DYNAMIC EARTHQUAKE BEHAVIOUR OF SHELL ROOFS

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SYNOPSIS: This paper presents an approximate numerical analysis which is capable of analysing a thin shell of arbitrary geometry, boundary conditions and applied loading. The shell is idealized as an assemblage of plane, triangular finite elements which have a refined displacement field for their membrane and flexural deformations. The direct stiffness method is used to formulate the matrix equations of equilibrium and a direct solution technique is employed to solve the simultaneous linear equations. The response to an arbitrary earthquake acceleration history is performed by a superposition of normal mode responses and the method of obtaining the mode shapes and frequencies is outlined. Examples are presented to illustrate the capability of this approach.

DISCRETIZATION of the STRUCTURE and the DISPLACEMENT FIELD The behaviour of the actual shell is assumed to be approximated by that of a "discretized structure" formed as an assemblage of bounded, plane, simply connected regions or 'finite elements'. The triangular element is the most suitable for this shell analysis as it has not only the simplest possible geometry, a feature that is very desirable for practical reasons, but it is also the only simple two-dimensional element that enables a doubly-curved shell to be approximated as closely as possible by this discretized shell of flat elements. The shell is defined by the global coordinates of the nodes, or nodal points, which are the points used to define the elements and by which the adjoining elements are interconnected.

Every finite element has a displacement field which is constrained to belong to a finite class of functions, each of which is continuous within the region of the element and which satisfies approximate continuity of deformations across the interfaces between adjoining elements. These coordinate functions are known as the "displacement shapes" and their associated amplitudes are the "generalized coordinates or displacements" and they may be regarded as kinematic degrees of freedom. This provides a displacement field, for the connected, discretized shell, that is continuous, piece-wise differentiable and can be made to satisfy any kinematic boundary conditions.

If the deformation of the shell is limited to that of small displacements and strains then the flexural and membrane actions within each individual element are uncoupled, and the coupling of these actions throughout the shell is achieved by the assemblage of the elements into the non-planar discretized shell. It is this uncoupling of the flexural and membrane actions within each element that enables the separate formulation

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of the flexural and membrane properties as those of a plate bending and a plane stress finite element respectively and superposing the resulting actions to obtain the shell element.

The major requirements of the shell element are that at each node the fundamental degrees of freedom should contain all three translational and all three rotational degrees of freedom as this would permit the congruent transformation of the element stiffness to a common global coordinate system as required by the direct stiffness method. It is also desirable that the displacement fields of both the plate bending and the plane stress elements should be similar algebraic functions so as to obtain the best continuity of the displacement fields between two adjoining, non-coplanar elements. In addition there should be a variation of the force resultants within the element so as to facilitate an easy interpretation of the resulting force distribution within the shell. Finally it is desirable that the triangular element should possess nodes only at the three corners of the element, firstly from a programming point of view, with respect to the order of the matrix algebra to be solved, and secondly, in that while the corner nodes of a triangle can lie in the middle surface of a doubly curved shell, any nodes along the sides of the element need not do so.

The finite element developed by Clough and Tocher [1] and later reformulated by Felippa [3] for plate bending best satisfies all the specified requirements for the plate bending components of the shell element and thus it was selected as the plate bending element. This element, using cubic displacement functions to produce a completely compatible plate element with three degrees of freedom at each of the three corner nodes, the degrees of freedom being the transverse displacement and the two plate rotations and the above authors have shown considerable success with this element in plate bending analyses.

The requirements of the plane stress element are that it has no more than three external nodes, and that the fundamental degrees of freedom at these nodes contain the two in-plane translations and the rotation about an axis normal to the plane of the element, this is the third rotation component missing from previous finite element shell analyses. If, in order to match the plate element, a cubic displacement function is used, this will automatically satisfy the requirement of a variation of internal forces, as these being proportional to the strain, will have a quadratic variation within the element. Felippa [1] suggested a cubic displacement function for a triangular element, having the nodes at the corners as well as a node at the entroid of the element. As the strain in this case has a quadratic variation it is known as the Quadratic Strain Triangle (QST).

The QST Plane Stress Element: The coordinate system used to define the element geometry is the cartesian coordinates shown in Fig. 1, while to define the displacement strain and stress fields within the element a natural or triangular coordinate system Fig. 2 is used. This system, as shown by Felippa [2] greatly reduces the computational effort involved in the formation of the element stiffness matrix because of the intrinsic relationship to the triangular element. Fig. 3 shows the nodal system used and the corresponding displacements, which are naturally classified into two categories

(a) The 18 fundamental degrees of freedom, the displacements and the global derivatives at each corner of the element; these values can specify the boundary displacements uniquely.

(b) The 2 additional degrees of freedom, the centroid displacements which will be eliminated later by a condensation process.

The nodal displacement vector $\{\bar{r}\}$ is then arranged as follows

$$\{\bar{r}\}^T = \langle \bar{u}^T \; \bar{v}^T \rangle$$

with

$$\{\bar{u}\}^T = \langle u_1 \; u_{x1} \; u_{y1} \; u_2 \; u_{x2} \; u_{y2} \; u_3 \; u_{x3} \; u_{y3} \; u_c \rangle$$

$$\{\bar{v}\} = \langle v_1 \; v_{x1} \; v_{y1} \; v_2 \; v_{x2} \; v_{y2} \; v_3 \; v_{x3} \; v_{y3} \; v_c \rangle$$

where

$$u_x = \frac{\partial u}{\partial x} \; ; \quad u_y = \frac{\partial u}{\partial y} \; ; \text{ etc.}$$

The best possible single displacement field for the triangular element is the complete cubic interpolation formula $\{\phi\}$ expressing the displacements $\{\bar{u}\}$ and $\{\bar{v}\}$ in the triangular coordinates as well as satisfying the general requirements of all displacement finite element analyses [2] by possessing the necessary rigid body motion and uniform strain states

i.e. $$f(\zeta_i) = f\{\phi\}^T$$

where

$$\{\phi\} = \begin{Bmatrix} \zeta_1^2(\zeta_1 + 3\zeta_2 + 3\zeta_3) - 7\zeta_1\zeta_2\zeta_3 \\ \zeta_1^2(a_3\zeta_2 - a_2\zeta_3) + (a_2 - a_3)\zeta_1\zeta_2\zeta_3 \\ \zeta_1^2(b_2\zeta_3 - b_3\zeta_2) + (b_3 - b_2)\zeta_1\zeta_2\zeta_3 \\ \zeta_2^2(\zeta_2 + 3\zeta_3 + 3\zeta_1) - 7\zeta_1\zeta_2\zeta_3 \\ \zeta_2^2(a_1\zeta_3 - a_3\zeta_1) + (a_3 - a_1)\zeta_1\zeta_2\zeta_3 \\ \zeta_2^2(b_3\zeta_1 - b_1\zeta_3) + (b_1 - b_3)\zeta_1\zeta_2\zeta_3 \\ \zeta_3^2(\zeta_3 + 3\zeta_1 + 3\zeta_2) - 7\zeta_1\zeta_2\zeta_3 \\ \zeta_3^2(a_2\zeta_1 - a_1\zeta_2) + (a_1 - a_2)\zeta_1\zeta_2\zeta_3 \\ \zeta_3^2(b_1\zeta_2 - b_2\zeta_1) + (b_2 - a_1)\zeta_1\zeta_2\zeta_3 \\ 27\zeta_1\zeta_2\zeta_3 \end{Bmatrix}$$

and thus u and v are expressed in triangular coordinates as

$$u(\zeta_i) = \{\phi\}^T\{\bar{u}\} \quad , \quad v(\zeta_i) = \{\phi\}^T\{\bar{v}\}$$

The linearized strain displacement equations are then

$$\{\epsilon(\zeta_i)\} = \begin{Bmatrix} \epsilon_x(\zeta_i) \\ \epsilon_y(\zeta_i) \\ \gamma_{xy}(\zeta_i) \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \end{Bmatrix} = \begin{bmatrix} \phi_x^T & \cdot \\ \cdot & \phi_y^T \\ \phi_y^T & \phi_x^T \end{bmatrix} \begin{Bmatrix} \bar{u} \\ \bar{v} \end{Bmatrix} = [B]\{\bar{r}\}$$

where the vectors $\{\phi_x\}$ and $\{\phi_y\}$ are the derivatives of $\{\phi\}$ with respect to x and y respectively with

$$\frac{\partial \phi_i}{\partial x} = \frac{1}{2A} \frac{\partial \phi_i}{\partial \zeta_k} b_k \quad ; \quad \frac{\partial \phi_i}{\partial y} = \frac{1}{2A} \frac{\partial \phi_i}{\partial \zeta_k} a_k$$

Assuming a linear elastic, isotropic material, the stresses are derived from the strains by the constitutive relations

$$\{\sigma(\zeta_i)\} = \begin{Bmatrix} \sigma_x(\zeta_i) \\ \sigma_y(\zeta_i) \\ \tau_{xy}(\zeta_i) \end{Bmatrix} = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & \cdot \\ \nu & 1 & \cdot \\ \cdot & \cdot & \frac{(1-\nu)}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x(\zeta_i) \\ \epsilon_y(\zeta_i) \\ \gamma_{xy}(\zeta_i) \end{Bmatrix} = [C]\{\epsilon(\zeta_i)\}$$

The internal strain energy U for the element can now be expressed as

$$U = \frac{1}{2} \int_{VOL} \{\epsilon(\zeta_i)\}^T \{\sigma(\zeta_i)\}\, dv$$

and for a constant thickness h

$$U = \frac{1}{2} h \{\bar{r}\}^T \left[\int_A [B^T][C][B]\, dA \right] \{\bar{r}\}$$

The external work done by the nodal forces $\{\bar{P}\}$ is

$$V = \{\bar{r}\}^T \{\bar{P}\}$$

The principle of minimum potential energy then gives that

$$\delta(U - V) = 0$$

$$\left[h \int_A [B^T][C][B]\, dA \right] \{\bar{r}\} = \{\bar{P}\}$$

by definition therefore the stiffness matrix of the element $[\bar{k}]$ is given by

$$[\bar{k}] = \left[h\int_A [B^T][C][B]\,dA\right]$$

The structure of the nodal vector is now modified and rearranged as follows

(a) The cross derivation of u and v are combined to form

$$\epsilon_{xy} = \gamma_{xy}/2 = (v_x + u_y)/2 \quad \text{, the shear strain}$$

$$\theta_z = \omega_{xy} = (v_x - u_y)/2 \quad \text{, the rotation of an element fibre}$$

about the z axis in the theory of small elastic deformations.

(b) The displacement vector is rearranged, the centroid displacements u_c, v_c are moved to the bottom of the vector to expedite the condensation process i.e.

$$\{r^*\}^T = \langle r_1^T\ r_2^T\ r_3^T\ r_c^T \rangle$$

where

$$\{r_i\}^T = \langle u_i\ v_i\ \epsilon_{xi}\ \epsilon_{yi}\ \gamma_{xyi}\ \omega_i \rangle,\ i = 1,2,3$$

$$\{r_c\}^T = \langle u_c\ v_c \rangle$$

The stiffness matrix (20 x 20) $[K^*]$ associated with $\{r^*\}$ is then partitioned as follows and the elimination of $\{r_c\}$ by the condensation procedure leads to the condensed (18 x 18) stiffness matrix $[K]$ of the element associated with the displacement vector $\{r\}$ where

$$\{r\}^T = \langle r_1^T\ r_2^T\ r_3^T \rangle$$

Let

$$[K^*] = \left[\begin{array}{c|c} K_{11} & K_{10} \\ \hline K_{01} & K_{00} \end{array}\right] \begin{array}{l} \text{18 rows} \\ \text{2 rows} \end{array}$$

Then

$$[K] = [K_{11}] - [K_{10}][K_{00}^{-1}][K_{01}]$$

The last two columns of $[K^*]$ must however be retained to recover the displacements of the centroid node during the computation of the stresses.

The Shell formed by the Shell Elements. This shell finite element has now nine degrees of freedom at each of its three corner nodes and they consist of the three translations and three rotations as well as the three membrane in-plane strains. Using the Direct Stiffness method to form the complete stiffness matrix of the discretized shell requires that the element stiffnesses have to be transformed to the common coordinate system of the complete structure; this is usually a cartesian coordinate system.

It is a simple process to transform the translation and rotation components directly to this global system as the associated stiffnesses are complete but the strain vectors are incomplete in that they only represent the three in-plane components and any transformation could result in all six strain components of three dimensional elasticity. These are undesirable from two points of view. Firstly that this would mean that each node on the shell would have twelve degrees of freedom, six displacements and six strains; this greatly increases the computational effort and the storage problems in solving the matrix equations. Secondly, it is possible, that if all the elements surrounding a node were co-planar, then the stiffness matrix would become singular.

However, in general, the elements in the discretized shell are not far from coplanar so that if the strain components are transformed to a coordinate system tangent at each node of the shell then the components normal to the tangent surface would be very small and could be neglected. Thus the resulting system is one where at each node in the complete shell there are six degrees of freedom in the global coordinate system of the shell and three degrees of freedom in an approximate tangent surface coordinate system at that node.

The structure of the analysis of the shell is as follows:

(1) From the stiffness matrices of each shell element in the element coordinate system to the appropriate coordinate systems for the total shell.

(2) Using the Direct Stiffness method form the Shell Stiffness and apply the Boundary Constraints.

(3) Form the load vector or vectors for the loads at the nodes.

(4) The matrix equation may now be written as

$$[K]\{r\} = \{R\}$$

where $[K]$ = shell stiffness matrix

$\{r\}$ = vector of (node) translations, rotations and strains

$\{R\}$ = vector of nodal applied forces and moments

(a) Reduce the stiffness matrix, which is symmetric

$$[K] = [L][D][L^T]$$

where $[L]$ is a lower triangular matrix

$[D]$ is a diagonal matrix, of multipliers

(b) Reduce the load vector $\{R\}$ by a forward substitution to obtain $\{y\}$

$$[L][D]\{y\} = \{R\}$$

(c) Back substitution to obtain the displacements $\{r\}$

$$[L^T]\{r\} = \{y\}$$

(5) Once the nodal displacements have been found the element deformations can be computed and the element internal force can be calculated.

DYNAMIC RESPONSE OF THE SHELL

As the shell is assumed to be linearly elastic the most efficient method of obtaining the response to a time varying load is to use a modal response superposition. This is because, in general, only the first few lower modes of vibration make any significant contribution to the total response of the shell and this enables a significant reduction in the computational effort to be achieved in the analysis.

The free vibration mode shapes and frequencies are obtained by solving the equation of motion

$$[M]\{\ddot{r}_n\} + [K]\{r_n\} = \{0\} \qquad n = 1, 2, \ldots, N$$

where $[M]$ is the diagonal matrix of lumped masses at the nodes

$\{r\}$ = mode shape

N = Total number of degrees of freedom,

and for a linearly elastic structure, the motion in free vibration is simple harmonic so that

$$\{\ddot{r}_n\} = \left\{\frac{\partial^2 r}{\partial t^2}\right\}_n = -\omega_n^2\{r_n\}$$

where ω_n is the natural frequency of vibration of mode shape $\{r_n\}$ and the equation of motion becomes

$$-\omega_n^2[M]\{r_n\} + [K]\{r_n\} = \{0\}$$

The Modified Rayleigh-Ritz technique is the most suitable method of reducing the number of degrees of freedom of the system from N to S. This is achieved by selecting a set of S assumed mode shapes $\{\bar{r}_i\}$ in which the nodal translation components are specified and they may be written in the form

$$\{\bar{r}_i\} = [G]\{\alpha_i\} \qquad i = 1, 2, \ldots, S$$

with $\{\alpha_i\} = \{0\}$ except that the ith term = 1.0.

Therefore every column of $[G]$ is a Rayleigh-Ritz mode shape which is then multiplied by the mass matrix to produce, for a unit frequency, the matrix of inertia forces. These loads are then applied to the shell and solved as a set of S static load vectors to produce the deflections $\{r_i\}$ caused by these forces.

The matrix of inertia forces $[P]$ is given by

$$[P] = [M][G]$$

and

$$\{r_i\} = [K^{-1}][P]\{\alpha_i\} = [\Psi]\{\alpha_i\} \qquad i = 1, 2, \ldots, S$$

where $[\Psi]$ is the deflection matrix

The shell force resultants $\{S_i\}$ caused by the inertial forces can also be evaluated

$$\{S_i\} = [\beta]\{\alpha_i\} \qquad i = 1, 2, \ldots, S$$

where $[\beta]$ is the force matrix.

Rewriting the equation of motion in terms of the generalized coordinates

$$[K^{**}]\{\alpha_i\} - \omega_i^2 [M^{**}]\{\alpha_i\} = \{0\} \qquad i = 1, 2, \ldots, S$$

where the generalized mass and stiffness matrices are given by

$$[M^{**}] = [\Psi^T][M][\Psi]$$

$$[K^{**}] = [\Psi^T][K][\Psi] = [\Psi^T][M][G]$$

$$= [\Psi^T][P]$$

As the eigenvalue routines are for symmetric matrices and also the equation must be transformed to the standard eigenvalue form, let $[\Gamma]$ be the normalized mode matrix of $[M^{**}]$ such that

$$[\Gamma^T][M^{**}][\Gamma] = [M^*]$$

with $[M^*]$ = diagonal $[m_1^* \; m_2^* \; . \; . \; . \; . \; m_s^*]$ where m_n^* are the eigenvalues of the generalized mass matrix. This then implies

$$[M^{*-1/2}] = \text{diagonal}\,[1/\sqrt{m_1^*}, \ldots\ldots, 1/\sqrt{m_s^*}]$$

and since

$$[M^{*-1/2}]^T[\Gamma]^T[M^{**}][\Gamma][M^{*-1/2}] = [I]$$

$$[M^{*-1/2}]^T[\Gamma]^T[K^{**}][\Gamma][M^{*-1/2}] = [K^*] \quad \text{symmetric}$$

this leads to the standard eigenvalue form

$$[K^*]\{\alpha_i\} = \omega_i^2\{\alpha_i\} \qquad i = 1, 2, \ldots, s$$

The squares of the frequencies ω_i^2 are the eigenvalues of $[K^*]$ and if $[\psi]$ is the normalized mode matrix then

$$[\Phi] = [\Gamma][M^{*-1/2}][\psi]$$

is the matrix of normal modes since

$$[\Phi]^T[M^{**}][\Phi] = [I]$$

and $$[\Phi]^T[K^{**}][\Phi] = [\omega^2]$$

and the mode shapes in the global cartesian coordinates follow as

$$[V] = [\Psi][\Phi]$$

It may be noted that this approach with the double eigenvalue solution, once for the generalized mass matrix and then for the frequencies, does not use the stiffness matrix $[K]$ to obtain the generalized stiffness matrix. This is a significant advantage since it was triangularized to solve for the improved trial shapes and its reformation would be a lengthy process. The conventional Rayleigh-Ritz technique has the disadvantage that in compressing the data, i.e. in reducing the number of degrees of freedom of the system from N to S, has the deleterious effect that the spectral condition number (λ max/λ min) of $[M^*]$ is usually quite large and the process may become unstable when very small roots are present. The modified technique, though requiring two eigenvalue solutions, is more stable and as the order S of the system is small, the extra computational effort of the extra eigenvalue solution is of no practical significance.

The major difficulty is in the selection of the trial shape matrix $[G]$ for if the essential characteristics of the shell are not represented in these trial shapes, then the improved mode shapes cannot represent these characteristics either. The most reliable method for obtaining these trial shapes is to select a coarse mesh idealization of the shell and to solve the eigenvalue problem

$$-\omega_n^2[M]\{r_n\} + [K]\{r_n\} = \{0\} \qquad n=1,2,\ldots,N$$

As the mass matrix represents a lumped mass system, the three rotation and the three strain degrees of freedom can be condensed from the system. This is most efficiently achieved by forming the flexibility matrix by solving the static analysis for a unit load applied at every translational degree of freedom. As a subset of this flexibility matrix $[F]$, relating the translations due to forces associated with these translational degrees of freedom, can be selected. In terms of this condensed flexibility matrix the equation of motion in free vibration becomes

$$[F][M]\{\hat{r}_i\} = \frac{1}{\lambda_i}\{\hat{r}_i\} \qquad i=1,2,\ldots,N_r$$

where N_r is the number of free translational degrees of freedom.

Upon introduction of $\{x_i\} = [M^{1/2}]\{\hat{r}_i\}$

where $[M^{\frac{1}{2}}]$ = diagonal$[\sqrt{M_1}, \sqrt{M_2}, \ldots\ldots \sqrt{M_{N_r}}]$.

the equation reduces to the standard eigenvalue form

$$[A]\{x_i\} = \beta_i\{x_i\} \qquad i=1,2,\ldots,N_r$$

where $[A] = [M^{1/2}]^T[F][M^{1/2}]$

$\beta_i = \dfrac{1}{\lambda_i} = \dfrac{1}{\omega_i^2}$ leads to the frequencies ω_i

and the translational mode shapes are recovered from

$$\{\hat{r}_i\} = [M^{-1/2}]\{x_i\}$$

THE RESPONSE to an ARBITRARY BASE ACCELERATION

The equation of motion for the shell subjected to the base acceleration $\ddot{u}_g(t)$ is

$$[M]\{\ddot{r}(t)\} + [C]\{\dot{r}(t)\} + [K]\{r(t)\} = -\{M\}\ddot{u}_g(t)$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices respectively, $\{M\}$ is a vector of the lumped masses. In terms of the generalized coordinates $\{\alpha_i\}$ where

$$\{r_i\} = [\Psi]\{\alpha_i\}$$

this becomes

$$[M^{**}]\{\ddot{\alpha}(t)\} + [C^{**}]\{\dot{\alpha}(t)\} + [K^{**}]\{\alpha(t)\} = -[\Psi^T]\{M\}\ddot{u}_g(t)$$

where

$$[M^{**}] = [\Psi^T][M][\Psi]$$

$$[C^{**}] = [\Psi^T][C][\Psi]$$

$$[K^{**}] = [\Psi^T][K][\Psi]$$

and by introducing the normal coordinates $\{Y_i\}$ where

$$\{Y_i\} = [\Phi^T]\{\alpha_i\}$$

and the assumption that the form of the damping matrix $[C]$ is such that

$$[\Phi^T][\Psi^T][C][\Psi][\Phi] = 2\lambda[\omega]$$

with λ = the proportional critical damping, which is small and is the same for all modes. This leads to the equation of motion in the following form

$$[I]\{\ddot{Y}(t)\} + 2\lambda[\omega]\{\dot{Y}(t)\} + [\omega^2]\{Y(t)\} = \{P_n\}\ddot{u}_g(t)$$

where $\{P_n\}$ is the normal force vector

$$\{P_n\} = -[\Phi^T][\Psi^T]\{M\} = -[V^T]\{M\}$$

The acceleration history is generated at equal intervals of time and the normal response $\{Y(t)\}$ is integrated by assuming a linear acceleration during the time interval. Once the normal response has been obtained the deflection and acceleration response can be obtained as follows

$$\{r(t)\} = [\Psi][\Phi]\{Y(t)\} = [V]\{Y(t)\}$$

$$\{\ddot{r}(t)\} = [V]\{\ddot{Y}(t)\}$$

and finally the force response in the shell is computed.

$$\{S(t)\} = [\beta][\Phi]\{Y(t)\}$$

The response of each earthquake component can be computed and the responses superposed to obtain the complete response and if desired envelopes of the responses can be evaluated for easy interpretation of the results.

CYLINDRICAL SHELL SUBJECTED TO EARTHQUAKE

As an example of the capability of this method of analysis a cylindrical shell as shown in Fig. 5 was subjected to the 1940 El Centro earthquake acceleration record. Two forms of the shell were used, one with free edges, and the other with a 4 foot deep and 1 foot thick edge beams. The mesh idealizations selected for the Modified Rayleigh-Ritz analysis are shown in Fig. 6. The first step is however to find the trial mode shapes and this, as outlined previously, was achieved by taking a coarse mesh idealization, the mesh refinement was half that shown in Fig. 6, and carrying out the direct eigenvalue analysis for the frequencies and modes of free vibration. These mode shapes were then used as the trial shapes for the Modified Rayleigh-Ritz analysis and the improved frequencies, and those of the earlier analysis are shown in Table 1.

Each shell was also subjected to static analyses, first for a dead and live load analysis and then for a 0.1 g lateral load, these being used as comparisons with the earthquake response. The shell was then analysed for its response to the vertical component of the earthquake and to the North-South component, acting in a direction transverse to the longitudinal axis of the shell. The results of these analyses are shown in Table 2. In both analyses where ten mode states were used for each the vertical and lateral loadings, it was found that the significant component of the response was provided by the first four or five modes and this gives considerable justification for using the Rayleigh-Ritz technique to reduce the number of degrees of freedom of the system.

CONCLUSIONS:

This particular finite element analysis has shown its capabilities in being able to analyse any arbitrary shell geometry [5] provided the coordinates of the nodes can be defined, with any arbitrary loading and boundary conditions, it also has been extended to cover dynamic loading such as the excitation caused by earthquake accelerations. The analysis can be extended further, to cover variations in element thickness and to account for variations in material properties and to the case of orthotropic shell.

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MODES FOR VERTICAL MOTION				MODES FOR LATERAL MOTION		
MODE	m	DIRECT	RAYLEIGH-RITZ	m	DIRECT	RAYLEIGH-RITZ
1	1	0.852	0.835	1	0.994	1.020
2	1	2.882	2.982	1	2.383	2.492
3	3	3.089	3.318	3	2.995	3.329
4	1	5.178	5.318	1	4.575	4.661
5	5	5.609	6.301	5	5.633	6.306
6	1	7.026	6.876	3	7.565	7.632
7	3	8.113	8.563	7	9.259	9.693
8	3	8.606	9.050	1	10.502	10.070
9	3	9.347	10.066	3	11.856	11.149
10	9	13.041	14.937	3	12.710	13.479

FREE EDGED CYLINDRICAL SHELL

MODES FOR VERTICAL MOTION				MODES FOR LATERAL MOTION		
MODE	m	DIRECT	RAYLEIGH-RITZ	m	DIRECT	RAYLEIGH-RITZ
1	1	1.957	1.979	1	1.938	1.937
2	1	3.053	2.949	1	3.518	3.335
3	1	6.427	6.037	1	4.586	4.321
4	1	7.453	7.386	3	8.689	8.156
5	3	9.134	8.815	1	9.054	8.219
6	3	10.479	9.465	3	11.725	11.425
7	1	12.271	10.781	3	13.483	11.925
8	3	14.161	13.850	5	15.091	14.055
9	5	14.980	14.328	1	16.400	14.287
10	3	16.413	14.948	5	17.525	14.584

CYLINDRICAL SHELL WITH EDGE BEAMS

TABLE-1, MODAL FREQUENCY CHARACTERISTICS

NATURAL FREQUENCIES, CYCLES/SECOND
COMPARISON OF DIRECT EIGENVALUE APPROACH
WITH THE MODIFIED RAYLEIGH-RITZ TECHNIQUE

m = No. of half waves in the longitudinal direction of the shell roof.

	VERTICAL LOADING		LATERAL LOADING	
	STATIC	EL-CENTRO	STATIC	EL-CENTRO
		DEFLECTION OF CROWN (INS)		
Horizontal	-	-	0.0115	0.0478
Vertical	3.7630	0.2272	-	-
		DEFLECTION OF EDGE (INS)		
Horizontal	-12.4150	0.4054	0.4091	1.7690
Vertical	-21.9800	0.6703	0.7041	2.8935
	FORCES AT EDGE OF SHELL (LB/IN OR LB IN/IN)			
Long. Force	13389.5	545.8	- 345.2	1399.8
Arch. Force	4.1	0.9	0.3	0.7
Shear Force	0.0	0.0	0.0	0.0
Long. Moment	-911.4	62.3	24.9	152.5
Arch. Moment	4.8	5.7	- 0.4	6.8
Twisting Moment	46.5	6.3	- 1.3	11.7

CYLINDRICAL SHELL WITH FREE EDGES

	VERTICAL LOADING		LATERAL LOADING	
	STATIC	EL-CENTRO	STATIC	EL-CENTRO
		DEFLECTION OF CROWN (INS)		
Horizontal	-	-	-0.00006	0.1376
Vertical	0.7073	0.2172	-	-
		DEFLECTION OF EDGE (INS)		
Horizontal	-0.8781	0.1374	0.1111	0.8433
Vertical	-1.6782	0.1726	0.1534	1.0709"
	FORCES AT EDGE OF BEAM (LB/IN OR LB IN/IN)			
Long. Force	17218.9 •	1851.3 •	-1066.6	8045.8
Arch. Force	-9.9	1.5	0.2	2.4
Shear Force	0.0	0.0	0.0	0.0
Long. Moment	-2652.9	473.6	395.3	3015.7
Arch. Moment	67.8	67.0	-68.6	433.3
Twisting Moment	- 30.0	27.2	27.8	180.4

CYLINDRICAL SHELL WITH EDGE BEAMS

TABLE-2. COMPARISON OF RESULTS AT THE MIDSPAN OF THE SHELL

Note 1. EL-CENTRO RESULTS ARE ABSOLUTE VALUES

- FORCES AT EDGE OF SHELL ARE - 1498.3 and 856.0 lb/in respectively

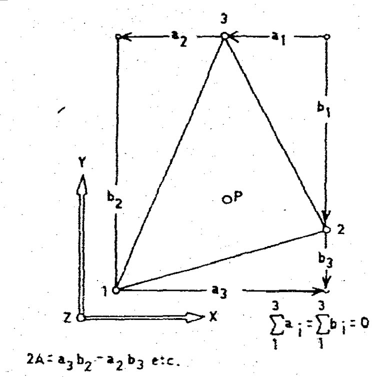

FIG. 1.
Global Dimensions.

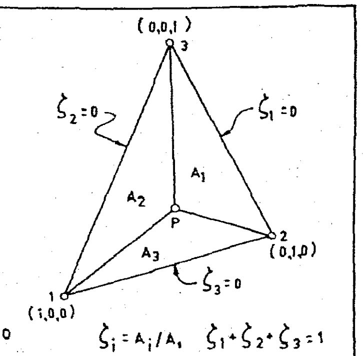

FIG. 2.
Triangular or Natural Coordinates.

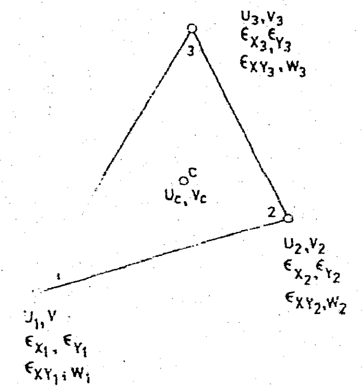

FIG. 3.
Nodal System (Nr) for displacements.

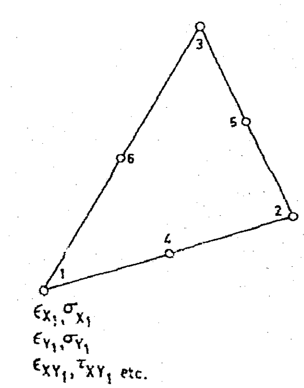

FIG. 4.
Nodal Systems. (N_ϵ) & (N_σ)

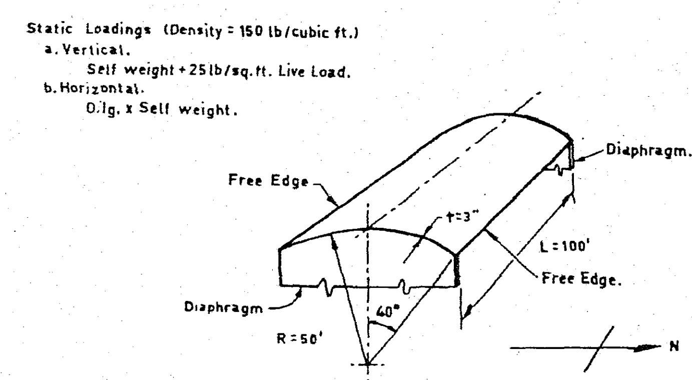

FIG. 5 FREE EDGED CYLINDRICAL SHELL.

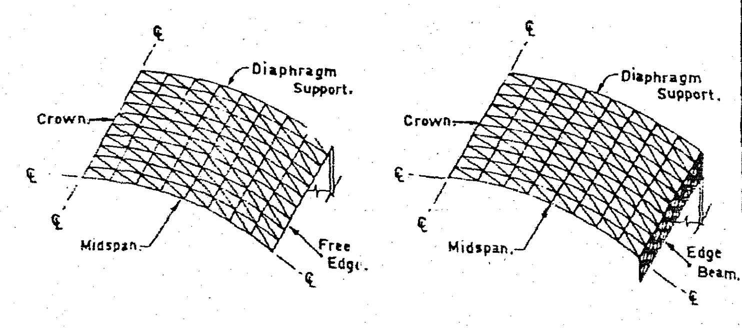

FIG. 6 CYLINDRICAL SHELL
FINITE ELEMENT IDEALIZATIONS.