STRUCTURAL DYNAMICS OF CANTILEVER-TYPE BUILDINGS

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SYNOPSIS

Structural and dynamic properties of buildings vary with the ratio of the stiffness of the horizontal to the vertical members and elements. Many contemporary buildings have low ratios, and perform more as cantilevers than as frames. Cantilever-type buildings are investigated over the entire range of relative stiffnesses. Traditional design methods may result in serious inadequacies. A Modified Cantilever Method is proposed to determine rapidly the natural periods of buildings having little moment restraint from the floor systems. Useful concepts and structural-dynamic data are provided for cantilever-type buildings, which type has been subject to severe earthquake damage in Chile, Anchorage and Caracas.

GLOSSARY OF TERMS

A_v	=	effective shear area = gross area/α; in^2
D_f	=	lateral flexural deflection of top of building loaded laterally with its own weight; inches
D_s	=	lateral shear deflection of top of building loaded laterally with its own weight; inches
E	=	the effective dynamic modulus of elasticity; kip/in^2
H	=	total height of building, inches
I	=	moment of inertia about the axis normal to the plane of loading; in^4
L	=	the member span, center to center of intersecting members; L' = the clear member span; inches
M	=	moment; in-kips (For ΣM see equation 3)
ORM	=	the overall resisting moment provided by axial forces in the vertical members; in-kips
OTM	=	overturning moment, or cantilever moment, due to lateral forces; in-kips
T_i	=	natural period of mode i; sec
W	=	total weight of the building; kips
α	=	factor for effective shear area based upon the shape;(2,3,5) dimensionless
β_i	=	ratio of period i computed with tapered flexural stiffness to that computed with average stiffness; dimensionless

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λ = ratio of period computed rigorously as a lumped mass system to period by equation 11; dimensionless

ρ = index of joint rotation, or the ratio of horizontal member stiffness to vertical member stiffness at the midheight story (see equation 1)

ρ' = ρ when clear spans are used as per equation 2

$\sum$ = the summation symbol

φ = ratio of period for a pure cantilever ($\rho = 0$) computed as lumped massed system to the period computed as a uniform bar

Subscripts

a,b,c refer to case (a), (b), or (c) in Figure 14

f refers to flexure

s refers to shear

C refers to column, or to vertical member or assembly

G refers to girder, or to horizontal member or assembly

INTRODUCTION

There are important reasons for considering cantilever-type buildings, including the fact that many contemporary buildings tend to respond to ground motion more as a system of slightly restrained vertical elements than as traditional rigid frames. Another reason is that buildings with spandrel or girder damage or hinging tend to function as pseudo cantilevers in subsequent earthquake response.(1) In addition, much can be learned about building dynamics including important new parameters and time-saving methods of analysis.

Whether or not a building tends to perform as a rigid frame, as a vertical cantilever, or as some combination of the two depends upon the relative stiffness and strength of the horizontal and the vertical elements. The horizontal elements may be beams, girders, joists, spandrels, floor slabs, or various combinations. The vertical elements may be columns, piers, pilasters, walls, core units (such as around stairwells and elevator shafts), or various combinations. The tendency in recent years has been to make the horizontal elements much less rigid than the vertical elements. This changes or negates much in accepted structural design practice and certain concepts regarding dynamic properties of buildings. Damage in recent earthquakes indicates that design practices must be improved for cantilever-type buildings or for structures that might become cantilever-type during earthquakes.

BASIC TERMS AND SYMBOLS

A paper by Jacobsen in 1939[2] provided excellent data on distributed mass models of buildings which are constructed essentially of solid walls. These might be termed pure cantilever buildings, the limiting case in a whole spectrum of cantilever types in current building configurations. The other end of the spectrum is the traditional rigid frame building with deep spandrels or girders, often with stiffnesses equal to or greater than column stiffnesses. Stiffness is defined as the ratio EI/L for each member, under the assumption herein that the members remain normal to each other at the joints. For simplicity, E can be assumed constant for a building, either because there is only one material or because the analyst can use transformed areas with a single modulus of elasticity. It is necessary to have a simple index to define the degree of horizontal to vertical stiffness in a building. As previously proposed[3,4] let

$$\rho = \frac{\sum_{1}^{all} \frac{I_G}{L_G}}{\sum_{1}^{all} \frac{I_C}{L_C}} \qquad (1)$$

The summations are taken for all members in the midheight story. If the girders or columns change in that story, average values from the adjacent stories are used. Stiffness ratios vary with height, although not to a great degree in most cases, especially in cantilever-type buildings. ρ is very easy to obtain and is a most useful index in this problem. If a wall should be solid, or have only insignificant openings, and be the only vertical resisting element, ρ would not exist and can be considered as zero. This would be a pure cantilever. However, if there should be some horizontal (floor) framing either between walls in the direction under consideration or elsewhere in the story, the numerator would exist and ρ would have a value other than zero, even though very small. ρ is always computed for the entire story including all members in that story.

Figure 1 indicates some partial building elevations together with approximate ρ-values based upon arbitrary assumptions as to the widths (normal to the elevation) of the horizontal and vertical members shown. Elevation (e) represents failed spandrel sections, and (f) represents a contemporary building of wall sections and flat slab floors.

In computing ρ, the height of the columns, walls, or piers would normally be taken as the full story height floor to floor, and the length of the horizontal elements, center to center of the vertical elements. In a few cases, the walls are punctured with openings of such size as to create short spans for which shear deformations would be significant. In such cases, clear spans would be used in equation 2.

$$\rho' = \frac{\Sigma\left(\frac{L'_C}{I_C} + \frac{30}{A_{vC}L'_C}\right)}{\Sigma\left(\frac{L'_G}{I_G} + \frac{30}{A_{vG}L'_G}\right)} \qquad (2)$$

ρ-values were computed for many rigid frames and real buildings,(3) and rigorous analyses were conducted with the aid of computer facilities to obtain the relationship of structural and dynamic characteristics to ρ and other parameters. Buildings may be generally classified as shear, shear-frame, cantilever-frame, framed cantilever, or cantilever depending upon ρ (or ρ') values and related characteristics. Another paper(4) has presented the effects of joint rotation, overall flexure and base compliance for the shear and shear-frame categories, which may generally be considered to have ρ-values greater than 0.10. This paper is concerned with the three cantilever types of buildings with ρ-values generally less than 0.10. It also discusses another type of building, the braced frame.

A basic concept in structural engineering is that the algebraic sum of the moments of all forces in a system in equilibrium must be zero, and all points in the system must have a set of forces and stresses that satisfy this law as well as the requirement for the algebraic sum of the forces along each axis to equal zero. The relative amount of moment resistance of any type in a building frame varies as ρ varies. At any horizontal plane including the base of the building

$$OTM + ORM + \Sigma M = 0 \qquad (3)$$

In this equation, clockwise rotation is considered positive. All moments must, of course, be in consistent units. With this equation it is apparent that for a pure cantilever, ORM would be zero and therefore $\Sigma M = -OTM$. For a rigid frame or for a combination of framing and walls, piers and/or core units -- given a diaphragm rigid in its own plane -- there will be both ORM and ΣM. The relative amount of each -- and therefore the shears in the girders and the axial forces in the columns -- will vary if ρ varies.

ANALYSIS OF CANTILEVER-TYPE FRAMES

Various cantilever-type frames were analyzed rigorously under static lateral forces to obtain moments, shears, and axial forces, and also natural periods and mode shapes. In some cases girder stiffnesses were varied to obtain various ρ-values and in other cases column stiffnesses were varied. Figure 2 shows an 8-story frame used for 22 runs with various combinations of column areas and moments of inertia. Figure 3 is a 16-story frame also computed for 22 column conditions. For these models the column properties were assumed constant from top to bottom and for all columns in each run. Column widths were assumed to be 14 inches parallel to the loading. ρ-values for each frame were varied from 1.50 to 0.0001. For each value, columns were alternately allowed to deform axially (column "shortening") in overall flexure, and not to deform axially. E was taken as 30,000 kip/in^2.

Structural Variations with ρ. Figures 4 and 5 show the relationship of ρ and $\sum M$ for frames having various values of ρ, for the 8 and 16-story models. It is apparent that approximate methods of analysis which assume points of inflection at midheight of columns, or even somewhere in the lower story, are quite erroneous for ρ-values somewhat less than 0.10. Lower, and upper, story moments can be much greater than indicated by approximate methods of rigid frame analysis, as has been previously noted.[1,6,7] This is a very important point for buildings with long spans or relatively flexible floor systems; especially in the lower and in the upper stories.

Figure 6 shows the axial force in the exterior column solely from lateral forces for various stories of the 8-story frames. It is to be noted that the lower story is sensitive to ρ changes even at high values while the other stories (except the second) have essentially constant column axial forces until ρ becomes less than about 0.01. Below this value the structure is approaching the pure cantilever condition and the column axial forces therefore approach zero. Figure 7 shows similar data for the 16-story frames. The results are similar.

Figure 8 shows the shear in the exterior girder for various floors of the 8-story frames and Figure 9 for the 16-story frames. The lower levels are the most sensitive to ρ-values over the whole range. Figures 6, 7, 8, and 9 are each for the cases in which the columns are allowed to deform axially. Some of the local variations are due to frame geometry effects such as flexural rotation of the columns distorting girders.

Figure 10 indicates the ratio of the moment resisted by flexure, $\sum M$, to the overturning moment, at the base for various ρ-values. From equation 3, using absolute values for convenience,

$$\frac{ORM}{OTM} = 1.00 - \frac{\sum M}{OTM} \qquad (4)$$

The values $\sum M/OTM$ are plotted directly in Figure 10, and ORM/OTM may be scaled between the curves and the ratio value of 1.00. Data are shown for the base levels of the 8-story and the 16-story frame. These curves, or similar curves for other structures more representative for specific problems, are useful for various purposes including the consideration of the effects of possible ρ variations during earthquakes. If the girders should hinge, ρ would decrease and thus increase the flexural moment demands on the columns or piers while the axial forces, per se, in the vertical members would decrease. On the other hand, if the columns should hinge before the girders, ρ would increase and the columns would have less flexural moment with increased axial forces to develop the necessary RM values. The relative hazards of one type of failure over another would depend on the particular interaction characteristics of the column section and the numerical values of moment and axial forces.

Dynamic Variations with ρ. The ratios of natural periods, T_1/T_2 and T_1/T_3, vary with ρ as shown in Figure 11. It is apparent that for the very low ρ-values the system is approaching the classical ratios of 6.27

and 17.6 for uniform cantilever bars in flexure. For high ρ-values the ratios are approaching the shear bar values of 3 and 5. Joint rotation is reduced with high ρ-values indicating greater relative girder stiffness; thus story "shear" predominates.(3,4) On the other hand, for buildings with ρ-values much less than unity the shear building concept (8,9) without correction (3,4,10) involves serious error in period determinations.

Figure 11 indicates the categories of buildings associated with the relative stiffness of the horizontal and the vertical elements. In the upper range where the curves are almost vertical there is the "shear" building. Immediately below this is the "shear-frame" building in which joint rotation plays an important part. In the next range, flexure and shear are both important and there may be no points of inflection in the columns of the lower story or two. This range is designated herein as the "cantilever frame". With lower ρ-values, overall flexure tends to dominate and there may be no column inflection points in many of the lower stories. This is termed here the "framed cantilever" range. Finally, at very low values of ρ, the building may be considered a cantilever for most purposes. The divisions between these categories are arbitrary and must be considered as subject to variation, and overlapping, depending upon the particular matter of interest. The popular but misleading term "shear wall" has no relationship to these categories. On the contrary, a building with "shear" walls with or without a frame will have low ρ-values and will tend to be a cantilever-type wherein flexure will often dominate over shear per se. The "ρ" approach is general.

Only the shear building is subject to analysis by popular, approximate dynamic or static methods. For all others, joint rotation and overall flexure are significant or dominant, and points of column moment inflection may be far from the positions indicated by approximate frame analysis procedures. The dynamic characteristics of building types 1 and 2 have been covered in a separate paper.(4) The other three types are considered herein.

The mode shapes also vary with ρ as shown in Figure 12 for the fundamental and the second modes of the 16-story frame. The low ρ-values indicate flexural performance. The second mode nodal point is much higher with the low ρ-values and the fundamental mode shape is curved from the vertical position as shown. It has been found, however, that a highly tapered shear stiffness from top to bottom can also produce a mode shape as shown (for the low ρ-values) even in pure shear.(3) Thus a so-called "flexural" curvature of a building may or may not represent dominant overall flexure and low ρ-values. The ρ-value, however, is a real indicator of the relative importance of shear and flexure. The participation factors for top story deflection (6) are also shown in Figure 12. There is considerable variation with the ρ-value.

Error in Approximate Methods of Frame Analysis. The technical literature and most textbooks on structural frame analysis contain considerable data on various approximate methods of stress analysis of rigid frames for buildings. Probably the most popular for several decades have been the "Portal Method" and the "Cantilever Method", although there are many others. Most of these methods are based upon assumptions as to the locations of points of inflection in girders and columns and/or as to shear distribution or axial forces in the columns. Enough assumptions are made to make indeterminate structures subject to analysis by ordinary equations of equilibrium. These methods have been and still are widely used in building design.

In most cases with traditional type buildings or buildings of the shear or shear-frame types -- in other words, buildings with ρ-values greater than say 0.10 -- the results obtained are satisfactory. However, with low ρ-values the results can be seriously in error and dangerous in design as indicated in Figures 4 and 5 where the points of inflection may not exist at all in many stories. This phenomenon, which has been reported previously,(6) occurs in cantilever-type buildings. Low ρ-values exist where the spans are long and/or the floor systems are shallow, where planned vertical structural members such as core units, walls, piers or massive columns dominate the response, or where non-structural filler walls or partitions are placed in a building in such manner as to dominate the response, at least until damage occurs and possibly allows the frame to act somewhat as the designer intended. However, girder hinging in earthquakes can also cause low ρ-values and cantilever-type response. (1)

Cantilever-type buildings must be treated as such and either be analyzed by rigorous methods or with proper allowances for the nature of the cantilever-type system. This becomes very important in not only design but in the analysis of existing buildings for resistance to ground motion.

Table I is a comparison of shears, axial forces and moments for the lower stories of the exterior column of the building frame of Figure 2, loaded as shown. The values have been computed by the Portal Method, the Cantilever Method (which bears no relationship to the cantilever considerations of this paper), and by rigorous methods with computer aid. Only the lateral forces of Figure 2 are considered in this analysis. The low ρ-value column shears are greater than the approximate methods suggest, the axial forces are less, and the column moments are much greater. It would seem that cantilever-type buildings designed under traditional approximate methods can be grossly overstressed, and have been in recent destructive earthquakes. If this overstressing occurs with structures not designed to be ductile,(6) failure can be expected.

PERIOD DETERMINATIONS

Natural periods of any category of buildings may be computed by the lumped mass analogy, providing the computations are conducted rigorously and the effects of column or pier width and joint rotation are carefully considered. So-called "shear" methods are not appropriate for any but the shear category in Figure 11. The lumped mass computation may involve considerable, and sometimes unnecessary, labor for cantilever-type buildings. Even with digital computers it is necessary to prepare data cards for all members.

Pure Cantilevers. If ρ-values are zero or very close to zero, the structure can be considered a pure cantilever system, and the periods can be readily computed by hand methods. (2,3) The building may consist of several vertical cantilever walls or elements or it may be a single tube or hollow rectangle. The assumption may usually be made that the total mass, shear area, and moment of inertia are uniformly distributed over the height as outlined by Jacobsen. (2) For a pure cantilever with a fixed base

$$T_{1s} = 0.288 \sqrt{D_s}\ , \text{ sec} \qquad (5)$$

$$T_{2s} = T_{1s}/3 \qquad (6)$$

$$T_{3s} = T_{1s}/5 \qquad (7)$$

$$T_{1f} = 0.258 \sqrt{D_f}\ , \text{ sec} \qquad (8)$$

$$T_{2f} = T_{1f}/6.27 \qquad (9)$$

$$T_{3f} = T_{1f}/17.6 \qquad (10)$$

$$T_{isf} = \sqrt{T_{is}^2 + T_{if}^2} \qquad (11)$$

$$D_s = \frac{WH}{0.8\, A_v E} \qquad (12)$$

$$D_f = \frac{WH^3}{8\, EI} = \frac{A_v H^2 D_s}{(10)\, I} \qquad (13)$$

The above equations are shown for the case of a single vertical element, which may be of any shape. If there should be two or more vertical elements without significant connection between them so that they would bend individually (i.e., develop no significant shear between them due to flexure) and if they participate in parallel, then the sum of their individual A_v and I values would be used in the above equations for A_v and I respectively. However, if individual vertical elements have sufficient connecting elements to develop flexural shear for the levels of stress involved, then the gross I value for the composite elements would be employed. In this sense the term gross I includes the Ad^2 contribution of the combined elements about their common neutral axis. There may be such combined elements in a building together with many individual elements, as for example internal core-wall units and individual building columns. The determination of the effective I value thus requires considerable judgment. Each case must be considered on its own merits. The decision -- often a difficult one -- must be made whether the analysis is approximate or rigorous.

The periods of several real cantilever buildings were computed as above and the results compared to known measured periods with good agreement, as shown in Table II. The low ρ-values are due to stiff wall elements and/or flexible floor systems. There are many buildings with such characteristics. The ones shown in Table II are privately owned and are therefore not identified herein.

Most buildings of the cantilever-type are dominated by vertical walls, piers or core elements that do not tend to vary greatly with height in the building. Thus the assumption of uniform properties is generally acceptable. This may be checked and, if necessary, corrections made by reference to Figure 15 and equation 14.

The Modified Cantilever Method (MCM). In order to explore the effects of moment restraint on the natural periods of framed-cantilever, cantilever-frame,and cantilever buildings, the 8-story and the 16-story frame building models were used as follows. The lowest three natural periods and mode shapes were computed rigorously as lumped mass models with a wide range of ρ-values. In addition, the periods were computed by equations 8, 9, 10 and 13, ignoring the slight restraint of the relatively flexible girder systems. Shear was also considered but it was found to be negligible.

$$\text{Let } \lambda_i = \frac{T_i \text{ computed rigorously as a lumped mass system}}{T_{isf} \text{ by equation 11 (or its equivalent)}}$$

Figure 13 shows the values of λ_i versus ρ for the lowest three modes of both the 8 and the 16-story frames. Some λ_i values are greater than unity. This illustrates a condition that holds for all values, namely that a lumped mass system has different characteristics than a uniformly distributed system. Of course, as the number of masses increases the period ratios would approach unity.

A separate investigation was made as to the difference in periods between a uniform rod in pure flexure and for that same rod modeled by N equal lumped masses with the masses equally spaced starting at the top of the rod. The latter case, of course, simulates the lumping of a typical building. A building per se is neither a distributed rod nor a lumped system; it falls between these limits. There are several ways to conduct the computations for the lumped mass system representing the rod. Three were employed: (a) rigorous computation, (b) rigorous determination of D_f as a lumped mass system followed by the use of equations 8, 9 and 10 to get the periods, and (c) determination of story deformations as a flexural rod followed by period computation with lumped masses. The case (b) method gives periods slightly shorter than the rigorous method for a small number of masses. However, this difference is reduced as the number of masses increases. The only difference between procedures (a) and (b) is the assumption about mass distribution in getting the periods from known deformations. The results for all three methods are shown in Figure 14. The values are independent of the mode, i. The ratios shown in Figure 14, case (a) computation represent the asymptotic λ values as ρ approaches 0. For example, the 8-story values, for all modes, would approach 1.12, and the 16-story values would approach 1.06.

The 8-story and 16-story frames used for a major part of this study were assigned uniform column moments of inertia -- for each ρ-value -- over the height of the structure. Since these columns actually represent the core, pier or wall-type members that are usually dominant and essentially uniform in low ρ-value structures, this assumption best simulates real cantilever-type buildings. The Modified Cantilever Method (MCM) utilizes the midheight characteristic of ρ, I, and A_V and simulates most real structures with very low ρ-values. If I should vary or taper considerably with height, corrections can be applied based upon the results of a study of the stiffness taper parameter. Most tall buildings have essentially uniform weight distribution with height even if the stiffness varies. The reason is that there are many more significant weight contributions than those of the structural members. The assumption of uniform weight distribution -- barring setbacks, of course -- is therefore retained. Very little has been done with the case of variable stiffness and uniform mass. However, Salvadori and Heer provided useful constants for cantilevers of constant mass and linearly varying flexural stiffness. (11) Their data were employed herein in a study that compared the periods of tapered stiffness flexural cantilevers to the periods computed as uniform stiffness cantilevers of average moment of inertia. With the assumption of linearly varying moment of inertia, the midheight value of I is the same as the average value.

Figure 15 summarizes the results of the tapered stiffness study and provides convenient correction factors, β_i, for the first three modes of vibration. The ratio of the true tapered-stiffness period to the period computed on the basis of assumed uniform moment of inertia, β_i, is plotted against the taper ratio, defined as the moment of inertia at the top of the building divided by that at the base. Assumptions are that I varies linearly and the mass is uniform. Only flexure is considered. β_i is very close to unity and therefore negligible for the second and third modes except for very highly tapered stiffnesses. The first mode is more sensitive and β_1 becomes important for taper ratios less than, say, 0.50.

It has been shown[3,4] that for tall buildings with ρ-values such as to indicate the shear or shear-frame types (see Figure 11) column axial deformation -- often termed column "shortening" -- may be significant in period determination and in stress analysis. This effect was also investigated for low ρ-value building frames. As may be expected, the effect of axial deformation becomes less with decreasing ρ-values. It essentially vanishes if ρ is less than 0.01 and is generally negligible for ρ-values less than 0.10. This effect is therefore negligible for cantilever-type buildings.

The Modified Cantilever Method (MCM) provides for the computation of approximate tall-building periods by the simple methods normally used for cantilever beams. It applies to buildings of the cantilever, framed-cantilever, or cantilever-frame types having low ρ-values.

$$T_i = (T_{isf})\,(\lambda_i)\,\left(\frac{\varphi_x}{\varphi_a}\right)\,(\beta_i) \qquad (14)$$

The values of T_{isf} are computed with equation 11 (or if shear is negligible, with equations 8, 9, and 10); λ_i data are obtained from Figure 13 or an equivalent figure; φ_x/φ_a data from Figure 14; and β_i data from Figure 15. In the above, φ_x refers to case (a), (b) or (c) in Figure 14.

Example of MCM. Find the natural periods of a 16-story building having $\rho = 0.005$ and $T_{1f} = 3.00$ sec. (Shear is negligible.) Assume case (b) of Figure 14 is appropriate, and that the ratio of $I_{top}/I_{base} = 0.6$.

From Figure 13, $\lambda_{i=1,2,3} = 0.40, 0.68, 0.88$

From Figure 14, $\varphi_b/\varphi_a = 1.04/1.06$

From Figure 15, $\beta_{i=1,2,3} = 0.93, 0.96, 0.99$

Using equations 14, 9, and 10 as appropriate:

$$T_1 = (3.00)\ (0.40)\ (1.04/1.06)\ (0.93) = 1.09 \text{ sec}$$

$$T_2 = (3.00/6.27)\ (0.68)\ (1.04/1.06)\ (0.96) = 0.31 \text{ sec}$$

$$T_3 = (3.00/17.6)\ (0.88)\ (1.04/1.06)\ (0.99) = 0.15 \text{ sec}$$

BRACED FRAMES AND FILLER WALLS

Some buildings have diagonal braces that considerably alter the response characteristics from those of rigid frames. If all panels are braced, axial stresses dominate and the structure becomes very rigid. The moment taken by the individual columns is greatly reduced and therefore the overall resisting moment is increased. If the braces are sufficiently strong and rigid, the entire frame performs as a unit cantilever member. It develops essentially all of the overturning moment (OTM) as though ρ were zero and the diagonal braces were merely internal shear distributors for the overall cantilever unit.

Diagonal struts were added to the frames of Figures 2 and 3. They were each assigned pinned ends so as to develop no moment. The brace areas were constant at 20 square inches. Two rigid frame systems were employed, one with $\rho = 0.10$ and the other with $\rho = 1.00$. These frames with the struts were then subjected to analysis. In some of the runs, the braces were omitted from the lowest story to simulate certain types of buildings. Table III gives some of the results obtained.

The stiffnesses of the frames are greatly increased with the bracing. Assigning an arbitrary stiffness of 1 to the non-braced frame, the stiffnesses at the top level are 7.7 and 34.1 for the $\rho = 0.10$ frames with the partial, and the full bracing, respectively. The bracing tends to dominate the characteristics and therefore the response to ground motion.

Let it now be assumed that the strut braces with pin ends simulate filler walls that offer compression value across the diagonals, that is until such time as the walls crack from tension normal to the compression diagonals. The great importance of these walls is apparent. For practical purposes if walls exist in all panels the structure tends to respond as a cantilever with $\rho = 0$. The difference in rigidity is so great, at all levels, that any other frames in the building without walls become relatively ineffective. The building becomes essentially a cantilever-type structure and should be treated as one in design and analysis.

The case with walls (braces) in all but the lowest story is interesting in that the first-story columns must resist shear, axial force and moment not only from the one frame (as shown in Table III) but additional amounts from any other non-walled frames in the building system. Moreover, the periods are reduced and therefore the response pattern is altered. Filler walls can not safely be ignored. It is also to be noted that when braces or walls occur in all but the lowest story, the overall building can be modeled reasonably well as a single degree of freedom system.

SUMMARY AND CONCLUSIONS

Many contemporary buildings have much greater stiffness in the vertical members than in the horizontal framing and tend to respond more as cantilevers than as conventional frames. An easily determined index, ρ , is provided to classify framed buildings into five basic types and to determine whether or not there are cantilever tendencies. The three cantilever types -- the cantilever-frame, the framed-cantilever, and the cantilever -- may be analyzed by the Modified Cantilever Method proposed herein. Correction factors are provided as part of the Modified Cantilever Method for the moment restraint of flexible floor systems, for variations between lumped mass and distributed mass systems, and for tapered stiffness cantilevers as compared to uniform cantilevers.

The use of traditional approximate methods of structural frame analysis can be seriously in error and lead to dangerous buildings if the building systems have low ρ-values and tend to respond as cantilever systems. Diagonal braces and filler walls also tend to create cantilever-type systems that may respond and be stressed in a much different manner than the designer or analyst would normally assume. Much earthquake damage in the last decade can be ascribed to cantilever-type buildings that were not treated as such in design, as well as to lack of designed ductility.

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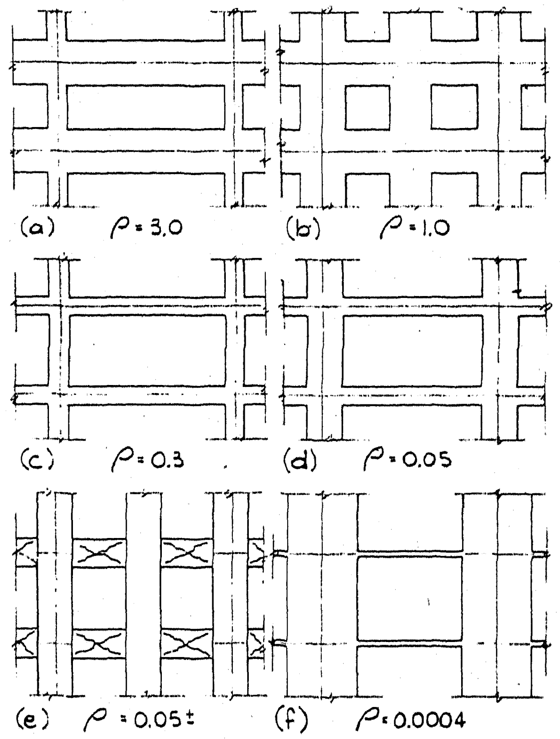


FIG. 1 - PARTIAL ELEVATIONS OF BUILDINGS

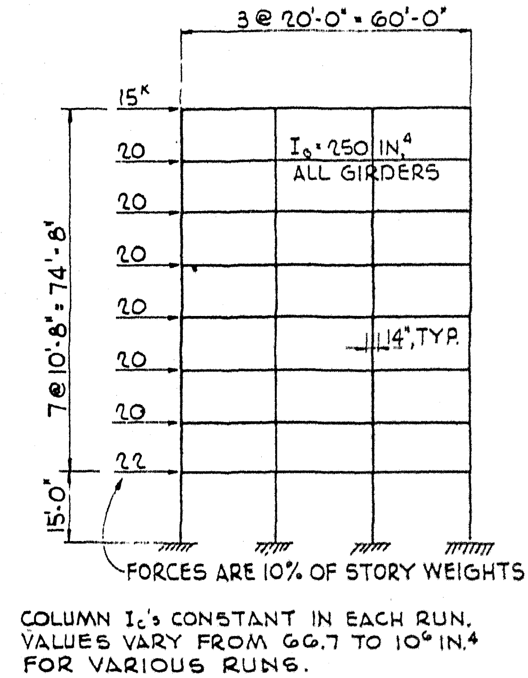


FIG. 2 - 8-STORY FRAME

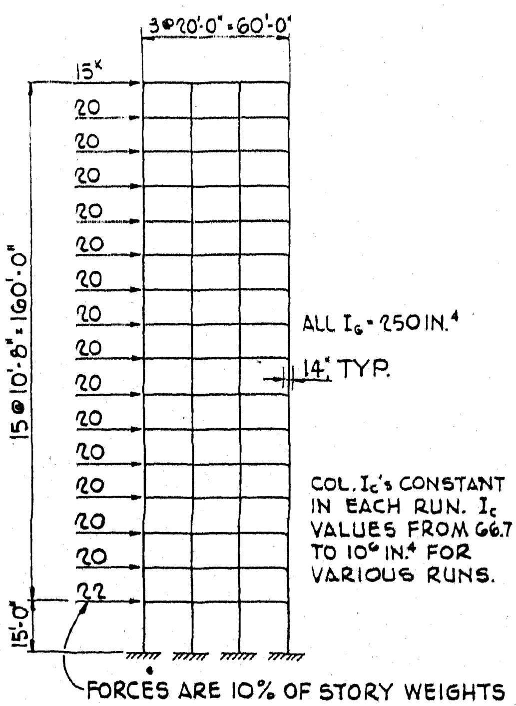


FIG. 3 - 16-STORY FRAME

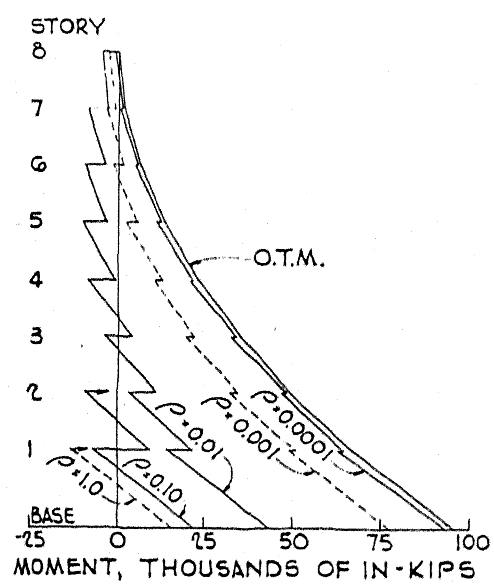


FIG. 4 - ΣM PER STORY, 8-STORY FRAME

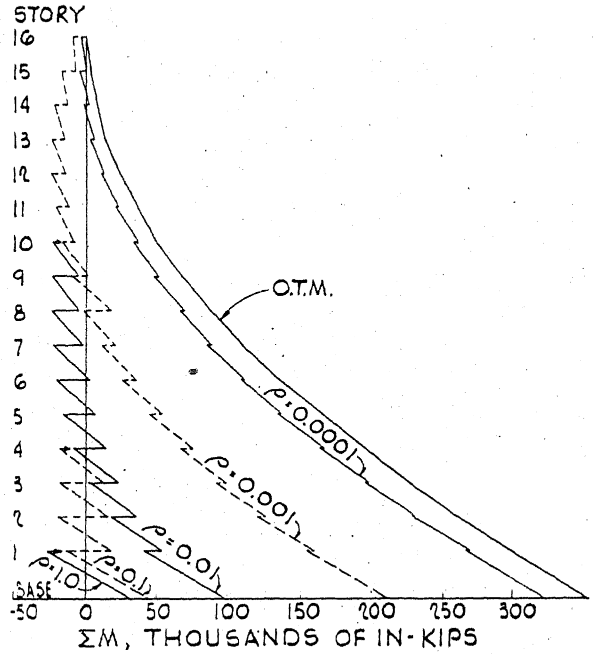


FIG. 5 - ΣM PER STORY, 16-STORY FRAME

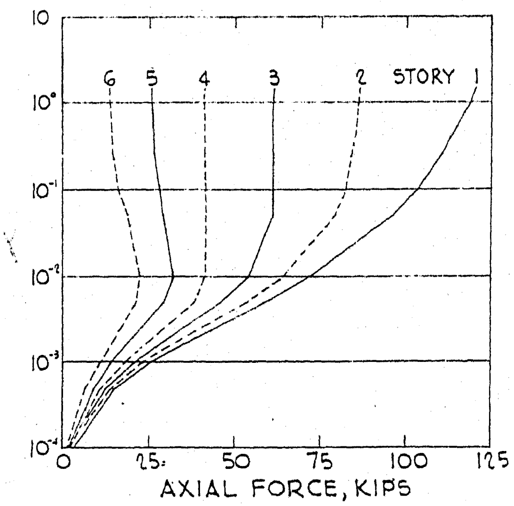


FIG. 6 - AXIAL FORCE IN EXTERIOR COLUMN FROM LATERAL FORCES, 8-STORY FRAME

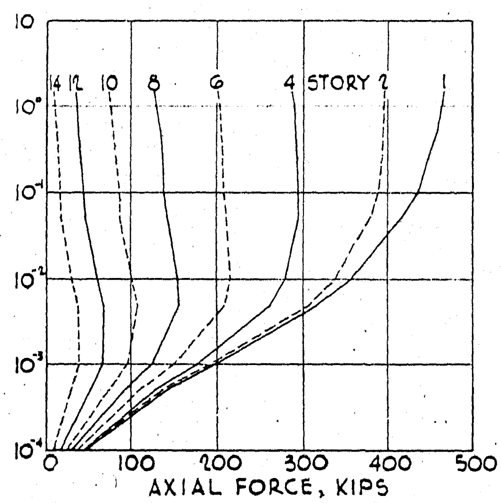


FIG. 7 - AXIAL FORCE IN EXTERIOR COLUMN FROM LATERAL FORCES, 16-STORY FRAME

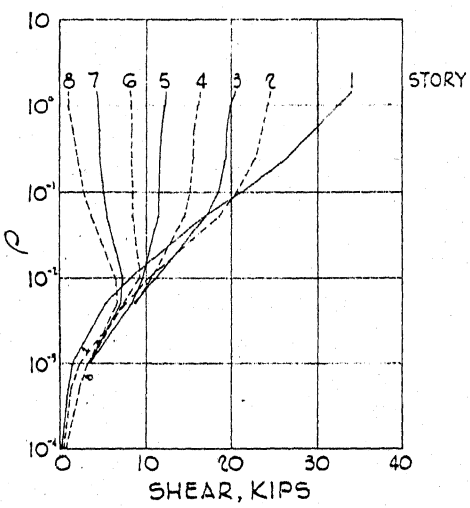


FIG. 8 - SHEAR IN EXTERIOR GIRDER FROM LATERAL FORCES, 8-STORY FRAME

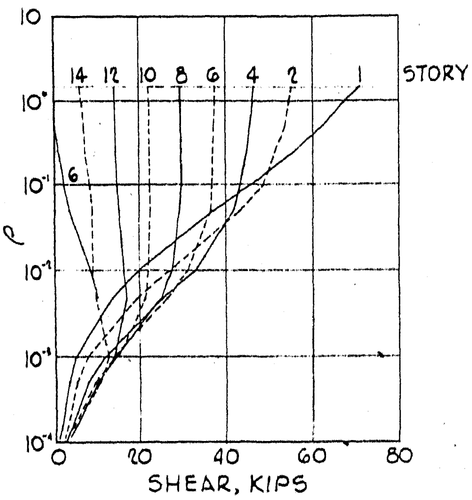


FIG. 9 - SHEAR IN EXTERIOR GIRDER FROM LATERAL FORCES, 16-STORY FRAME

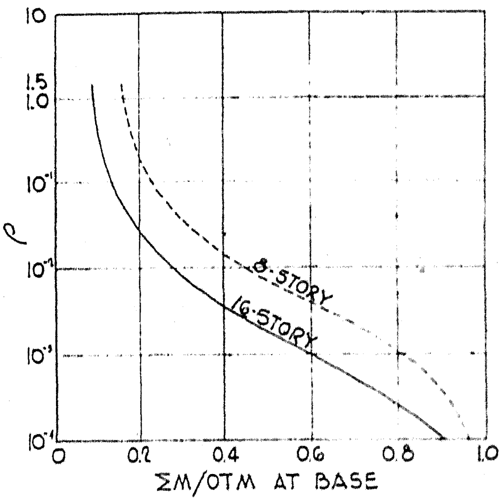


FIG. 10 - ρ VERSUS RATIO OF FLEXURAL MOMENT TO OVERTURNING MOMENT

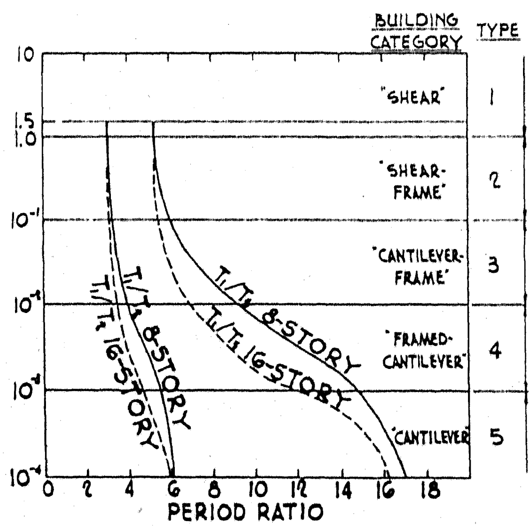
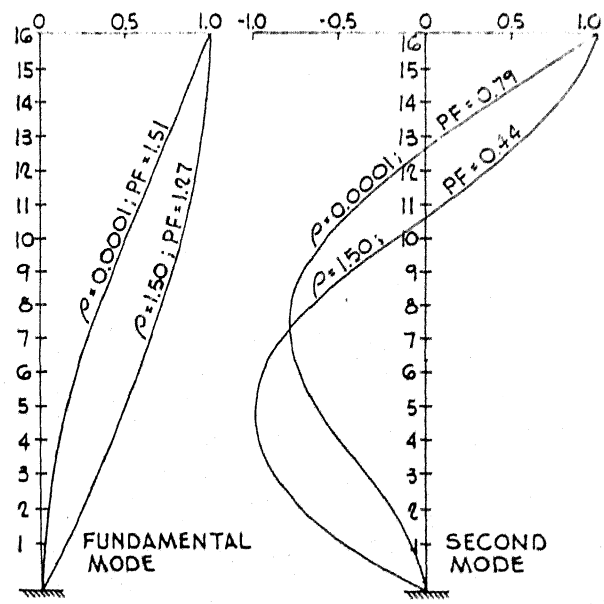


FIG. 11 - PERIOD RATIOS VERSUS ρ



PF REFERS TO PARTICIPATION FACTOR, TOP LEVEL DEFLECTION

FIG. 12 - MODE SHAPES, 16-STORY FRAME

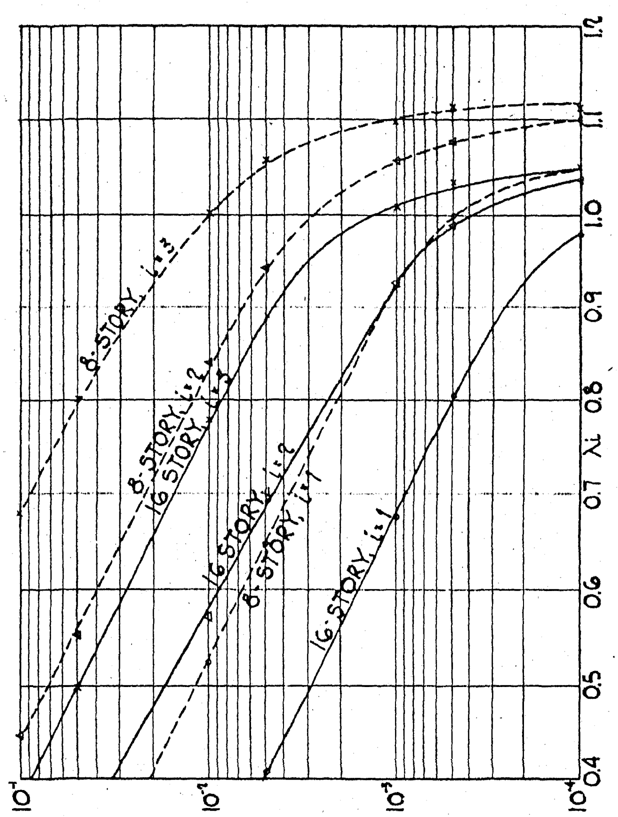


FIG. 13 - ρ VS. λi FOR 8 & 16-STORY FRAMES

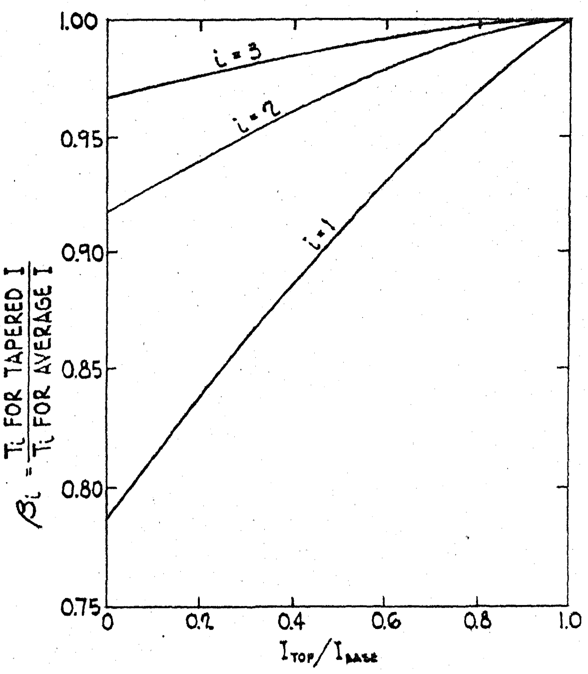


FIG. 15 - β_i VERSUS TAPER RATIO FOR FLEXURAL CANTILEVERS

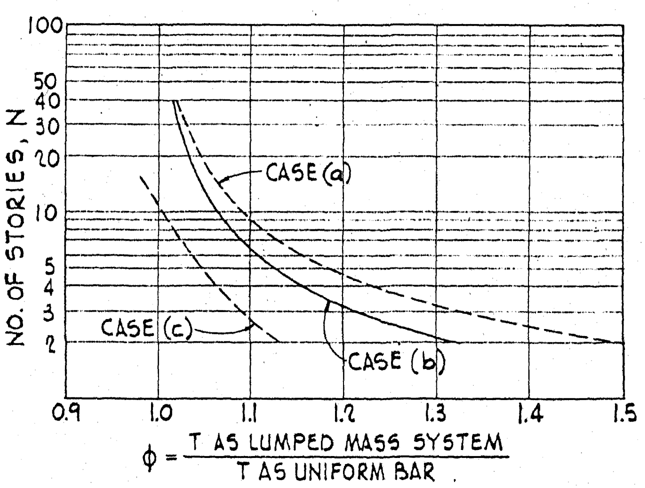


FIG. 14 - CORRECTION FACTOR FOR LUMPING WHEN ρ = 0

Table I - Comparison of Shears, Axial Forces and Moments in Column No. 1

Floor Level	Approximate Methods: "Portal" Method	Approximate Methods: "Cantilever" Method	Exact Values: ρ=1.0	ρ=0.1	ρ=.01	ρ=.001	ρ=.0001
		Shear, kips					
2							
	22.5	20.3	24.6	22.9	28.8	32.6	33.6
1							
	26.2	23.6	36.2	35.8	37.9	39.0	39.2
Base							
		Axial Force (Tension), kips					
2							
	81.3	73.2	86.2	82.4	64.8	23.7	3.3
1							
	112.9	101.7	118.8	103.6	72.9	25.5	3.5
Base							
		Moment, inch-kips*					
2 Top	1440	1296	+1734	+1556	-1138	-8283	-11,897
Bot	-1440	-1296	-1418	-1374	-4828	-12,458	-16,199
1 Top	2354	2124	2871	1219	-3853	-12,246	-16,172
Bot	-2354	-2124	-3651	-5227	-10,677	-19,260	-23,231
Base							
Lowest story with point of inflection	1	1	1	1	3	6	8

*Positive moment here refers to tension on the right side of the column.

Table II - Periods of Real Cantilever Buildings

Building Designation	ρ	Assumed E (kip/in.^{2})	No. of Stories	Fundamental Period, (sec.) Computed	Fundamental Period, (sec.) Measured
SN 38	0	800	14	0.52	0.50
SN 43	0.00010	3000	21	1.55	1.50
SN 44	0.00019	3000	9	0.25	0.26
SN 45	0.00074	3000	9	0.53	0.50
SN 55	0.00020	3000	9	0.31	0.32
SN 56	0.00070	3000	9	0.65	0.58

Table III - Data with and without Diagonal Braces

Diagonal Bracing	Frame: ρ	Frame: I_c (in.^{4})	Approx. T_1 (sec)	ΣM at Base (in.-kips)	Right Column, First Story: Shear (kips)	Axial (kips)	Base Moment (in.-kips)
None	1.00	100	4.47	15,300	36	119	3650
"	0.10	1000	2.68	21,700	36	104	5230
All panels but lower story	1.00	100	2.79	14,800	38	102	3610
	0.10	1000	1.06	16,300	39	91	4110
All panels	1.00	100	0.64	130	0	72	44
	0.10	1000	0.47	705	2	70	306