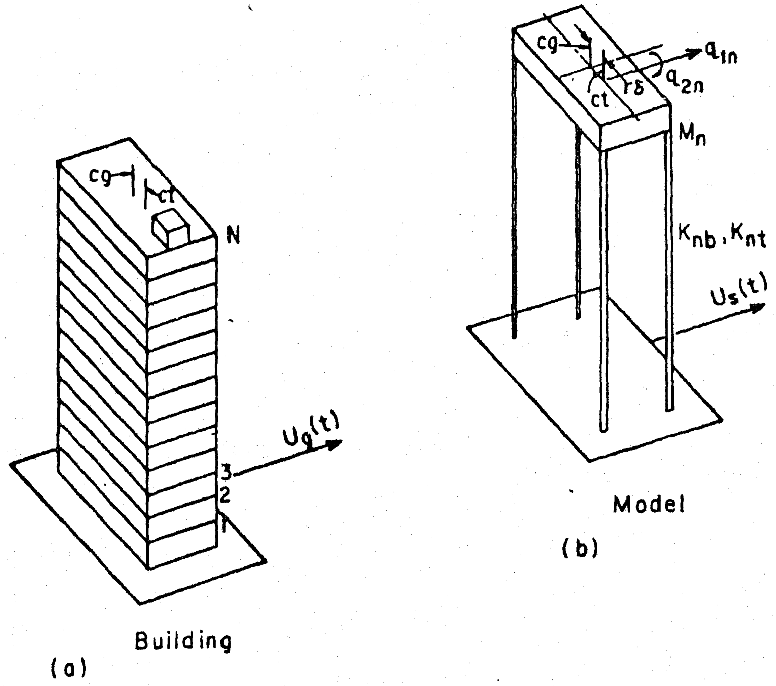


FIG. 2 BUILDING HAVING AN ECCENTRICITY $r\delta$ AND ITS IDEALIZED MODEL

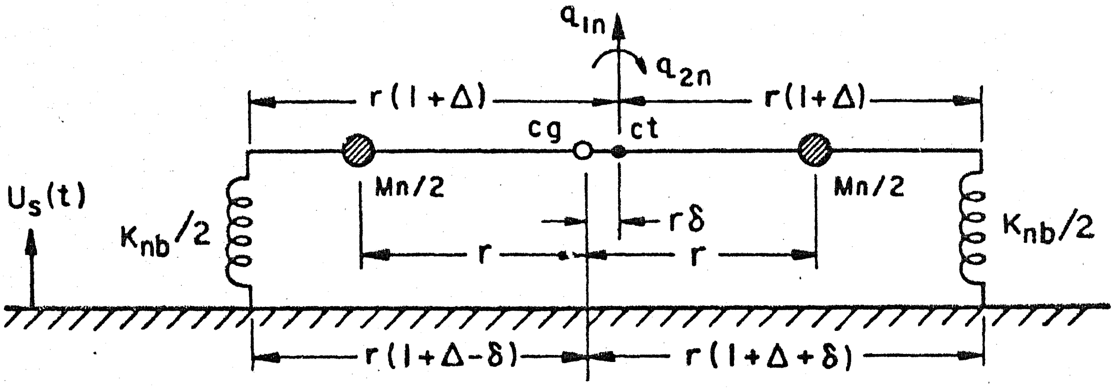
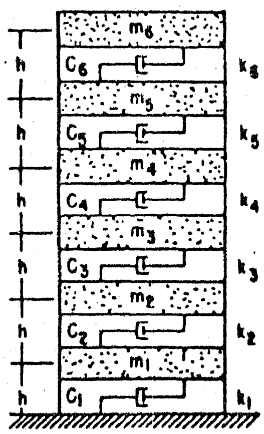


FIG. 3 IDEALIZED MODEL OF BUILDING HAVING AN ECCENTRICITY rδ



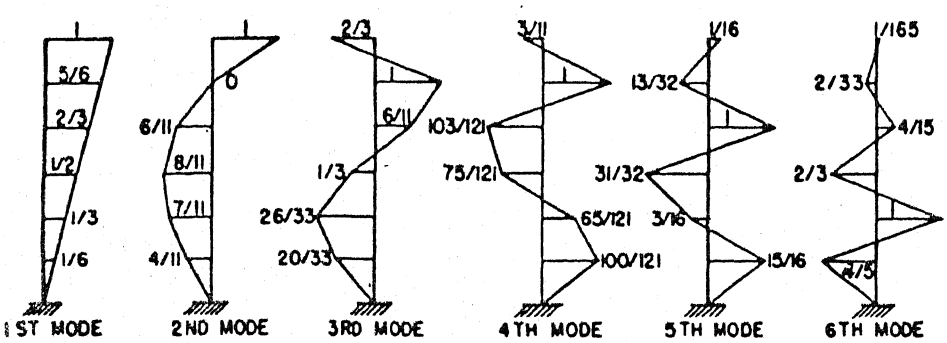
FLOOR	m_i/m_1	k_i/k_1	c_i/c_1	STORY
6	1			
		6/21	6/21	6
5	1			
		11/21	11/21	5
4	1			
		15/21	15/21	4
3	1			
		18/21	18/21	3
2	1			
		20/21	20/21	2
1	1			
		21/21	21/21	1

WHERE:

$$M = \sum_{i=1}^{6} m_i$$

$$k_1 = \frac{14\pi^2 M}{T_1^2}$$

$$c_1 = \frac{14\pi M\lambda}{T_1}$$

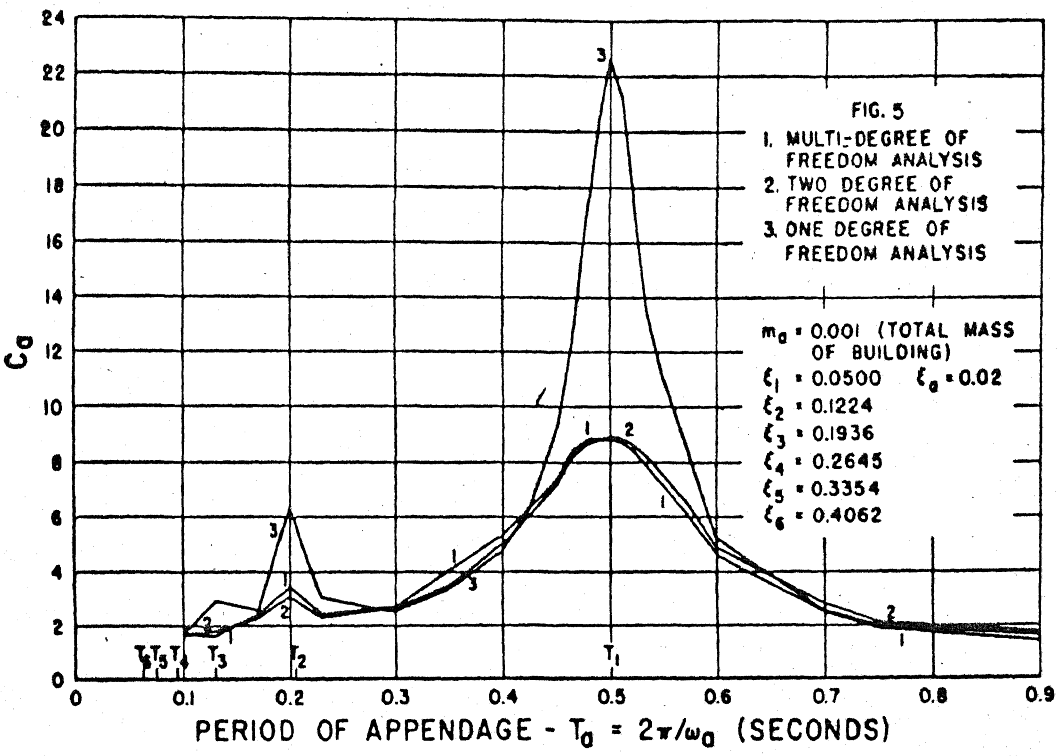


MODE NO.	$(T_i/T_1)^2$
1	1
2	1/6
3	1/15
4	1/28
5	1/45
6	1/66

$$T_1^2 = \frac{14\pi^2 M}{k_1}$$

LET $T_1 = 0.5$ SEC.

FIG. 4 - PROPERTIES OF 6-STORY BUILDING



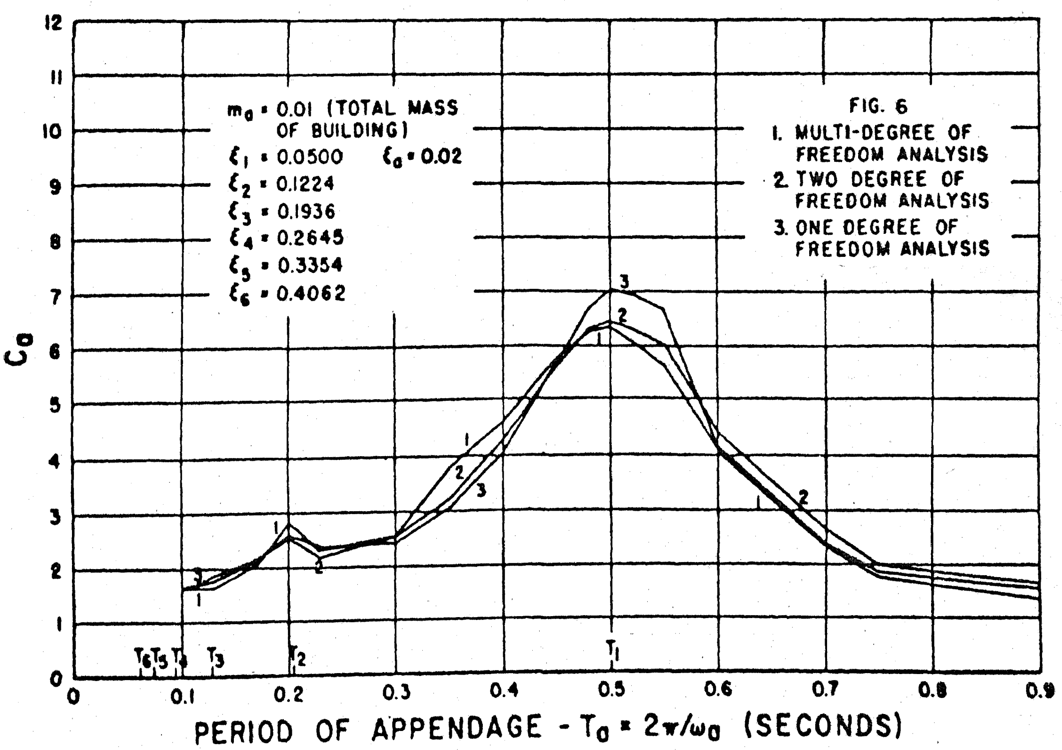


FIG. 5,6 - COMPARISON OF METHODS OF ANALYSIS

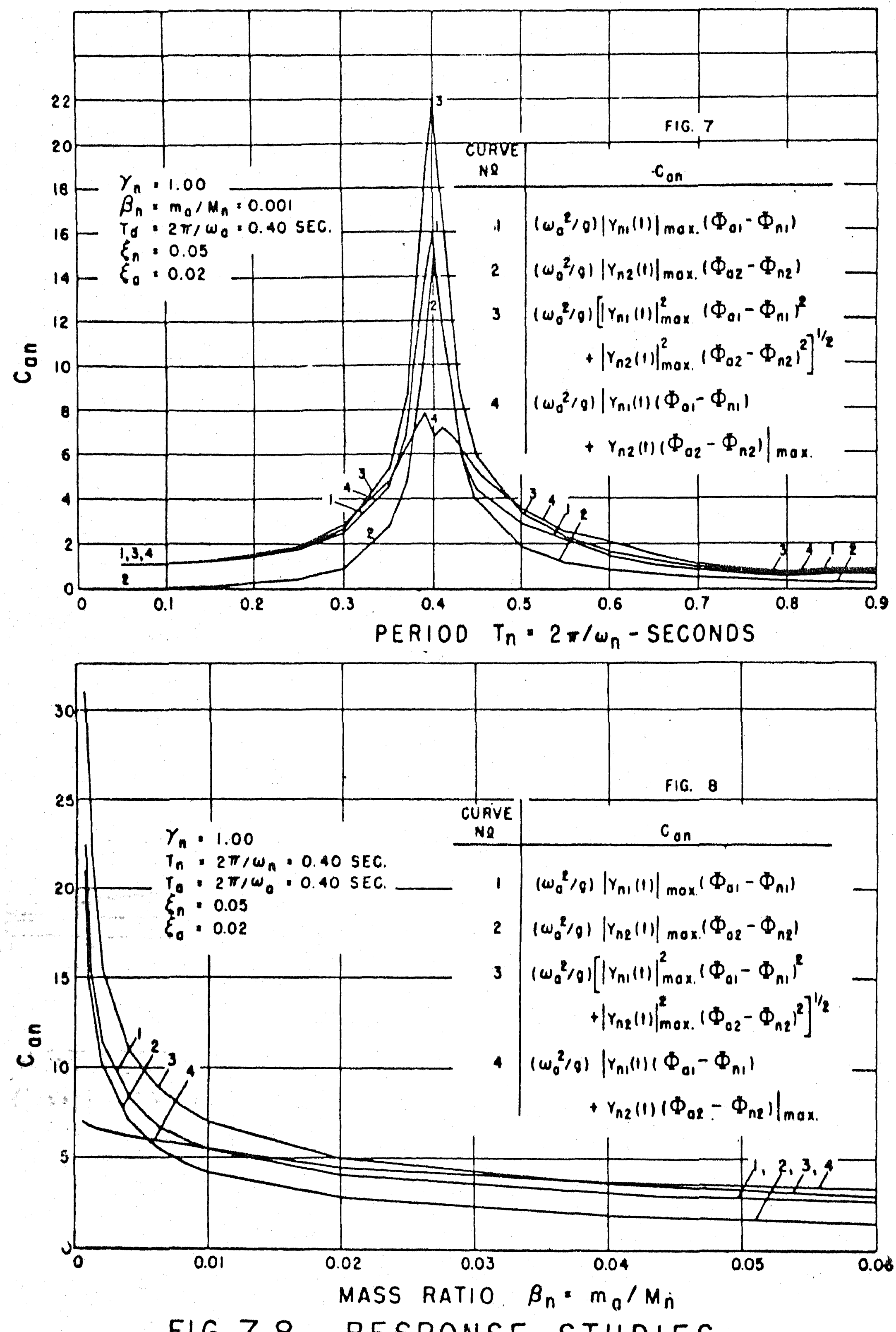


FIG. 7,8 RESPONSE STUDIES–
TWO MASS SYSTEM OF FIGURE 1.

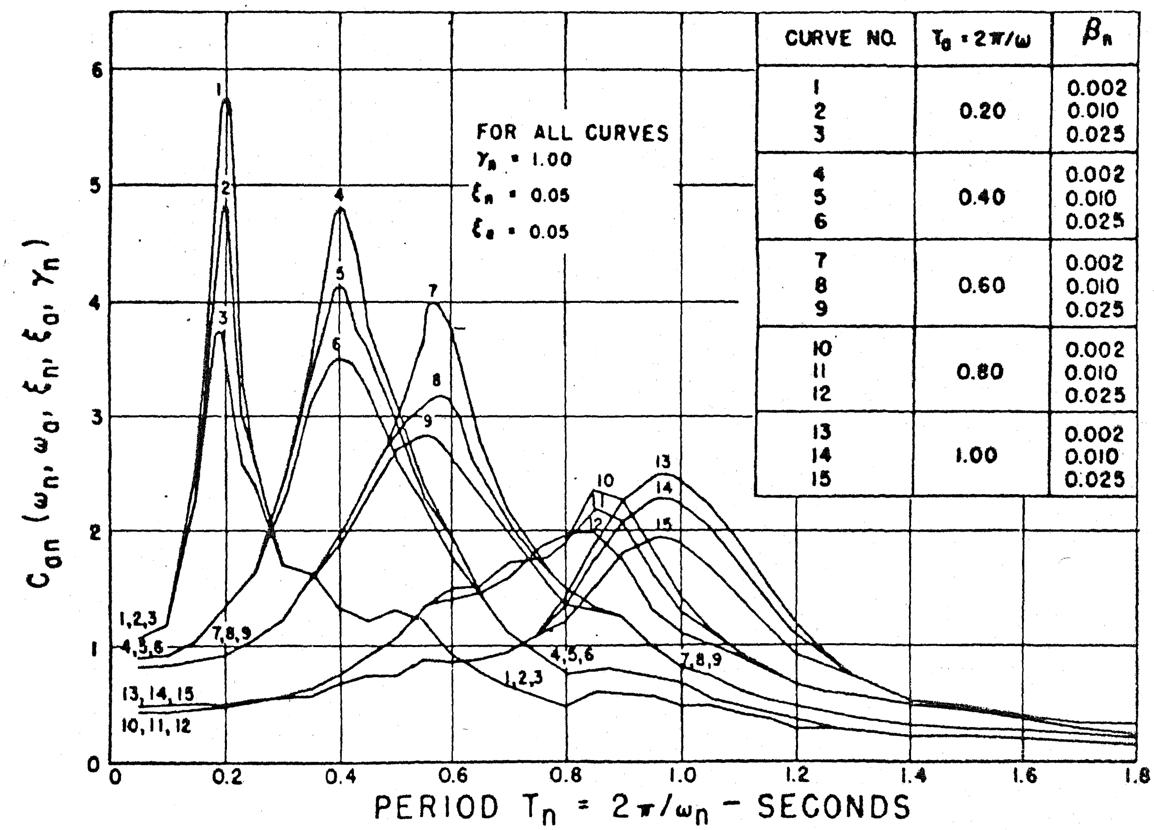


FIG. 9 —TWO DEGREE RESPONSE SPECTRA N-S COMPONENT, EL CENTRO EARTHQUAKE, 1940