EARTHQUAKE ANALYSIS OF SUSPENSION BRIDGES

by

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SYNOPSIS

This paper is concerned with the earthquake analysis of suspension bridges, in which the effects of large deflections are taken into account. The first part of the study deals with an iteration scheme for the nonlinear static analysis of suspension bridges by means of tangent stiffness matrices. The concept of tangent stiffness matrix is then introduced in the frequency equation governing the free vibration of the system. At any equilibrium stage, the vibrations are assumed to take place tangent to the curve representing the force-deflection characteristics of the structure.

The bridge is idealized as a three dimensional lumped mass system and subjected to three orthogonal components of earthquake ground motion producing horizontal, vertical and torsional oscillations. By this means a realistic appraisal is achieved for torsional response as well as for the other types of vibration. The modal response spectrum technique is applied to evaluate the seismic loading for the combination of these vibrations. Various numerical examples are introduced in order to demonstrate the method of analysis. The procedure described enables the designer to evaluate the nonlinear dynamic response of suspension bridges in a systematic manner.

1. GENERAL

1.1 Introduction:- The suspension bridge is a highly nonlinear three dimensional structure. As a consequence, in dynamic studies the governing nonlinear equations of motion are frequently simplified by introducing assumptions which linearize these equations[1]. These simplifying assumptions may however be avoided, and the nonlinear behaviour of the structure may thereby be taken into account in both static and dynamic analyses, by using an iterative solution employing tangent stiffness matrices. The iterative scheme has been applied previously by a number of authors in connection with the static analysis of suspension bridges[2,3,4]. In this paper the same operation is extended to solve the dynamic response problem of suspension bridges, which are idealized as three dimensional lumped mass systems vibrating due to earthquake ground motions. Only geometric nonlinearity is considered; the material is assumed to remain elastic.

The method proposed for the nonlinear vibration analysis of suspension bridges involves two distinct steps, which are summarized as follows:

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(1) Under the static action of the dead and live loads the equilibrium configuration and the internal stress resultants of all constituent elements of the structure are first determined through an iteration routine based on the Newton-Raphson method.

(2) The vibration of any point in the bridge, with respect to the static equilibrium position, is assumed to take place along the tangent to the curve defining the force-deflection characteristics of that point. The natural frequencies and mode shapes of the structure are obtained from a solution of the eigenvalue problem in which the frequency determinant is expressed in terms of the tangent stiffness matrix of the system. Once these fundamental dynamic properties are determined, the response spectrum concept can be used in conjunction with classical modal analysis to evaluate the seismic forces acting on suspension bridges during earthquakes.

Details of these two basic steps are given in the following sections.

2. METHOD OF ANALYSIS

2.1 Determination of the Equilibrium Configuration:- For the purpose of clarity, the method of analysis presented in this paper is introduced by reference to the simple nonlinear system shown in Fig.1. A schematic illustration of the iteration process used to obtain the static equilibrium geometry of this example structure is given in Fig.2. The following steps are involved:

(i) A linear stiffness analysis is performed under the action of the given external load, $P_1 = P_e$, yielding the straight line $\overline{oa}$. The slope of this line, K_1, is equal to the stiffness of the system in the unloaded position. The joint displacement, D_1, obtained from this first linear cycle of analysis is larger than the equilibrium displacement, D_{eq}.

(ii) The internal stress resultants of each member are calculated on the basis of the deformed geometry D_1 of the system using the nonlinear expressions given in Appendix I. The resultant of the first cycle internal forces, P_{1i}, is not in equilibrium with the given external load, as shown in Fig.2. The unbalanced joint load, P_2, is

$$P_2 = P_{1i} - P_e \tag{1}$$

(iii) In order to eliminate P_2 the displacement D_1 must be diminished. This is accomplished by loading the joint with a force P_2, applied in the direction of the resultant internal joint load P_{1i}, and performing a second linear stiffness analysis of the structure under the action of P_2. During this step the original stiffness K_1 is replaced by a tangent stiffness, K_2, which depends on the loaded and unloaded member lengths and also on their end deformations and stress resultants in the previous cycle (see Appendix I). The force-deflection characteristic of the structure is now represented by the straight line $\overline{bc}$, having slope K_2 and leading to a cycle displacement D_2, which reduces the initial displacement and brings the system closer to the actual equilibrium configuration.

(iv) With repeated application of steps (ii) and (iii) the unbalanced joint load is continuously diminished, as may be seen in Fig.2. The tangent stiffness matrix is successively altered after each cycle so as to include the latest geometry and internal stress resultants of the system. At the end of any jth cycle the unbalanced joint load is

$$P_{j+1} = P_{ji} - P_e \tag{2}$$

and the total joint displacement, D_{jt}, is equal to the algebraic sum of the individual cycle displacements or

$$D_{jt} = D_1 + D_2 + \ldots D_j \tag{3}$$

The iterative process is repeated until the unbalanced joint load is reduced to some acceptable value.

The above iterative scheme has been applied to the system shown in Fig.1, and the numerical results of each cycle are tabulated in Fig.3. Although seven cycles were required to reach an exact solution, the error in the deflection after the fourth cycle was only 1.6%.

2.2 Equilibrium Configurations for Suspension Bridges:- The preceding approach for establishing the equilibrium geometry of nonlinear structures is perfectly general and can be applied without variation to more complicated structures providing that the unbalanced loads are eliminated in every direction at every joint.

However, in the case of suspension bridges the entire dead load is carried by the hangers and main cable only. The stiffening girder is assumed to be unstressed and the towers, which carry only axial load, do not bend under dead load condition. Therefore, the geometry of the suspension bridge available to the designer is usually the dead load equilibrium geometry, and the unloaded lengths of the cables and hangers are unknown. Since these unloaded lengths appear in the tangent stiffness matrices, it is necessary to calculate them before establishing any subsequent geometric configuration as a result of added live loads. The unloaded geometry may be determined from a single cycle of linear analysis under the action of the known dead loads. The unloaded member lengths, L_o, may be obtained from Hooke's Law as

$$L_o = L/(1 - Q/AE) \tag{4}$$

where L = the member length in the known dead load equilibrium configuration, AE the axial rigidity of the member and Q = the member axial force due to dead loads.

With the application of live load the originally unstressed stiffening girder participates in the overall behaviour of the bridge, which subsequently acquires a new equilibrium geometry, as illustrated in Fig.4. It should be noted that since the stiffening girder is not stressed under the dead load condition, the unloaded lengths of the members in the girder are available from the known dead load geometry.

The number of iteration cycles needed to establish the equilibrium configuration is obviously dependent on the degree of nonlinearity and on the desired accuracy. In most studies of actual suspension bridges undertaken by the writers, four to six cycles were sufficient to eliminate the unbalanced joint loads to within an accuracy of about 0.3% of the maximum internal stress resultant.

2.3 Frequency Analysis using Tangent Stiffness Matrices:- The dynamic analysis of discrete mass structures is a topic which has received extensive treatment in the literature and is well known[5,6,7]. The nonlinear behaviour of suspension bridges during vibration about any static equilibrium configuration may be accounted for by replacing the linear stiffness matrix of the system, $[K]$, by a tangent stiffness matrix, $[K_T]$. This is equivalent to assuming that at any equilibrium stage the vibration of any point in the bridge takes place along the tangent to the curve representing the force-deflection characteristics of the point. This idea of tangential vibration is illustrated in Fig.5. Accordingly, the frequency determinant becomes

$$\mathrm{Det}\left|[K_T] - \omega^2 [M]\right| = 0 \tag{5}$$

where, M = mass matrix, and ω = the natural frequency of the system in any one of its normal modes. $[K_T]$ depends on the strains and the internal forces developed in the members at the static equilibrium position. The eigenvalues, ω, as well as the eigenvectors, can be obtained from a solution of Eq.5 using routine computer programs.

The concept of tangential vibration can be simply illustrated by application to the single degree of freedom suspended system shown in Fig.6. This structure is considered to be vibrating freely about static equilibrium Position 3, corresponding to some dead and live load combination. The dynamic displacement of the system from the equilibrium position is defined by z, measured positive downward. This deformation is represented by Position 4. Then, in accordance with D'Alembert's principle, from Fig.6c,

$$W - \frac{W\ddot{z}}{g} - F - \Delta F = 0 \tag{6}$$

From statics, the resultant of the cable forces, F, is equal and opposite to the sum of the dead and live loads, W. When the system is translated by the amount z into Position 4 the resultant of the cable forces is increased by an increment ΔF, in order to maintain equilibrium. This increment is simply the product of the tangent stiffness of the cables, K_T, and the displacement z. That is,

$$\Delta F = K_T z \tag{7}$$

where K_T is defined as the force required to deform the structure through a unit vertical displacement, measured with respect to the dead plus live load equilibrium geometry under consideration. With the substitution of $F = W$, $m = W/g$ and $\Delta F = K_T z$, Eq.6 is reduced to

$$m\ddot{z} + K_T z = 0 \tag{8}$$

The response of linear multidegree of freedom systems can be described in terms of a combination of a number of equivalent single degree of freedom systems whose behaviours are governed by a set of independent equations of the above form. For the nonlinear system, in which tangential vibrations are contemplated, the dynamic response can be obtained therefore by direct application of the standard modal superposition technique, once the nonlinearity of the structure has been taken into account inside the stiffness matrix and Eq.5 has been solved. The modal superposition approach has been applied to suspension bridges previously[1,8], but the nonlinearity of the structure was not taken into account inside the stiffness matrix.

3. APPLICATIONS

3.1 Idealization of Suspension Bridges:- Depending on the memory capacity of the computer available, the suspension bridge may be idealized as a plane or space frame composed of a series of straight line elements. While the plane frame idealization may be used for the study of the response to vertical and longitudinal ground motions, the three dimensional idealization is desirable for a realistic investigation of the torsional and horizontal vibrations of the deck due to ground motion perpendicular to the deck centerline. Idealized forms for typical tower, stiffening truss, cables, hangers and bridge deck components of a suspension bridge are shown in Fig.7; when assembled they form a three dimensional lumped mass system.

The main cable and hangers are considered as pure axial force members of constant cross-section, while the stiffening girder is assumed to be composed of beam-column elements between hangers. Loads are considered to act at the nodal points only. Since the stiffness matrix approach is quite general, it is not necessary to resort to any other simplifying assumptions. The influence of hanger extensions, cable point loads, degree of fixity at the tower base, continuity of the stiffening truss across the towers, and variations in moments of inertia can easily be taken into account.

In the case of a three dimensional idealization, all three rotations and the horizontal translations perpendicular to the bridge centerline must be suppressed at the cable-hanger junctions in order to prevent singularity in the stiffness matrix. This reduces the total number of degrees of freedom of the system. If necessary, to fit the capacity of the computer, this number may be further reduced by neglecting the axial length changes and the torsional rigidities of the flexural members.

3.2 Primary and Secondary Degrees of Freedom:- The joints of a vibrating structure normally have more degrees of freedom than the number of directions in which the lumped masses are considered to vibrate. For example, the lumped masses are not considered to vibrate in the rotational degrees of freedom. To distinguish between the vibrating and non-vibrating directions the degrees of freedom are classified into two groups:

(i) Primary degrees of freedom (P), which refer to the vibration directions of the lumped masses.

(ii) Secondary degrees of freedom (S), which refer to the directions along which no vibrations take place. The secondary degrees of freedom only contribute to the stiffness of the system.

The master stiffness matrix generated for the system contains both primary and secondary degrees of freedom and is therefore of higher order than the mass matrix of Eq.5. In order to make the matrices compatible, the secondary degrees of freedom are eliminated by means of a matrix partitioning process[(I)].

3.3 Types of Vibration:- An earthquake may excite a suspension bridge in any one or a combination of the following three types of vibration:

(A) Torsional vibration of the bridge deck, coupled with a lateral vibration of the towers, due to horizontal ground motion perpendicular to the centerline of the bridge. The torsional vibration is essentially a combination of the vertical and lateral motion of the bridge deck. Such vibration may also be developed due to lateral wind loading.

(B) Vertical vibration of the bridge deck, coupled with a horizontal vibration of the towers, due to horizontal ground motion parallel to the centerline of the bridge.

(c) Vertical vibration of the bridge deck, coupled with a horizontal vibration of the towers, due to vertical ground motion.

In all cases, vertical vibration of the towers and also longitudinal vibration of the bridge deck may be neglected, since their effects are relatively small.

These three types of vibration should be taken into account when performing an earthquake analysis of suspension bridges. Different aspects of this problem have been discussed in the literature by various authors[12,13,14].

4. NUMERICAL EXAMPLES

The procedures discussed in the preceding section are now demonstrated by application to two example structures.

The idealized N-S component of the 1940 El Centro earthquake spectrum[5] was used in the dynamic analysis of both structures; the spectrum values were multiplied by two-thirds when considering vertical excitation. The bridges were considered to have about 1% of critical damping in all modes, and the tower and anchorage supports were assumed to be subjected to the same ground motion. Computations were performed with the IBM 7044 at the Computing Centre at the University of British Columbia. Although the capacity of the program was sufficient to handle full scale suspension bridges, smaller hypothetical structures were selected for sake of simplicity and clarity of presentation.

4.1 Example 1:- A hypothetical suspension bridge was idealized as the two dimensional lumped mass system shown in Fig.8 and subjected to vertical ground motion only. The member properties of the bridge are summarized in Table 1. As discussed in Section 3.3, this excitation produced vertical vibration of the deck and horizontal vibration of the towers. The primary and secondary degrees of freedom corresponding to the vertical ground motion are also shown in Fig.8 for a typical cable, girder and tower joint.

(I) See, for example, Reference 7, p.44.

The static equilibrium position about which the bridge was assumed to vibrate was taken as the dead, plus one-half live load configuration, which was established by the iteration scheme outlined in sections 2.1 and 2.2. The mode shapes and natural periods for the first ten modes of vibration are given in Fig.9.

4.2 Example 2:- In order to obtain a realistic appraisal of the dynamic response of suspension bridges, especially towards lateral ground motion, a three dimensional idealization of the structure is desirable. For this purpose, the centre span of the preceding example was idealized into the twenty eight lumped mass system shown in Fig.10. The member properties are listed in Table 2. The bridge was subjected, non-concurrently, to the three types of vibration described in Section 3.3. The geometry supplied in Fig.10 was assumed to be the equilibrium geometry about which the vibration occurred.

The primary and secondary degrees of freedom of typical joints in the case of ground motion perpendicular to the bridge centerline are shown in Fig.10; the lumped masses of the bridge deck were assumed to have both vertical and horizontal motions, allowing the investigation of torsional vibrations.

The mode shapes, natural periods and participation factors for the first ten modes of vibration, corresponding to the Type A vibration of Section 3.3, are given in Fig.11. The mode shapes of Types B and C vibration are of the same form as those shown with the preceding example, Fig.9. Modes 6 and 8 indicate the presence of a torsional oscillation of the bridge deck. This potentially destructive vibration, which has caused suspension bridge failures, emphasizes the usefulness of the three dimensional idealization. Only two of the first ten modes of vibration were of a torsional character. It is possible that higher torsional modes would have a significant influence on the maximum response.

The inertia forces corresponding to each of the three types of ground motion considered acting independently are tabulated in Table 3.

4.3 Dynamic Response Due to Combination of Ground Motions:- The maximum dynamic response of a suspension bridge may be developed by an earthquake ground motion which produces concurrently the three types of vibration mentioned in Section 3.3. This maximum response may be estimated by a suitable combination of the inertia forces obtained from non-current analyses. For this purpose, the shears, moments, axial forces and deflections, etc., in the structure are first determined separately for each mode of each type of vibration by considering the respective modal inertia forces to be acting statically and performing a nonlinear analysis using the iteration method outlined in Section 2.1. The root mean square combination of these modal values constitute the desired maximum dynamic response.

5. CONCLUDING REMARKS

1. The concept of tangent stiffness matrix,used in conjunction with the standard modal superposition method, provides a systematic approach to the nonlinear dynamic analysis of suspension bridges.

2. For a realistic evaluation of the overall dynamic response of a suspension bridge, a three dimensional idealization is desirable. Such an idealization permits a study of the torsional oscillation of the bridge deck. In fact, significant vibrations of this type were observed due to earthquake ground motion perpendicular to the bridge centerline.

3. The general procedures described in this paper may supply useful information in the study of the aerodynamics of suspension bridges.

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1.1 The member end forces, f_1, f_2, ... f_6, of a plane frame based on the deformed geometry (Fig.12) may be written as follows[4]:

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{Bmatrix} = \begin{Bmatrix} (L_o - L)\ AE/L_o \\ (f_3 + f_6)/L \\ (4EI/L)\ (\delta_3 + \theta)\ s_1 + (2EI/L)\ (\delta_6 + \theta)\ s_2 \\ -(L_o - L)\ AE/L_o \\ -(f_3 + f_6)/L \\ (4EI/L)\ (\delta_6 + \theta)\ s_1 + (2EI/L)\ (\delta_3 + \theta)\ s_2 \end{Bmatrix} \qquad \text{A.1}$$

in which AE = the axial rigidity, EI = the flexural rigidity, L_o = unloaded length, L = loaded length in the last cycle, θ = additional end rotation at each end of a member due to unequal lateral end displacements given by

$$\sin \theta = (\delta_2 - \delta_5)/L_o \qquad \text{A.2}$$

s_1, s_2 = the stability correction factors[9].

For flexural members in compression:

$$s_1 = \rho\ (\sin \rho - \rho \cos \rho)/4(2 - 2 \cos \rho - \rho \sin \rho) \qquad \text{A.3}$$

$$s_2 = \rho\ (\rho - \sin \rho)/2\ (2 - 2 \cos \rho - \rho \sin \rho) \qquad \text{A.4}$$

For flexural members in tension:

$$s_1 = \rho\ (\rho \cosh \rho - \sinh \rho)\ /4(2 - 2 \cosh \rho + \rho \sinh \rho) \qquad \text{A.5}$$

$$s_2 = \rho\ (\sinh \rho - \rho)/2(2 - 2 \cosh \rho + \rho \sinh \rho) \qquad \text{A.6}$$

$$\rho = L \sqrt{(L_o - L)\ AE/L_o EI} \qquad \text{A.7}$$

1.2 The tangent stiffness matrix of an axial force member in space is[10,11]

$$[k] = \frac{AE}{L_o} \begin{bmatrix} \ell^2 & & & & & \text{Symmetric} \\ \ell m & m^2 & & & & \\ \ell n & mn & n^2 & & & \\ -\ell^2 & -\ell m & \ell n & \ell^2 & & \\ -\ell mn & -m^2 & -mn & \ell m & m^2 & \\ -\ell n & -mn & -n^2 & \ell n & mn & n^2 \end{bmatrix} - \frac{Q}{L} \begin{bmatrix} 1-\ell^2 & & & & & \text{Symmetric} \\ -\ell m & 1-m^2 & & & & \\ -n\ell & -mn & 1-n^2 & & & \\ \ell^2-1 & \ell m & n\ell & 1-\ell^2 & & \\ m\ell & m^2-1 & mn & -m\ell & 1-m^2 & \\ n\ell & mn & n^2-1 & -n\ell & -mn & 1-n^2 \end{bmatrix} \qquad \text{A.8}$$

in which, ℓ, m, n are the direction cosines of the member centerline and Q is the axial force, compressive positive, given by

$$Q = AE(L_o - L)/L_o \qquad \text{A.9}$$

The tangent stiffness matrix for a plane flexural member is[3,11]

$$[k] = \begin{bmatrix} Sg_1+Dj_1 & Smn-Dj_5 & -C_inq_1 & -Sg_1-Dj_1 & -Smn+Dj_5 & -C_jnq_1 \\ Smn-Dj_2 & Sg_2+Dj_6 & C_imq_1 & -Smn+Dj_2 & -Sg_2-Dj_6 & C_jmq_1 \\ -C_ij_3 & C_ij_7 & A_is_1 & C_ij_3 & -C_ij_7 & Bs_2 \\ -Sg_1-Dj_1 & -Smn+Dj_5 & C_inq_1 & Sg_1+Dj_1 & Smn-Dj_5 & C_jnq_1 \\ -Smn+Dj_2 & -Sg_2-Dj_6 & -C_imq_1 & Smn-Dj_2 & Sg_2+Dj_6 & -C_jmq_1 \\ -C_jj_4 & C_jj_8 & Bs_2 & C_jj_4 & -C_jj_8 & A_js_1 \end{bmatrix}_{XYZ} \qquad \text{A.10}$$

(6 x 6)

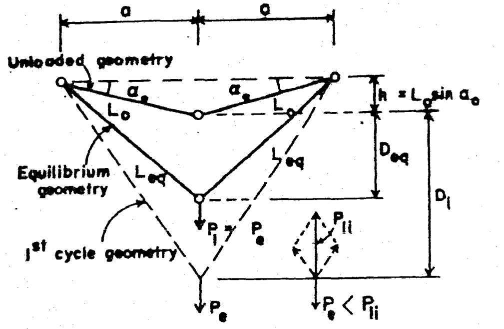
in which

$$\begin{cases} A_i = A_j = 2B = 4EI/L \\ C_i = C_j = 6EI/L^2 \\ D = 12EI/L^3 \\ S = AE/L \end{cases} \qquad \text{A.11}$$

$$\begin{cases} g_1 = \dfrac{L_b}{L_o} - (1 - m^2) \\ g_2 = \dfrac{L_b}{L_o} - (1 - n^2) \end{cases} \qquad \text{A.12}$$

$$\begin{cases} j_i = q_1(n^2 - wmn) & j_5 = j_2 \\ j_2 = q_1\left[mn - w(m^2 - n^2)/2\right] & j_6 = q_1\,(m^2 + wmn) \\ j_3 = j_4 = nq_1 & j_7 = j_8 = mq_1 \end{cases} \qquad \text{A.13}$$

$$\begin{cases} q_1 = (4s_1 + 2s_2)/6 \\ w = \delta_3 + \delta_6 + 2\theta \end{cases} \qquad \text{A.14}$$



DATA
$L_e = 100$
$a = 99.498743$
$\frac{P_e}{2AE} = 0.003$
$\frac{h}{L_o} = \sin \alpha_o = 0.1$
$D_{eq} = 10.12361$

FIG. 1.- SIMPLE SUSPENDED SYSTEM

CYCLE NO.	UNBALANCED JOINT LOAD	TANGENT STIFFNESS	RELATIVE DEFLECTION AT EACH CYCLE	FINAL DEFLECTION	MEMBER LENGTH
j	$P_j/2AE$	K_j	D_j	$D_{jt} = \sum_j D_j$	L_j
1	-3000.0×10^{-6}	100.0×10^{-6}	-30.00000	-30.00000	100.00000
2	23998.1	1972.3	12.16725	17.83275	107.23805
3	5939.0	1023.6	5.80228	12.03047	103.31825
4	1125.8	645.9	1.74306	10.28741	101.90849
5	88.6	545.3	0.16243	10.12498	101.54594
6	0.7	536.3	0.00137	10.12361	101.51362
7	0.0	536.2	0	$D_{eq}=10.12361$	101.51335

Fig.3.- ITERATION SCHEME FOR EXAMPLE STRUCTURE OF FIG.1.

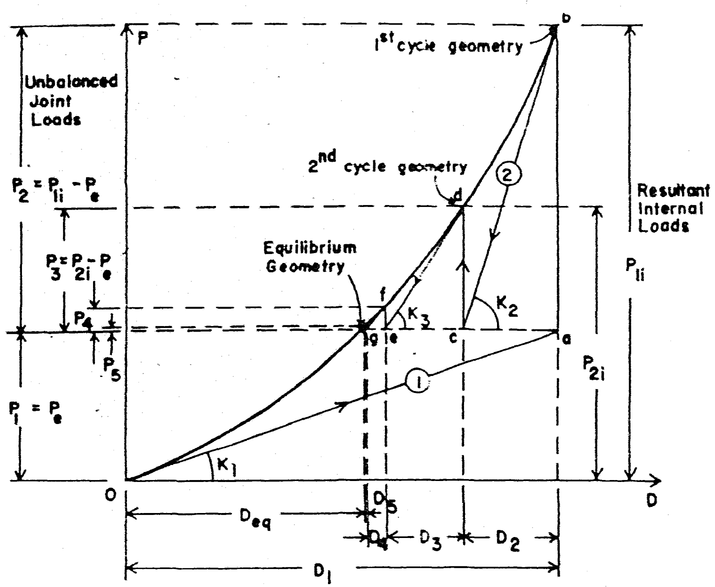


FIG. 2.- DETERMINATION OF EQUILIBRIUM CONFIGURATION

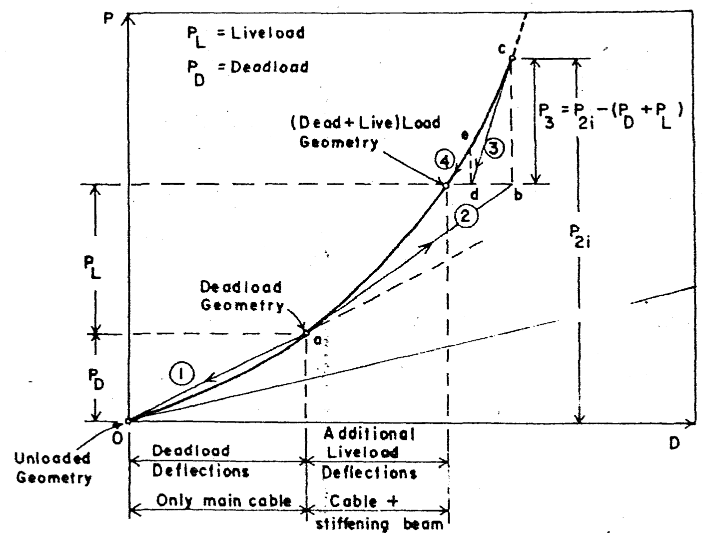


FIG. 4.- DEADLOAD AND LIVELOAD GEOMETRIES

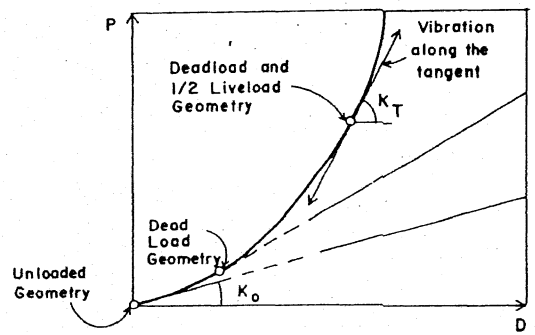


FIG. 5.- VIBRATION ALONG THE TANGENT LINE

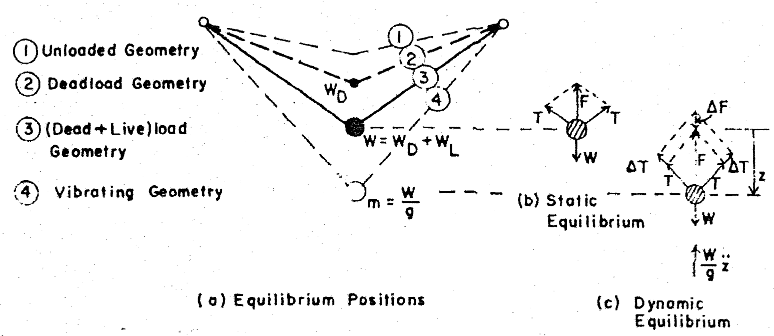


FIG. 6.- VIBRATION OF A NONLINEAR SUSPENDED SYSTEM

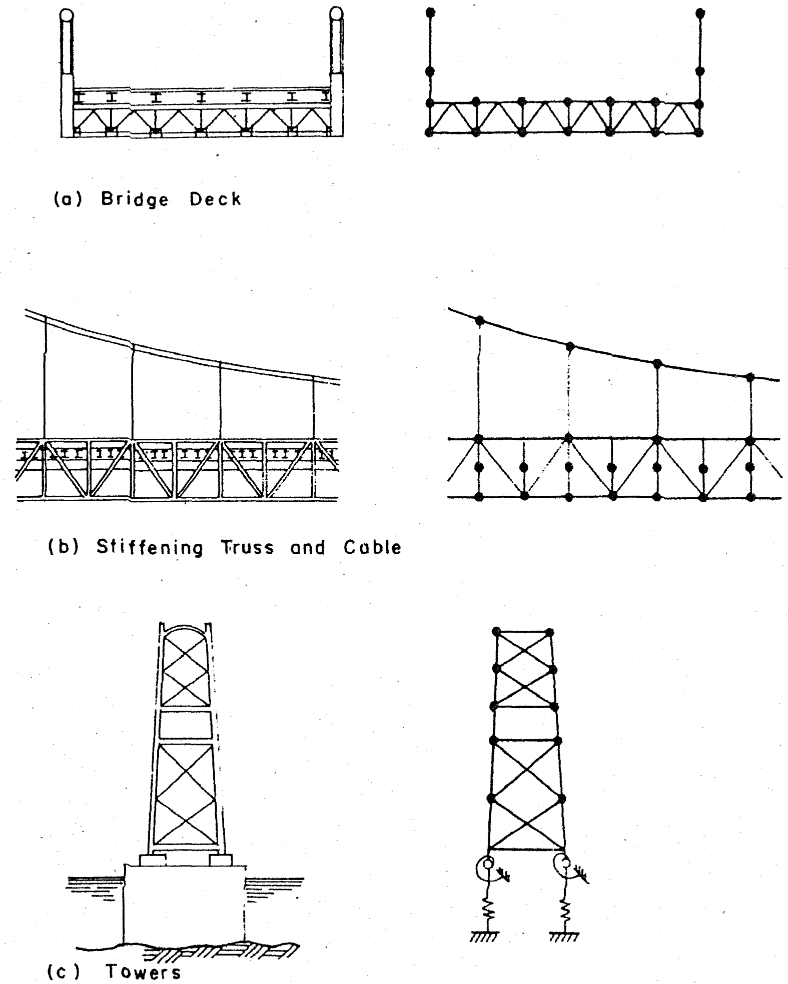


FIG. 7. — IDEALIZATION OF SUSPENSION BRIDGE

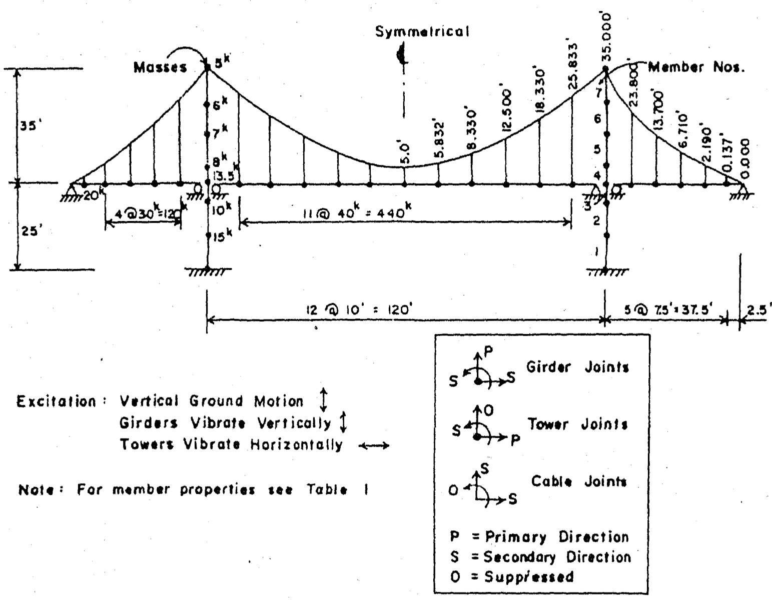


FIG. 8.—IDEALIZED SUSPENSION BRIDGE, EXAMPLE I.

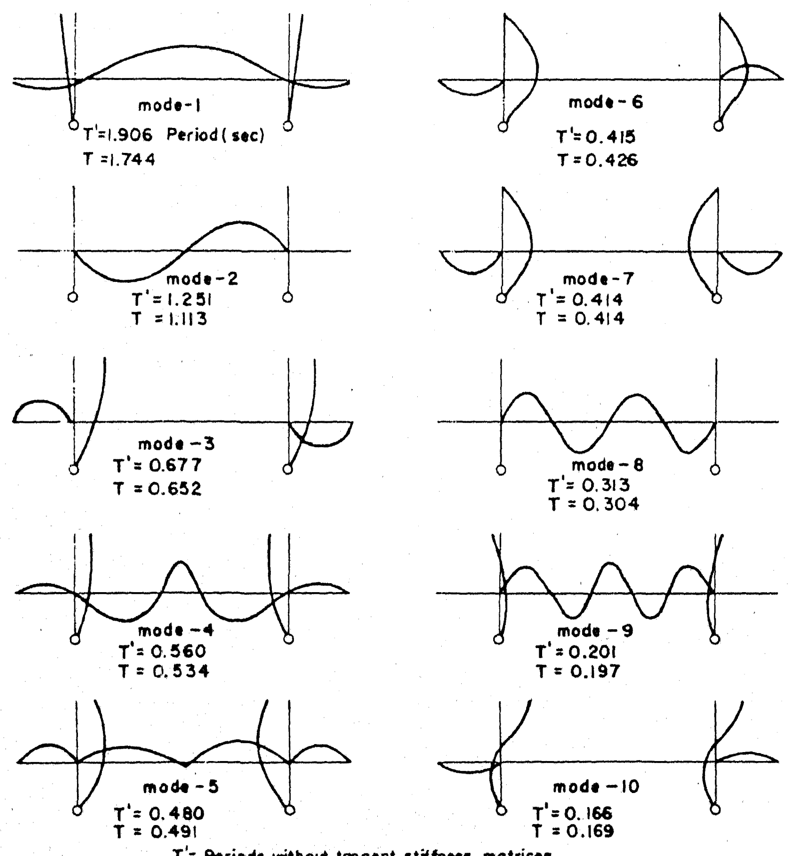


FIG. 9.— MODE SHAPES OF EXAMPLE I

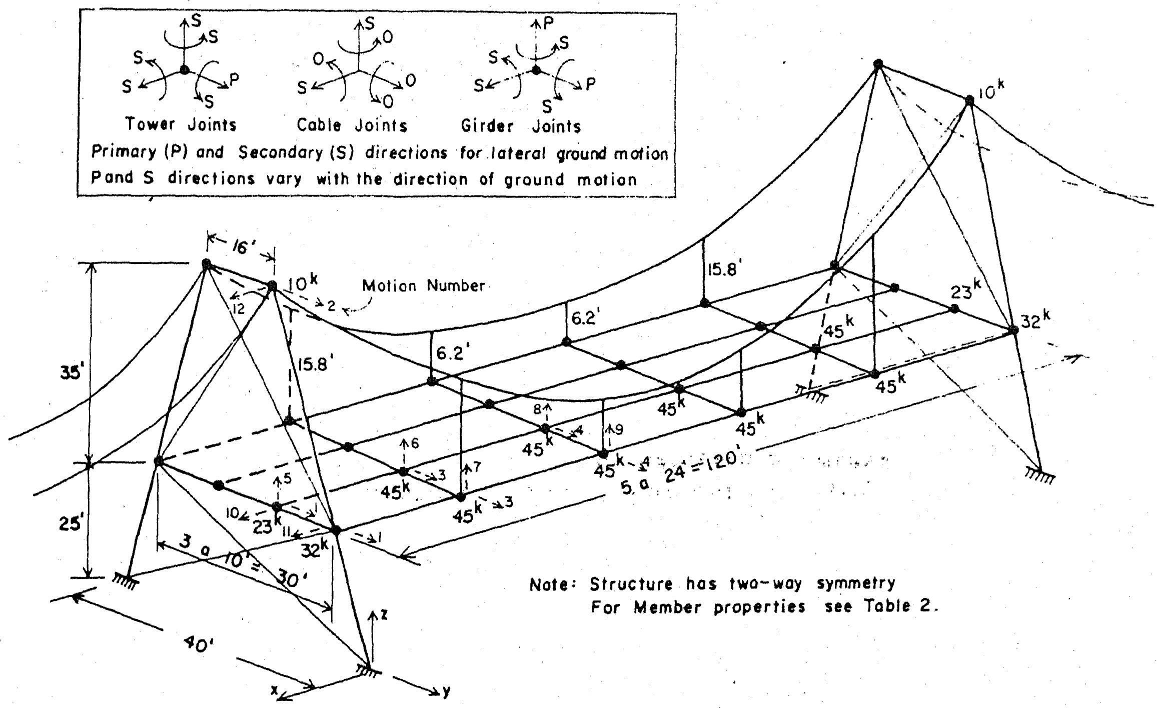


FIG. 10. – IDEALIZED SUSPENSION BRIDGE, EXAMPLE 2.

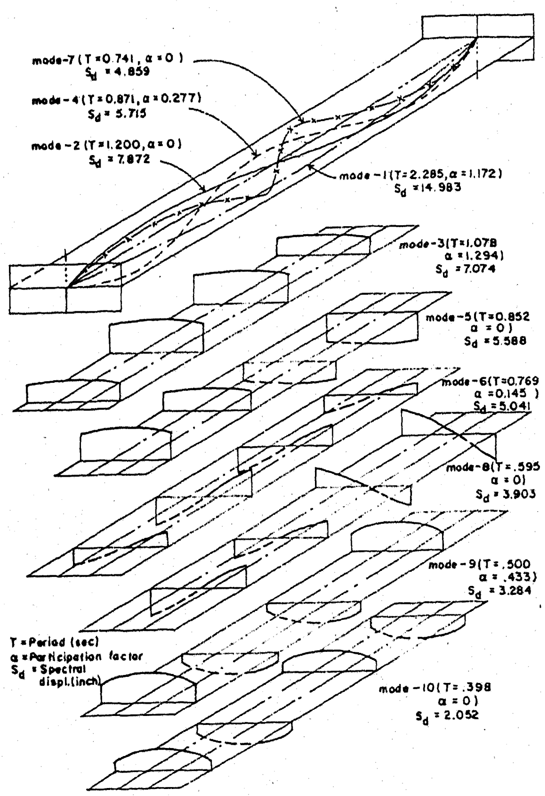


FIG. 11. – MODE SHAPES OF EXAMPLE 2

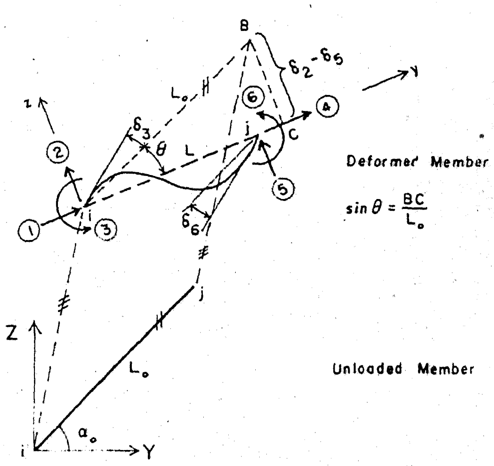


FIG. 12 – UNLOADED AND DEFORMED MEMBER

MEMBER	NO.	Moment of Inertia (in.^{4})	Area (in.^{2})	Length (ft.)
Tower	1	8000	50	10
	2	8000	50	10
	3	7000	45	5
	4	6000	40	5
	5	5000	35	10
	6	4000	30	10
	7	3000	25	10
Stiffening Girder		2000	30.0	As shown in Fig. 8.
Cable		0	1.00	
Hangers		0	0.79	

TABLE 1. - MEMBER PROPERTIES OF EXAMPLE 1, $E = 30 \times 10^6$ lb/sq.in

	Moments of Inertia max. (in.^{4})	min. (in.^{4})	torsional (in.^{4})	Area (in.^{2})
Tower legs	5414.3	406.9	12.15	42.68
Tower top beam	69.2	8.5	0	5.88
Tower bracing	98.0	98.0	0	16.73
Edge Longitudinals	3635.3	254.0	8.84	35.29
Internal longitudinals	1042.6	87.5	5.08	22.92
Transversals	248.6	53.2	1.53	13.24
Cables	0	0	0	1.00
Hangers	0	0	0	0.79

TABLE 2. - MEMBER PROPERTIES OF EXAMPLE 2, $E = 30 \times 10^6$ lb/sq.in.

** MOTION No.	MASS (kip)	* VIBR. TYPE	BRIDGE DECK MODAL SEISMIC INERTIA FORCES (kips) 1	2	3	4	5	6	7	8	9	10
3	45	A	9.6	0	0	9.6	0	0	0	0	0	0
4	45	A	15.5	0	0	-5.7	0	0	0	0	0	0
6	45	A	0	0	18.2	0	0	32.3	0	0	0	0
		B	18.1	0	0	32.6	0	0	-3.0	0	0	-1.0
		C	12.1	0	0	21.7	0	0	-2.0	0	0	-0.7
7	45	A	0	0	13.7	0	0	8.0	0	0	8.6	0
		B	13.7	0	0	8.0	0	0	4.0	0	0	-0.1
		C	9.1	0	0	5.3	0	0	2.7	0	0	0
8	45	A	0	0	39.1	0	0-	-14.2	0	0	0	0
		B	39.2	0	0	-14.2	0	0	-7.6	0	0	0.6
		C	26.2	0	0	-9.5	0	0	-5.0	0	0	0.4
9	45	A	0	0	31.6	-0.1	0	-4.4	0	0	0	0
		B	31.7	0	0	-4.4	0	0	9.4	0	0	-0.2
		C	21.1	0	0	-3.0	0	0	6.2	0	0	-0.1

* A = Horizontal Ground Motion Perpendicular to bridge axis
B = Horizontal Ground Motion Parallel to bridge axis
C = Vertical Ground Motion

** For motion numbers see Fig.10; for mode shapes of Type A vibration see Fig.11.
For all types of ground motion, the towers are assumed not to vibrate.
No motion is considered to take place in motion directions 3 and 4 (Fig.10), due to ground vibrations of Type B and C.

TABLE 3. - SEISMIC INERTIA FORCES, EXAMPLE 2.