SPECTRUM TECHNIQUES FOR TALL BUILDINGS

by

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SYNOPSIS

The linear response of tall buildings to earthquake motions is studied by means of average spectral properties of strong earthquakes and frequency characteristics and modal geometries which measurements show to be common to many tall buildings. Attention is focused on the effects of increasing height. It is found from use of simple models for the modes and frequencies of tall buildings that the maximum deflection and overturning moment tend to increase linearly with height, the average base shear tends to be independent of height, and the expected value of the maximum acceleration on the top of the structure decreases approximately as the inverse square root of height. It is concluded that as more information becomes available the common properties of tall buildings can be used to formulate simplified design procedures for such structures.

GLOSSARY

$a(\ell)$, $a_n(\ell)$	= acceleration at the top of the structure, contribution of n^{th} mode;
h	= average interfloor height = ℓ/N;
ℓ	= building height;
$m(x)$, m	= mass per unit length, constant mass per unit length;
M, M_n	= base overturning moment, contribution of n^{th} mode;
n	= mode index;
N	= number of stories;
r	= a parameter greater than zero;
S_{vn}	= velocity spectrum value for n^{th} mode;
S_v	= average velocity spectrum value, independent of frequency;
t	= time;
T_n	= natural period of n^{th} mode = $2\pi/\omega_n$;

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$v(x, t)$ = lateral deflection;

$v(\ell)$, $v_n(\ell)$ = top floor deflection, contribution of n^{th} mode;

V, V_n = base shear, contribution of n^{th} mode;

x = vertical coordinate;

y_n = n^{th} normal coordinate;

$\ddot{z}(t)$ = ground acceleration;

α_n = participation factor (see Eq. 3);

β, C_1, C_2 = constants;

$\varphi_n(x)$ = n^{th} mode shape;

ζ_n = damping factor for n^{th} mode; and

ω_n = natural frequency of n^{th} mode.

INTRODUCTION

The number of tall buildings in the seismic regions of the western United States has increased rapidly in the past few years, most dramatically perhaps in Los Angeles where the recent removal of the 13 story height limit has allowed high-rise building to take place. In Los Angeles there are now several buildings over thirty stories and buildings over fifty stories are in the design stages. Similar construction also has taken place in San Francisco and other west coast cities. Because data on the earthquake response of tall buildings is very limited, there is widespread interest in the earthquake performance of these structures among engineers and other professional people, and among the general populace.

In this paper the well-known average spectral properties of strong earthquake motions (1) are used together with properties which appear to be common to many tall buildings to investigate the expected earthquake response of such structures within the elastic range, and in particular to examine the effects of increasing height.

PROPERTIES OF TALL BUILDINGS

For the tall buildings now being built in California and elsewhere, it is apparent that architectural, engineering, and economic considerations have dictated that most of these structures have certain common properties. The most obvious common property is that the buildings have a cross-section that is constant with height, which implies that the mass per story is approximately constant throughout the building. Secondly, it is clear that most of the buildings are rectangular in plan, or in some instances slightly cruciform. Typical plan dimensions for a rectangular building might be 100 ft. by 200 ft. The third property shared by most of the tall buildings because of code requirements is open, moment resisting frame construction. Both steel and reinforced concrete have been used, although steel frames are most common, especially for the tallest buildings.

Because of their similarities in form and construction it is not surprising that tall buildings also share many dynamic characteristics as evidenced by measured frequencies and mode shapes. Table I and Figs. 1-3 summarize data available for tall buildings for which several frequencies and mode shapes have been measured. The numbers on Figs. 1-3 refer to the buildings listed in Table I. All of these results are from forced vibration tests of structures or from measurements of wind induced motions. The results in Table 1 show that the ratios of higher frequencies to the fundamental frequency exhibit a pattern common to all the buildings, particularly the taller, steel frame structures, buildings 2, 3 and 4. Furthermore, the period ratios are very similar to those for the well-known shear beam. It is seen too that the fundamental period is approximately proportional to the number of stories, with a constant of proportionality of about 0.1 for the steel-frames structures and 0.06 or 0.07 for the concrete buildings.

The first mode responses shown in Fig. 1, which have been normalized with respect to the top floor deflection, are not too dissimilar considering the variety of structures included. It is significant, too that the shorter buildings tend to lie on the right in Figure 1, and the taller structures on the left. Figure 1 shows that, as a general rule, most fundamental modes of tall buildings tend to resemble straight lines more nearly than the first mode shapes of either bending beams or shear beams.

The second modes shown in Fig. 2 and the third modes in Fig. 3 also show strong similarities. (The points at $x/\ell = 0.7$ in Fig. 3 for building 2 are thought to be in error.) In Figs. 2 and 3 the modes are again normalized with respect to the roof deflection, with the exception of the third transverse mode of building 1. It has since been found that normalization by setting the integral of $\varphi^2(x)$ equal to unity would improve the agreement shown in Figs. 2 and 3.

The degree of similarity of the mode shapes shown in Figs. 1-3 is made more clear by Fig. 4 (5) which shows the first three modes of building 5 at various stages of construction. All measurements were taken after the structural frame of the building itself was complete. Building 5 is not free-standing in its final form although it was designed as such. Rather, it is connected structurally at the base and "non-structurally" at each level to a service tower and to a corridor connecting the building to an elevator tower and two other buildings. Although free-standing structures would be expected to exhibit less scatter in presentations such as Fig. 4, comparison of Figs. 1-3 with Fig. 4 does suggest that the difference in mode shapes of different tall buildings might not be much larger than the uncertainty involved in knowing the true mode shape of a particular structure.

The data for the comparisons discussed above are admittedly sparse and many more measurements need to be taken. However, certain trends in both the frequency distributions and the mode shapes are clearly present and it is possible to study the earthquake response of tall buildings by use of sets of simple modes and frequencies which approximate the measured results.

EARTHQUAKE RESPONSE

If the response of a structure to earthquake motion $\ddot{z}(t)$ is written as a sum of modal responses

$$v(x, t) = \sum_{n=1}^{N} y_n(t)\varphi_n(x) \tag{1}$$

in which $\varphi_n(x)$ is the mode shape and y_n the normal coordinate, then it is well-known that the normal coordinates are governed by

$$\ddot{y}_n(t) + 2\zeta_n\omega_n\dot{y}_n(t) + \omega_n^2 y_n(t) = -\alpha_n\ddot{z}(t) \tag{2}$$

with

$$\alpha_n = \frac{\int_0^{\ell} m(x)\varphi_n(x)dx}{\int_0^{\ell} m(x)\varphi_n^2(x)dx} \tag{3}$$

The maximum value of y_n can be found from the response spectra for the intensity of motion under consideration. In terms of the velocity spectrum value appropriate for each mode,

$$y_n\Big|_{max} = \frac{\alpha_n S_{vn}}{\omega_n}; \qquad \dot{y}_n\Big|_{max} = \alpha_n S_{vn}; \qquad \left[\ddot{y}_n + \alpha_n\ddot{z}(t)\right]_{max} = \alpha_n\omega_n S_{vn} \tag{4}$$

With the maximum values of the normal coordinates known, the contributions of each mode to response parameters of more direct engineering interest can be found. Thus, for example, the maximum deflection, base shear, overturning moment and top floor acceleration contributed by the n^{th} mode are, respectively:

$$v_n(\ell) = \frac{\alpha_n S_{vn} \varphi_n(\ell)}{\omega_n} \quad ; \quad V_n = \alpha_n^2 \omega_n S_{vn} \int_0^\ell m(x) \varphi_n^2(x) dx \; ;$$

$$M_n = \alpha_n \omega_n S_{vn} \int_0^\ell x m(x) \varphi_n(x) dx \; ; \quad a_n(\ell) = \alpha_n \omega_n S_{vn} \varphi_n(\ell) \tag{5}$$

In Eq. 5 the influence of modal geometry appears in the integrals and in α_n; the frequency characteristics of the structure in ω_n; and the excitation frequencies and structural damping in the spectral values, S_{vn}.

For the application of Eq. 5 to tall buildings, the mass distribution, m(x), can be taken as a constant, m. Also, for design purposes and for the purpose of investigating general trends in the response, it is reasonable to take the damping to be constant in all modes of response for the tall buildings and to assume that the average velocity spectra S_{vn} has a constant value, S_v. This approximation tends to overemphasize the effects of modes with periods shorter than about 0.5 sec. Further, for purposes of establishing the general effect of increasing height, the fundamental period is taken to be proportional to the number of stories over the range of interest.

$$T_1 = \beta N \tag{6}$$

APPROXIMATION BY SHEAR BEAM MODES AND FREQUENCIES

As a first approximation to the frequency and modal properties of tall buildings, consider the uniform cantilever shear beam for which

$$\frac{\omega_n}{\omega_1} = (2n-1) \quad ; \quad \varphi_n(x) = \sin \frac{(2n-1)\pi x}{2\ell} \tag{7}$$

The frequencies of the shear beam agree well with those for tall buildings, and the higher mode shapes are fairly close to those measured for structures. However, the fundamental mode is obviously inaccurate and this mode will be examined separately below.

With this simple set of frequencies and mode shapes some trends in the response of tall structures can be found by evaluating the individual modal contributions of Eq. 5. These modal responses are then combined to approximate the total behavior of the structure.

It is important to note that approximating the modes and frequencies of the tall buildings by those of a shear beam is a matter of mathematical convenience and does not imply that shear beams are suitable models for tall buildings in all respects.

Maximum Deflection

With the use of Eqs. 6 and 7

$$v_n(\ell) = \frac{2S_v \beta N}{\pi^2(2n-1)^2} \tag{8}$$

Hence, the total displacement is bounded by

$$v(\ell) \leq \frac{2S_v \beta N}{\pi^2} \left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots\right\} \tag{9}$$

And an estimate of the most probable value of $v(\ell)$ (9) is given by

$$v(\ell) \approx \frac{2S_v \beta N}{\pi^2} \left\{1 + \frac{1}{81} + \frac{1}{625} + \cdots\right\}^{\frac{1}{2}} \tag{10}$$

Equations 9 and 10 show, as might be expected, that the maximum deflection is proportional to the number of stories and that the value is determined almost exclusively by the first mode response.

Base Shear

Letting the height of the building ℓ equal N times the story height h, it is found that the base shear is given by

$$V_n = \frac{16mhS_v}{\beta\pi} \cdot \frac{1}{(2n-1)} \tag{11}$$

Thus,

$$V \leq \frac{16mhS_v}{\beta\pi} \left\{1 + \frac{1}{3} + \frac{1}{5} + \cdots\right\} \tag{12}$$

$$V \approx \frac{16mhS_v}{\beta\pi} \left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots\right\}^{\frac{1}{2}} \tag{13}$$

The series in Eq. 12 is not divergent because the number of terms cannot exceed N. These results show that on the average, the base shear for the tall buildings tends to be independent of height. Also, the first mode is, in general, the largest contributor, but Eqs. 11 and 12 indicate that in a particular earthquake it would be possible for higher mode contributions to be very important, depending on the amplitude of the spectra at the frequencies of the structure.

Overturning Moment

Under the foregoing conditions

$$M_n = \frac{32mh^2 S_v N}{\beta\pi^2} \frac{1}{(2n-1)^2} \tag{14}$$

and

$$M \le \frac{32mh^2 S_v N}{\beta\pi^2} \left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots\right\} \tag{15}$$

$$M \approx \frac{32mh^2 S_v N}{\beta\pi^2} \left\{1 + \frac{1}{81} + \frac{1}{625} + \cdots\right\}^{\frac{1}{2}} \tag{16}$$

Equations 14-16 show that the overturning moment tends to increase linearly with height, and that its value is determined essentially by the first mode response. This condition reaches an extreme when the fundamental mode is a straight line in which case it can be shown that the contribution of the higher modes to overturning moment vanishes. (10)

Top Floor Acceleration

For the acceleration at the top of the structure it is found that

$$a_n(\ell) = \frac{8S_v}{\beta N} \tag{17}$$

$$a(\ell) \le \frac{8S_v}{\beta N} \left\{1 + 1 + 1 + \cdots\right\} \tag{18}$$

$$a(\ell) \approx \frac{8S_v}{\beta N} \left\{1 + 1 + 1 + \cdots\right\}^{\frac{1}{2}} \tag{19}$$

It is necessary to know the number of terms in the series in Eqs. 18 and 19 before these expressions can be evaluated. The number of terms is equal to the number of modes excited by the earthquake, and it is reasonable to take the number of terms to be proportional to the number of stories.

This leads to

$$a(\ell) \le C_1 S_v \tag{20}$$

$$a(\ell) \approx C_2 \frac{S_v}{\sqrt{N}} \tag{21}$$

in which C_1 and C_2 are constants.

Equations 17-19 indicate that the magnitudes of the individual modal contributions to the top floor acceleration tend to decrease with height, however, it is known that the number of modes which contribute significantly to the response increases with height. The equations show also that, on the average, all contributing modes are equally important to the top floor acceleration. Therefore, accelerograms recorded on the tops of tall buildings would be expected to show many different frequencies and would resemble a ground motion record more than any single modal response.

The implications of Eqs. 20 and 21 are that for the tall buildings the maximum possible acceleration tends to be constant and the most likely value decreases with height. If S_v is replaced by the more exact values, S_{vn}, (with constant damping) the effects of higher modes are reduced and it is found that the maximum possible acceleration also decreases with increasing height and the expected value decreases faster than specified by Eq. 21

Comparison with Measured Results

In general, the top floor and midheight accelerations are the only structural responses recorded in tall buildings during earthquakes and these accelerograms are not plentiful for strong ground shaking. Fig. 5 shows a combination of calculated and measured linear responses of structures to strong ground motion. This figure gives the ratio of the maximum top floor acceleration to the maximum ground acceleration as a function of the number of stories in the structure. All possible data are not shown in Fig. 5. A number of other buildings between 5 and 15 stories show values between about 2 and 4. The very high magnifications shown by buildings during weak earthquake motions are excluded in Fig. 5 inasmuch as these high values do not appear during stronger shaking.

The data are too few to draw definite conclusions but the trend with height is in agreement with the analysis given above. The values for 55 and 70 stories in Fig. 5 are from analyses of preliminary designs for structures of these heights, designs based on the 40 story Union Bank Building in Los Angeles. The results of these analyses were obtained through the courtesy of A. C. Martin and Associates, Los Angeles, California.

The type of recorded motion expected in the upper floors of tall buildings during strong earthquakes is illustrated by Fig. 6 which shows the ground acceleration and 20th floor acceleration calculated for the linear response of a 20-story building to the N-S Component of the El Centro 1940 earthquake. (11) It is seen that several frequencies contribute to the motion as indicated by the above analysis. Also it is clear that in this example the building amplifies the maximum acceleration by a factor of about four. Because the damping in the structure used to prepare Fig. 6 decreases for the higher modes, it is thought that the amplification in Fig. 6 is too high. Based on Fig. 5, a factor of two to three seems more reasonable.

OTHER APPROXIMATE MODE SHAPES

For many important response variables the fundamental mode either dominates the response or is a major contributor, therefore it is important to assess the effects of different fundamental mode shapes on the response. For the response parameters chosen for study here, the influence of the mode geometry comes through the following integrals.

$$\int_0^{\ell} m(x)\varphi_n(x)dx \quad ; \quad \int_0^{\ell} m(x)\varphi_n^2(x)dx \quad ; \quad \int_0^{\ell} xm(x)\varphi_n(x)dx \tag{22}$$

in which $m(x)$ is a constant for the tall buildings.

The effects of fundamental mode geometry can be made clear by introducing the generalized mode shape

$$\varphi_1(x) = \left(\frac{x}{\ell}\right)^r , \quad r > 0 \tag{23}$$

The values of α_1 and the integrals of Eq. 22 are shown in Table II for varying values of r. The results show that a shear beam first mode is closely modeled by Eq. 23 with $r = \frac{1}{2}$ and a bending beam first mode is approximated well by $r = 3/2$. Intermediate values of r, near unity, would describe the fundamental modes of tall buildings shown in Fig. 1.

Using values from Table II, it is not difficult to determine the effect of different fundamental mode shapes on the contribution of the first mode to the top floor deflection, base shear, overturning moment and top story acceleration. These results, summarized in Table III, show that a change in geometry from shear mode to bending mode can change the values of first mode effects by as much as 23 percent. The different values of first mode contributions for shear-type modes and straight line modes can vary as much as 18 percent. For the top floor deflection and the overturning moment which are determined almost exclusively by the fundamental mode, a straight line approximation to the mode shape leads to

$$v(\ell) \approx \frac{2.4 S_v \beta N}{\pi^2} \tag{24}$$

from Eq. 10 and

$$M \approx \frac{31 m h^2 S_v N}{\beta \pi^2} \tag{25}$$

from Eq. 16. Similar approximations for the base shear and top floor acceleration in tall buildings are not made because higher modes must be considered also.

In order to estimate better the effects of modal geometry it would be useful to have available a set of simple mathematical functions approximating closely the observed mode shapes shown in Figs. 1-4, and higher modes as well. If m(x) is constant and the first mode is linear, the use of Legendre polynomials or other orthogonal polynomials appears promising, and efforts in this direction are currently in progress.

SUMMARY AND CONCLUSIONS

The geometry of tall buildings and their measured dynamic properties show that the tall structures possess approximately constant mass per unit height, similar frequency distributions and similar mode shapes. By modeling the observed properties by the modes and frequencies of a shear beam, and using the well-known average spectral properties of strong motion earthquakes, it was shown for tall buildings that, on the average, the top story deflection and base overturning moment tend to increase linearly with height; the base shear is independent of height; and the expected value of the top floor acceleration tends to decrease as $N^{-\frac{1}{2}}$.

The shear beam modes and frequencies are adequate for showing trends in the response because the trends are determined primarily by the distribution of frequencies and the higher mode shapes, which are modeled sufficiently well by the shear beam properties. However, the constants in Eqs. 8-19 are only approximate because the lowest modes of tall buildings, particularly the fundamental, are not modeled accurately by shear beam modes. The effect of the fundamental mode geometry was studied by means of a simple function that can vary over a wide range of mode shapes, including those of bending and shear beams, and the nearly linear fundamental modes shown by many tall buildings. For the first mode effects the shear mode gives results differing from a straight line mode by up to 18 percent.

Because the tall buildings share many properties, it is concluded that as more information becomes available it will be possible to develop simple approximations for the important mode shapes and frequencies. With these approximate dynamic characteristics, and with design spectra appropriate to the expected ground motions, simplified design techniques for specifying earthquake loadings on tall buildings can be developed which are founded on measured dynamic properties of structures and measured characteristics of strong ground motions.

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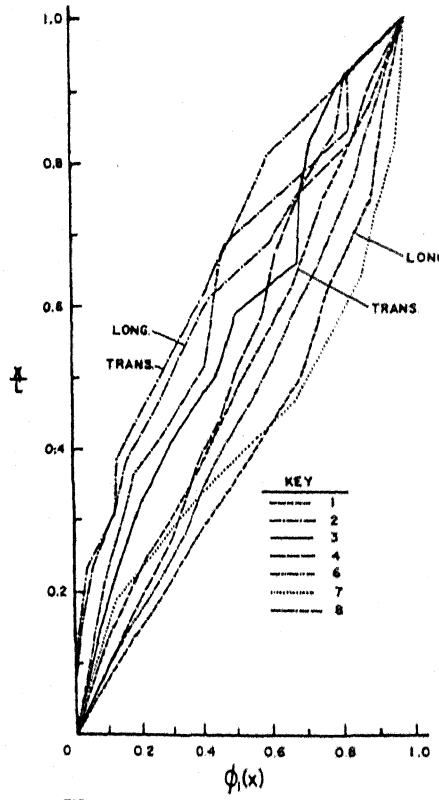

FIG. 1 - MEASURED FUNDAMENTAL MODE SHAPES OF TALL BUILDINGS (SEE TABLE 1).

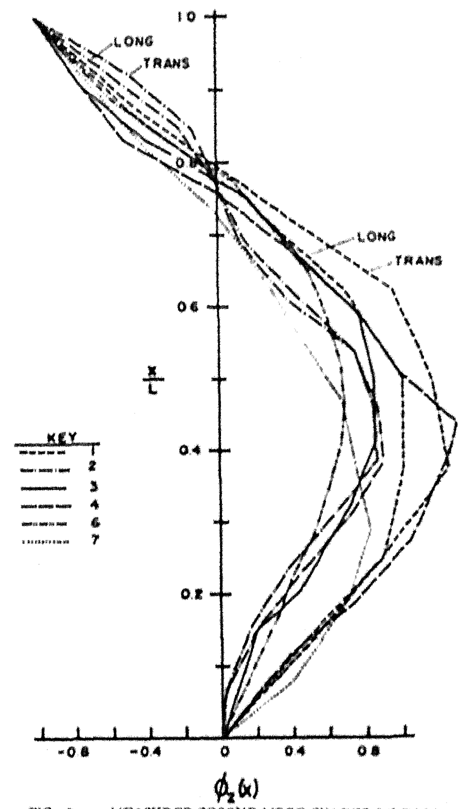

FIG. 2 - MEASURED SECOND MODE SHAPES OF TALL BUILDINGS (SEE TABLE 1).

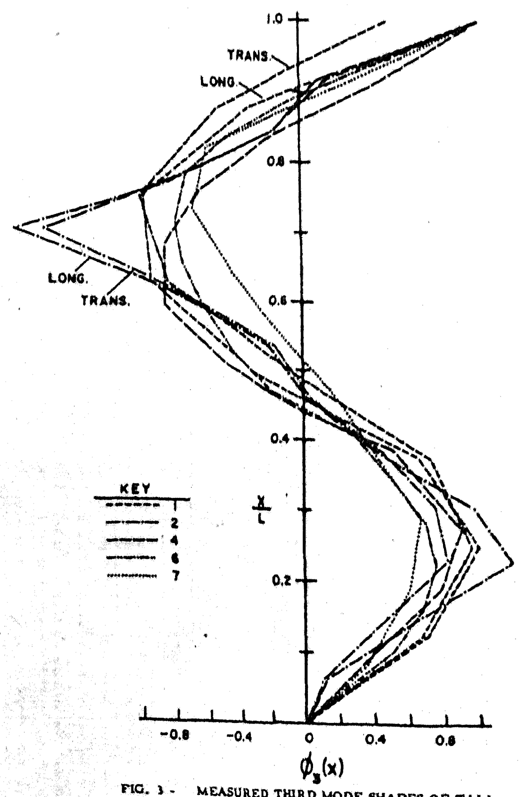

FIG. 3 - MEASURED THIRD MODE SHAPES OF TALL BUILDINGS (SEE TABLE 1).

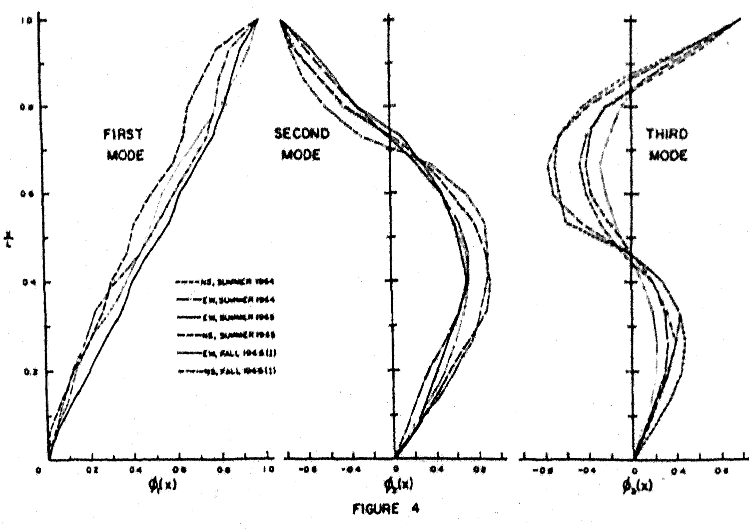

FIG. 4 - MEASURED MODE SHAPES OF A 15-STORY BUILDING (5).

TABLE I

MEASURED PERIODS OF TALL FRAMED STRUCTURES

Building	Stories	Type	Plan Dimensions (ft.)	Direction	T_1(sec.)	T_1/N	Period Ratios T_1/T_2	T_1/T_3	T_1/T_4	T_1/T_5	T_1/T_6
1. Jet Propulsion Laboratory (2)	9	Steel Frame	220x40	Long. Trans.	0.99 1.02	0.110 0.113	2.97 3.26	5.02 6.37	7.42		
2. Union Bank, (3) Los Angeles	40	Steel Frame	196x98	Long. Trans.	3.10 3.54	0.078 0.088	2.90 3.03	5.09 5.58	7.18 7.96	9.25 10.5	11.5 13.1
3. Canadian Imperial Bank of Commerce (4)	44	Steel Frame	140x100	Long. Trans.	4.65 4.65	0.106 0.106	2.90 2.72	5.06 4.78	7.71 7.06	9.61 8.93	11.9 11.1
4. CIL House (4)	34	Steel Frame	168x112	Long. Trans.	4.46 3.94	0.131 0.116	3.05 2.84	5.58 5.04	8.25 7.36	11.1 9.85	13.6
5. Univ. of Cal. Medical Center, East Bldg. (Summer 1964 Tests, frame and slabs only.) (5)	15	Steel Frame	107x107	E-W N-S	1.18 1.18	0.078 0.078	2.65 2.65	4.60 4.60	6.54 6.54	8.47 8.47	
6. Alexander Bldg. (6, 7) (Built in 1920's)	15	Steel Frame	68x60	Long. Trans.	1.27 1.37	0.085 0.091	3.10 3.05	5.29 5.27	7.47 7.21		
7. Ottawa Post Office Building (4)	10	Reinforced Concrete Frame	266x74	Long. Trans.	0.695 0.592	0.070 0.059	2.87 2.77	4.45 4.16	6.30 5.48	7.83 7.30	
8. Canadian Dept. of Health and Welfare Building (8)	18	Steel and Concrete Core, Steel Columns, Reinforced Concrete Floor Slabs	140x88	Long. Trans.	0.99 1.28	0.058 0.075	3.75 4.26	5.17 6.55			

TABLE II

FUNDAMENTAL MODE GEOMETRIES

Mode Type	$\int_0^l m(x)\varphi_1(x)dx$	$\int_0^l m(x)\varphi_1^2(x)dx$	$\int_0^l x\,m(x)\varphi_1(x)dx$	α_1
Shear beam	$\frac{ml}{1.57}$	$\frac{ml}{2}$	$\frac{ml^2}{2.47}$	1.27
$\varphi_1 = (\frac{x}{l})^{\frac{1}{2}}$	$\frac{ml}{1.5}$	$\frac{ml}{2}$	$\frac{ml^2}{2.5}$	1.33
Bending beam*	$\frac{ml}{2.5}$	$\frac{ml}{3.90}$	$\frac{ml^2}{3.46}$	1.56
$\varphi_1 = (\frac{x}{l})^{\frac{3}{2}}$	$\frac{ml}{2.5}$	$\frac{ml}{4.0}$	$\frac{ml^2}{3.5}$	1.60
$\varphi_1 = (\frac{x}{l})$	$\frac{ml}{2}$	$\frac{ml}{3}$	$\frac{ml^2}{3}$	1.50
$\varphi_1 = (\frac{x}{l})^r$	$\frac{ml}{r+1}$	$\frac{ml}{2r+1}$	$\frac{ml^2}{r+2}$	$\frac{2r+1}{r+1}$

*Values from use of Rayleigh's Principle

TABLE III

EFFECT OF MODE SHAPE ON FUNDAMENTAL MODE RESPONSE

Mode Type	Relative Value of First Mode Contribution to Total Response: Top Floor Deflection	Base Shear	Overturning Moment	Top Floor Acceleration
Shear beam	1	1	1	1
$\varphi(x) = (\frac{x}{l})$	1.18	0.93	0.97	1.18
Bending beam	1.23	0.78	0.88	1.23