the results obtained in the few investigations that have been carried out to date,[1,2] it appears that for a given ground motion the lateral displacements obtained from an elasto-plastic analysis are quite similar in magnitude to the displacements obtained from an elastic analysis. Therefore, it has been suggested--and it is generally accepted at present--that a seismic design for a building can be based on an elastic analysis made for a reduced acceleration spectrum corresponding to a selected value of ductility factor which can be mobilized by the appropriate choice of the structural system. This ductility factor has been usually selected as 4 to 6 for typical ductile framed structures. The authors have some reservations about the application of this approach in the case of reinforced concrete structures.

The large ductility requirement for lateral displacement can be achieved through localized inelastic deformations that occur at certain critical regions along the members of the structure. The ductility requirements for these local deformations may considerably exceed the ductility requirements for the lateral displacement, depending on the detailed design of the members of the structure. Therefore, requirement of a lateral displacement ductility factor although necessary is not by itself sufficient to prevent failure under an extreme earthquake. A recent study[1] has shown that results based on linear elastic response generally will over-estimate the ductile deformations in the columns and under-estimate them in the girders as compared to values obtained from an elasto-plastic analysis. In reinforced concrete structures the knowledge of accurate values for the required ductility at critical regions is particularly important as available rotation capacity is sensitive to types, amount and detailing of the reinforcement.

The moment-rotation relationship at critical regions differs significantly from the bi-linear elasto-plastic idealization usually assumed in the analytical studies. From results obtained in a series of tests on reinforced concrete frames that have been carried out at the University of California, Berkeley, since 1959,[3-7] as well as from detailed analysis of results obtained by other investigators,[8-16] it has been observed that the behavior of reinforced concrete structural members under repeated load reversals is quite different from that expected from elasto-perfectly plastic elements. Significant loss of stiffness occurs as the number of cycles of severe alternating strains is increased. Although the observed loss of stiffness does not prevent the properly reinforced critical region from developing its ultimate strength, the initial stiffness at load reversal decreases and the deformation at which the carrying capacity is reached increases as the number of alternating loading cycles increases. Therefore, the real problem in reinforced concrete structures properly designed in accordance with current requirements appears to be one of deterioration of stiffness rather than of strength.

Because the earthquake response of any structure is sensitive to variations in its natural frequencies which in turn depend on the stiffness, it is clear that deterioration of stiffness has to affect the overall behavior of a structure.[17] This was clearly shown in recent studies by Clough and Johnston[17] and by J. M. Eisenberg[18]. It was concluded[17] that the principal effect of the loss of stiffness resulting from inelastic deformation is an increase in the period of vibration which leads to a modified type of response behavior. For long period structures, the change of period T tends to eliminate resonance with the earthquake input and thus to diminish the response activity significantly. For short period, the loss of stiffness leads to increased amplitude of displacement. In spite of these significant effects the overall ductility requirements are not materially different from those obtained with the idealized elasto-plastic mechanism except for structures with $T < 0.5$ second. It should be noted that these conclusions were obtained from studies on single-degree of freedom systems. It is not clear what the effect can be on multi-degree of freedom system as it depends on many factors. However, there is no doubt that a considerably different distribution of internal forces occurs due to the stiffness degradation. Some regions which, based on the assumed elasto-plastic behavior, are not expected to suffer inelastic deformation may actually undergo such deformations with the corresponding danger of failure.

The objective of this paper is to discuss the prediction of the expected stiffness deterioration of a reinforced concrete region which undergoes repeated reversal of bending and the possible implication of these localized deteriorations in the design of multi-story buildings for strong motion earthquakes.

MECHANISM OF STIFFNESS DETERIORATION. Large inelastic deformations usually are local in nature and occur at certain concentrated regions of the frame members. At the present time the generally accepted practice calls for design of framed structures with "strong" columns and "weak" girders, which contemplates that the columns will be essentially free from large inelastic deformations and that such deformations will be concentrated in the girders of the system. The following discussion is concerned with this type of a system.

The exact location of the regions at which inelastic deformations in the girders will be concentrated depends on the arrangement of steel reinforcement as well as on the loading. In general, these deformations occur at the girder ends, i.e., at the girder-column joint and/or at some section near mid-span. These regions will be called "critical" regions.

The instantaneous stiffness at the critical region is defined in the usual way as the slope of the general force-deformation curve. The behavior of the girder will be largely controlled by the bending moment and therefore the significant stiffness is that for flexure. Therefore, the instantaneous stiffness at time τ during loading for a particular critical region is defined as:

$$K = dM/d\phi \qquad (1)$$

where M is the moment and ϕ is the corresponding average curvature. Thus, the determination of stiffness K requires determination of $M - \phi$ variations during the loading history.

The moment and the curvature at critical regions of reinforced concrete members subjected to large inelastic deformations are sensitive to (a) the inelastic behavior of steel reinforcement which often exhibits pronounced Bauschinger effect, (b) the degree of cracking in the concrete, (c) the effectiveness of composite action (bond) between steel and concrete, (d) the possibility of slip or loss of effective anchorage, and (e) the presence of shear deformations and shear (diagonal) cracking. These factors are all sensitive to the stress history of the structure during the earthquake--and often lead to a decrease of stiffness in successive cycles of loading. This decrease is referred to as "degradation" or "deterioration" of stiffness.

The role of some of the above factors in the observed stiffness deterioration has been discussed in previous publications,[4,19,20] and is summarized below with reference to a cantilever beam, Fig. 1. If a doubly-reinforced concrete member is loaded well into the inelastic range-causing yielding in the tensile steel--the major flexural crack--denoted as C_1^t in Fig. 1(a)--will not close completely on unloading, Fig. 1(b). The degree of opening will depend on how far the tensile steel was strained into the plastic range during first loading. If the tensile steel was strained well beyond the initial yielding, a crack C_2^b may originate on the bottom side during unloading.

If the member is then loaded in the opposite direction, the critical section which has already been cracked will offer considerably less resistance to rotation than during the first loading. This decrease in resistance may be caused by the fact that the two faces of the former crack C_2^t are not in perfect contact. The crack at the top may close or not, depending on the peak value of the reversed load P_3 in comparison with P_1, on the amount of top and bottom reinforcing steel and other factors. Because the concrete in the two faces has undergone a process of disruption a reduction in the stiffness of the critical region should occur, even if the crack closes.

If the load P_3 in the reversed direction reaches the same peak value as P_1, the width of the crack C_3^b would be larger than C_1^t -- observed under P_1. It is evident that if the member is now unloaded, the critical cross-section will be cracked throughout $C_4^t - C_4^b$ and the width of the crack will depend mainly on the amount of yielding of the steel, the effectiveness of composite action (bond) between steel and concrete and, to a lesser extent, on the degree of the concrete disruption.

At the start of a new cycle of alternating load, the original doubly-reinforced concrete section will behave as a steel cross section represented by the tensile and compressive steel reinforcement. If the

1. Linear distribution of strains over the depth of the section.

2. Elasto-plastic stress-strain relationship for reinforcing steel, with or without pronounced Bauschinger effect on load reversal.

3. Idealized elasto-plastic stress-strain relationship in compression, and no tensile strength for concrete. It was assumed that the behavior of concrete in compression is independent of its prior tensile or compressive strain history, and that the cracks in concrete close when the strain with reference to the initial unloaded state becomes zero.

4. Shear effects, axial load effects, and time-dependent processes such as shrinkage, creep, as well as changes in moisture and temperature were neglected.

Also, it was assumed that "average" strains between two adjacent cracks may be used to evaluate the stresses at the cracked section and the curvature. These assumptions imply that concrete does not participate in resisting any tension, that the steel reinforcement in tension is entirely free to slip with respect to the surrounding concrete and therefore no strain compatibility between steel and concrete is imposed when the reinforcement is in tension. The deformations calculated using these assumptions in some cases were in good agreement with test results, but in others differed significantly from the observed values. Therefore, a somewhat more precise behavior model is desirable, particularly for evaluation of the discrepancies between test results and calculated values.

In reinforced concrete elements plane sections no longer remain plane after cracking as shearing deformations are produced in the concrete tensile zone and slippage on the steel reinforcement takes place. Also, longitudinal stresses and strains vary between cracks and the neutral axis follows an oscillatory curve. The cracked beam and the strains in concrete and steel reinforcement are shown in Fig. 2.

Deformations and crack widths can be defined if the strain field is completely defined. Because the behavior of reinforced concrete members considered in this study is non-linear and includes significant plastic deformation it is necessary to define strain and curvature incrementally. It can be shown that the average curvature ϕ at time τ of strain history can be expressed as:

$$\phi = [\alpha^t(\varepsilon^t_{so}) - \alpha^b\ (\varepsilon^b_{so})]\ /\ d \qquad (2)$$

where ε^t_{so} and ε^b_{so} are the top and bottom steel strains at the cracked section, d_s is the distance between top and bottom steel, and the coefficients α^t and α^b are "loading functions" which characterized the composite action ("bond effectiveness") for a particular loading history. These loading functions are defined by strength was given by a linear elastic stage on first loading, ideal plastic deformation at yield, a linear elastic unloading stage followed by a curvilinear stress-strain relationship when the stress is reversed, then again a linear elastic unloading followed by the same curvilinear relationship upon second reversal, and thereafter repeating this pattern of linear unloading and curvilinear reversed loading. The curvilinear portion of the stress-strain diagram was taken as:

$$|\sigma|(\text{ksi}) = 64.5 - 52.7\ (0.838)^{1000\ \epsilon} \tag{5}$$

The compressive strength of concrete f_c was given as 5.42 ksi, and Hognestad's basic stress-strain curve was adopted, i.e., a parabola for the ascending branch of the stress-strain diagram and a linear descending branch. Again, no experimental data was available for unloading and reloading the concrete. As a first approximation it was assumed that for increasing compression strain the basic curve can be used to determine corresponding stresses. For decreasing compression strain the unloading from the basic curve follows a straight line parallel to the initial tangent modulus, which was taken as $E_c = 3.9 \times 10^3$ ksi. Unloading to zero stress leads to a residual compression strain, and any further reduction in compression strain takes place without generating any tension in concrete. Furthermore, it was assumed that concrete will crack when extension (under zero stress) exceeds the extensibility of concrete. For practical purposes, it was assumed that cracking will first take place when the net strain becomes tensile with reference to the original state.

On reloading it was assumed that the concrete stresses were defined by the original basic curve as described above, i.e., concrete $f_c - \epsilon$ relationship was unaffected by the previous strain history. The authors wish to emphasize the inadequacy of this assumption - but in the absence of conclusive data it is difficult to justify any other assumption, particularly with respect to the response of previously cracked concrete to reloading in compression.

Calculations of curvature were carried out using Eq. (2) with a loading cycle which starts with an initial inelastic deformation, followed by unloading and a reversal to inelastic deformation, and then again unloading and a reversal to the inelastic range was assumed. Although this does not follow the loading sequence used in the laboratory tests, the values of calculated curvature in this initial load cycling are approximately in agreement with the test results. For example, the values of the calculated moment and curvature at the end of the 2nd loading are 3328 kip-in and 592×10^{-6} rad/inch and of the approximately corresponding test values are 3150 kip-in and 500×10^{-6} rad/inch, Figs. 3(a) and 3(b).

Test results[4-8,10-16] as illustrated by the example shown in Fig. 3(a) indicate that values of initial stiffness (at $M = 0$) for successive then it can be shown that:

$$(dM/d\phi) = \frac{(A_s^b)\,(\bar{E}_s^b)\,(d_s)^2}{\left[1 + \frac{A_s^b\,\bar{E}_s^b}{A_s^t\,\bar{E}_s^t}\right]\left(1 + \frac{\ell_a}{\ell_o}\right)} \tag{9}$$

Considering the experimental results shown in Fig. 3(a), the values of the initial stiffness (at $M = 0$) can be obtained as slopes of the appropriate M-ϕ curves at successive loading cycles. Also, using numerical values $A_s^t = 4$ sq. in., $A_s^b = 2$ sq. in., $\ell_o = 10$ in., and $d_s = 15.12$ in., for the joint considered:

$$dM/d\phi = \frac{457.2\,(\bar{E}_s^b)}{\left[1 + \frac{\bar{E}_s^b}{2\,\bar{E}_s^t}\right]\left(1 + \frac{\ell_a}{10}\right)} \tag{10}$$

Values of $\bar{E}_s^t$, $\bar{E}_s^b$, and ℓ_a can be only approximated, as the behavior of the particular reinforcing steel under cyclic loading condition and bond deterioration in the joint tested are not known precisely. The authors believe that for the particular conditions it is reasonable that bond deterioration may take place over the entire straight portion of the bent bars (7.8 inches long) and over a portion of the circular arc of the hook. Thus an estimate of $\ell_a = 10$ inches seems reasonable.

The values of tangent moduli $\bar{E}_s^t$ and $\bar{E}_s^b$ at zero stress in the steel reinforcement depend on the preceding strain history. On a reloading cycle the stress in the top bar changes from compression to tension, and the stresses in the bottom bar change from tension to compression. At the peak load preceding the current stress transition the stresses in the bottom bars are double the stresses in the top bars, i.e., if the bottom bars are at tension yield stress the top bars are in compression at half the yield value. Because the effect of non-linear unloading is greater for the bars undergoing the larger amount of unloading, it follows that the bottom bars are more sensitive to this non-linearity and that $\bar{E}_s^b < \bar{E}_s^t$. It may be assumed that for the case of $M = 0$ and $\sigma_s = 0$, $\bar{E}_s^t = E_s = 27.3 \times 10^3$ ksi and that $\bar{E}_s^b$ may vary between this value and about half the value. Thus $0.5 < (\bar{E}_s^b/\bar{E}_s^t) < 1.0$ would represent a reasonable estimate.

Using the upper, lower, and intermediate values for this ratio, the values of initial stiffness were calculated using Eq. (10) and values of $\ell_a = 0$ and 10 inches. The calculated values are compared with test data, as shown in Table I.

It can be seen that the non-linear stress-strain relationship of steel reinforcement, without the anchorage bond deterioration, can not closely spaced ties. However, inelastic deformations can take place even in case of complete confinement when there are voids in the concrete and especially when weak aggregates are used which can crush.

CONCLUDING REMARKS. Reinforced concrete frame buildings designed for earthquake loads must be able to accommodate large inelastic lateral displacements in a ductile manner. This ductility can be achieved through localized inelastic deformations at critical regions of the structure. Depending on the design of the members and their connections the required ductility factor (usually taken in the range of 4 to 6), may not by itself be sufficient to ensure satisfactory performance of the building under a strong earthquake because the ductility requirements for local deformations at the critical regions may considerably exceed the ductility requirements for lateral displacements.

While strength and ductility requirements at these critical regions can be met even after several cycles of inelastic alternating strains, loss of stiffness occurs as the number of cycles of severe load reversals is increased. Stabilization of stiffness may be achieved for critical regions located at midspan of girder, but no stabilization is observed for critical regions simulating connections of girders to exterior columns. This progressive loss of stiffness may introduce several problems in the design of reinforced concrete multi-story frames.

First, the loss of stiffness may lead to an over-estimation of the possible energy absorption capacity of the structure. If ductility at any cycle is defined in the usual manner as the ration of the maximum deformation which develops at the ultimate load to the deformation at which the member first starts to yield under constant load, Fig. 3(a), it would appear that the loss of stiffness increases ductility and therefore, the loss of stiffness would look beneficial. However, the energy absorption capacity, defined by the area under the load-deformation curve, is not increased by the observed deterioration in cycles shown in Fig. 3(a).

Secondly, the observed localized deterioration in stiffness may be important in multistory buildings because it may lead to a reduction of load carrying capacity of the building due to the additional moments introduced by the increased deflections.

In reinforced concrete structures subjected to dynamic actions of an alternating character there is another factor which may considerably accentuate the deterioration of bond effectiveness--and consequently the stiffness--already observed in quasi-static loading tests. Experience gained in the examination of structures that have been subjected to actions producing rapid increases and quick reversals in the stresses in the reinforcement shows that bond deteriorates quite rapidly and is far less reliable under dynamic conditions than it is in cases of quasi-static loading. On the other hand, the effect of stiffness deterioration observed in laboratory tests of subassemblies may be significantly reduced in a complete building system when contribution of floors, roof, partitions and cladding is considered.

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TABLE I - VALUES OF ($dM/d\phi$) AT RELOADING ($M = 0$)

Estimated $\bar{E}_s^b/E_s$	Calculated ($dM/d\phi$) kip-in.^{2}		Reloading Cycle	Measured ($dM/d\phi$) kip-in.^{2}
	$\ell_a = 0$	$\ell_a = 10$ in.		
1.0	8.32×10^6	4.16×10^6	3rd	3.8×10^6
0.75	6.76×10^6	3.38×10^6	8th	2.6×10^6
0.50	5×10^6	2.5×10^6	12th	1.6×10^6

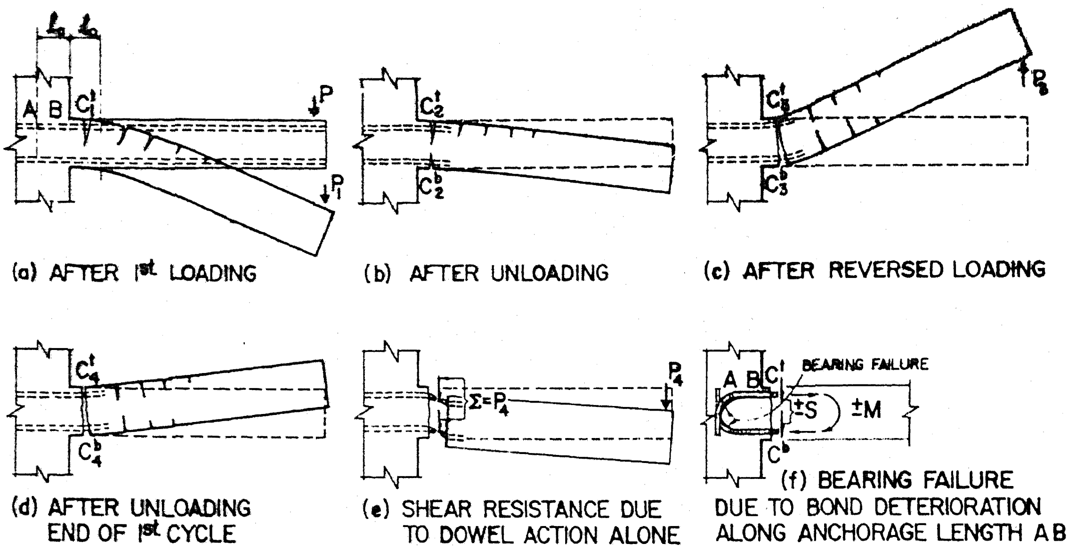

FIG. I EFFECT OF ALTERNATING LOADING ON REINFORCED CONCRETE

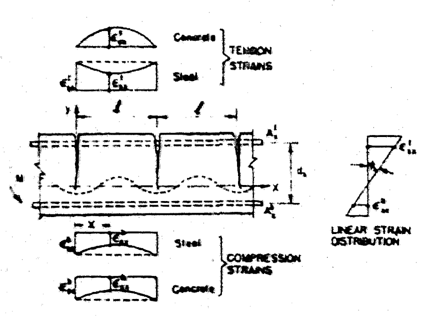

FIG 2 VARIATION OF STRAINS BETWEEN CRACKS

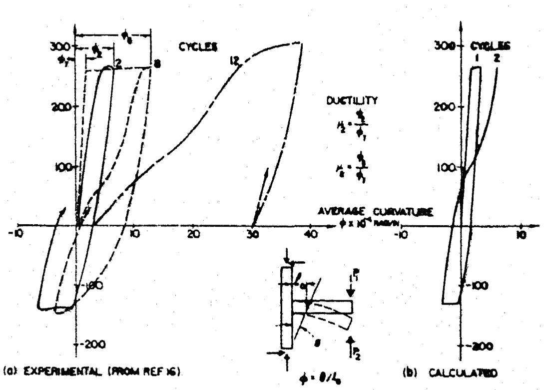

FIG 3 MOMENT - CURVATURE FOR BEAM-COLUMN JOINT