USE OF RESONANCE METHOD IN MECHANICAL MODELLING OF SEISMIC EFFECT ON STRUCTURES

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SUMMARY

The paper presents a method by which the seismic inertial load adequate stresses are determined in respect with separate modes of natural vibrations of the model of a structure.

These vibrations are excited in the resonance regime and the acceleration level of the model base with each mode of vibration is determined according to the standard spectral diagram of dynamic coefficient adopted in the USSR codes of seismic design. Thus, although vibrations of the model of a structure occur conformably to the sinusoidal law, the test method in current use permits taking indirectly into account of the real character of seismic ground vibrations during destructive earthquakes.

The paper sets forth the analytic rationale of this method, as well as examples of the realization of the method as applied to arch dams.

DETERMINATION OF SEISMIC INERTIAL LOAD ON AN ARCH DAM WITH THE AID OF THE SPECTRAL METHOD OF THE EARTHQUAKE RESISTANCE

To simplify the exposition of the nature of the application of the spectral method of the theory of seismic stability to the problem of designing an arch dam we consider that the masses m_K of the arch dam weight are concentrated along the medial surface of the dam at separate points (descrete scheme of mass distribution).

Let such points in the dam be n . Consider now any point K and assume that at this point the external dynamic force P_{uK} acts in the direction of the vector U_K of the elastic horizontal dam displacement at the point K under consideration.

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Resolving the elastic displacement and the external force into a series according to natural modes of dam vibrations, we can write

$$U_K(t) = \sum_{i=1}^{n} U_{iK}(t) \tag{1}$$

$$P_{uK}(t) = \sum_{i=1}^{n} P_{iuK}(t) \tag{2}$$

where t is the time

i is the mode number of natural vibrations.

Assuming that the modes of natural vibrations Φ_{iuK} are known, then the external load in the i th form, with due allowance for (2), may be represented as:

$$P_{uK}(t) = \sum_{i=1}^{n} a(t) m_K \Phi_{iuK} \omega_i^2 \tag{3}$$

where $a_i(t)$ is the factor of series expansion;

ω_i is the frequency of natural vibrations of the dam in the i th mode.

Using the conditions of the orthogonality of natural modes and the well-known operation connected with it, we obtain

$$a_i(t) = \frac{1}{\omega_i^2} \frac{\sum_{j=1}^{n} P_{uj}(t) \Phi_{iuj}}{\sum_{j=1}^{n} m_j \Phi_{iuj}^2} \tag{4}$$

Now we turn to the expression of external force which can also be written as follows

$$P_{uK}(t) = m_K W_{uK}(t),$$

where $W_{uK}(t)$ is the projection of the horizontally directed vector of the seismic acceleration of the structure as a non-deforming body along the direction of the displacement vector $U_K(t)$ of the medial dam surface as an elastic body.

It follows that

$$W_{uk}(t) = W(t)\cos(\widehat{U_k, W}) \tag{5}$$

Here $W(t)$ is the modulus of the ground seismic acceleration vector which is determined from the instrument recording of earthquake (from the accelerogram).

We denote $W(t) = Wf(t)$

where $W = |W(t)|_{max}$ and $|f(t)| \leqslant 1$

we finally obtain

$$P_{uk}(t) = m_k W f(t)\cos(\widehat{U_k, W}) \tag{6}$$

and after substituting this value into (4), we obtain

$$a_i(t) = \frac{1}{\omega_i^2} W f(t)\, b_i \tag{7}$$

where

$$b_i = \frac{\sum\limits_{j=1}^{n} m_j \Phi_{iuj} \cos(\widehat{U_j, W})}{\sum\limits_{j=1}^{n} m_j \Phi_{iuj}^2} \tag{8}$$

The differential equation of the point K motion, with due account of (1) and (7), can be written as follows

$$\ddot{U}_{ik}(t) + \frac{\varepsilon_i}{m_k}\dot{U}_{ik}(t) + U_{ik}(t)\,\omega_i^2 = -Wf(t)\, b_i \Phi_{iuk}$$

(ε is the coefficient of diffusion of elastic energy).

The solution of this equation may be represented with allowance for the notation

$$b_i \Phi_{iuk} = \eta^{*}_{iuk} \tag{9}$$

in the following form

$$U_{ik}(t) = -\frac{K_c g}{\omega_i^2}\eta^*_{iuk}\omega_i\int_0^t f(\xi)\, e^{-\frac{\delta_i}{2\pi}\omega_i(t-\xi)}\sin\omega_i(t-\xi)\,d\xi \tag{10}$$

where $K_c = \frac{W}{g}$ is the coefficient of seismicity;

δ_i is the logarithmic decrement of natural vibrations.

As is known

$$\omega_i\int_0^t f(\xi)\, e^{-\frac{\delta_i}{2\pi}\omega_i(t-\xi)}\sin\omega_i(t-\xi)\,d\xi = \beta_i(t) \tag{11}$$

is the function of the time of the coefficient of dynamic.

Thus, the dam elastic displacement and the corresponding seismic force acting at the point K of the medial dam surface, with account of (1), (10) and (11), can be written as follows

$$U_k(t) = K_c g \sum_{i=1}^{n} \frac{1}{\omega_i^2}\beta_i(t)\eta^*_{iuk}$$
$$S_{uk}(t) = K_c q_k \sum_{i=1}^{n} \beta_i(t)\eta^*_{iuk} \tag{12}$$

Here $q_k = m_k g$

According to (8) and (9)

$$\eta^*_{iuk} = \Phi_{iuk}\frac{\sum_{j=1}^{n} q_j \Phi_{iuj}\cos(\widehat{U_j, W})}{\sum_{j=1}^{n} q_j \Phi_{ius}} \tag{13}$$

Computation may prove to be more convenient by considering the mass of the dam as distributed along the medial surface. Then, instead of (12) and (13) the following formulae may be used

$$S_u(x,y,z,t) = K_c q(x,y,z) \sum_{i=1}^{n} \beta_i(t) \eta^*_{iu}(x,y,z) \tag{14}$$

$$\eta^*_{iu}(x,y,z) = \Phi_{iu}(x,y,z) \frac{\int_\Omega q(x,y,z) \Phi_{iu}(x,y,z) \cos(\hat{U,W}) d\Omega}{\int_\Omega q(x,y,z) \Phi^2_{iu}(x,y,z) d\Omega} \tag{15}$$

Here x, y and z denote the coordinates of the point considered on the medial surface;

$q(x,y,z)$ is the dam weight per unit area of the medial surface at the point with the coordinates x, y and z;

$d\Omega$ is the elementary area of the medial surface.

Hence, the formulae (12),(13), (14) and (15) completely solve the problem of determination of the seismic load on the arch dam.

However, elimination of the time parametr t from the formulae (12) and (14) is difficult in practice. Various methods have been proposed for the solution of this question.

The following formulae obtained from a probability approach to the theory of seismic stability are recommended by the present writers

$$|S_{uk}| = K_c q_k \sqrt{\left(1 - \sum_{i=1}^{N} \eta^*_{iuk}\right)^2 + \sum_{i=1}^{N} \beta_i^2 \eta^{2*}_{iuk}} \tag{16}$$

$$|S_u(x,y,z)| = K_c q(x,y,z) \sqrt{\left[1 - \sum^{N} \eta^*_{iu}(x,y,z)\right]^2 + \sum_{i=1}^{N} \beta_i^2 \eta^{2*}_{iu}(x,y,z)} \tag{17}$$

being the number of natural vibration modes taken into account and is usually sufficient for $N = 3$ to 5.

The second external load component, normal to the force $P_{uk}(t)$ represented by the formula (6), is to be considered in order to have complete characterization of the seismic inertial load acting on the concrete arch dam.

In other words, if $P_{uk}(t)$ according to (6), and consequently, the seismic load according to (16) or (17) are normally directed at the medial dam surface, then the load component we are interested in, will be directed tangentially at the same surface. Thus, the tangential component of the external load may, analogously with (6), be written as follows

$$\mathcal{T}_K(t) = K_c q_K f(t) \sin(\widehat{U_K, W}) \tag{18}$$

METHOD OF MODELLING RESEARCH INTO EARTHQUAKE RESISTANCE OF AN ARCH DAM ON A SEISMIC PLATFORM

A technique of analytic computation of seismic inertial load acting on an arch dam was presented in the preceding section of this paper. However, practical implementation of this method as applied to actual dams is bound with some difficulties due to the complexity of dam design during its space action. Thus, estimation of the self-induced frequencies ω_i and the modes φ_{iu} (x , y , z) on which the dynamic coefficients β_i and the modes of vibrations η_{iu}(x , y , z) determining the normal (radial) component of the seismic load by the formula (16) or (17) proves to be difficult.

Furthermore, it is difficult to design the stress condition of the dam, for in this account should be taken of both the normal (radial) and tangential components of the seismic load, the distribution regularity of which along the medial dam surface is somewhat complex.

With the aim of overcoming these difficulties we have recourse to modelling investigation. In this case, the most convenient method is, of course, that with aid of which the model will be tested on the seismic platform capable of reproducing real seismic vibrations of the ground, certainly in a modelled form. It is only under this condition that we shall arive at a direct solution of the problem of directly measuring the state of stress of the model and consequently, of the dam itself.

However, such a way of solving the given problem, even at the present stage of development of technology is difficult, as it would be necessary to have a powerful three-component seismic platform of preset guidance.

Therefore, proposed in this paper is a method of modelling investigation that requires for its realization a seismic platform capable of reproducing only sinusoidal vibrations to be sure with a possibility of grade change of the frequency of platform vibrations.

As will be shown below, even with such restrictions of seismic platform indices, there is full possibility of carrying out modelling investigazions of problems of earthquake resistance of structures in full accordance with the spectral approach to seismic effect, as set forth in the last section of this paper.

All this is achieced by that the mode of vibrations $\varphi_{iu}(x, y, z)$ we are interested in, is reproduced in the resonance regime, whereas the absolute value of the amplitudes of vibrations of the dam in this mode is determined as follows.

On the basis of (12) for the i th mode of vibrations we can write the following formula for the elastic displacement of the model

$$U_{ik}(t) = K_c g \frac{1}{\omega_i^2} \beta_i(t) \eta^*_{iuk}$$

or the maximum value

$$U_{ik} = K_c g \frac{1}{\omega_i^2} \beta_i \eta^*_{iuk}$$

On the other hand, with the resonance vibrations

$$U^*_{ik} = A^*_i \frac{\pi}{\delta_i} \eta^*_{iuk}$$

where A^*_i is the amplitude of vibrations of the seismic platform corresponding to the resonance regime

δ_i is the logarithmic decrement of the natural vibrations of the model.

From the condition $U_{ik} = U^*_{ik}$ we obtain

$$K_c g \frac{1}{\omega_i^2} \beta_i = A^*_i \frac{\pi}{\delta_i}$$

hence

$$A^*_i = \frac{\delta_i}{\pi} K_c g \beta_i \tag{19}$$

where ω_i is now equal to the resonance frequancy of platform vibrations. Assuming that the amplitude of platform acceleration in the resonance regime equals $W_i^* = A_i^* \omega_i^2$ in accordance with (19) this parameter of platform vibrations may also be resorted to:

$$W_i^* = \frac{\delta_i}{\pi} K_c g \beta_i \qquad (20)$$

The right-hand members of the equalities obtained are assumed to be given.

Thus, the tension state of the model and hence of the dam itself in the i th mode of vibrations under consideration will be characterized by stresses in the model measured in the resonance regime satisfying the condition (19) or (20). Knowing the stresses σ_i (x , y , z) in separate forms the full stress in the dam may be computed by a formula similar to (17) and written as follows

$$|\sigma(x,y,z)| = \sqrt{\sum_{i=1}^{n} \sigma_i^2(x,y,z)}\ .$$

The immediate task of model investigation will thus be solved. Tasks of methodological character are incidentally solved; namely, with a view to specifying the method computation of self-induced frequencies and modes, the latter, being defined theoretically, may be compared with their experimental values. Thus, natural modes are ascertained by measuring the elastic displacements of the model in resonance regimes, whereas self-induced frequencies will be equal to resonance frequencies.

On the whole, the accuracy of a theoretical determination of seismic load and the state of stress of a dam may be verified. If we set ourselves the task of determining the frequencies and natural vibration modes on the model, and, approximately, of the qualitative aspect of the state of stress of a particular arch dam under design then a most simple procedure of model investigation can be applied.

The question here is of the generation of resonance vibrations in the dam model by a single powerful electro-magnetic vibrator of oriented action, without mounting the model on the seismic platform.

TESTING THE MODEL WITH THE APPLICATION OF A VIBRATOR

The possibility of the use, in principle, of vibrators of oriented action for model investigations into seismic stability directly follows from the equation (4). Taking into consideration the method for determination of seismic inertial forces, and taking account of the equality of inertial forces arising under the influence of a single vibrator fixed at the point f and of the inertial forces generated during the vibration of the model we obtain the required value of exciting force of the vibrator

$$P_{iuf}(t) = K_c g \frac{\sum_{j=1}^{n} m_j \Phi_{iuj} \cos(\widehat{U_j, W})}{\Phi_{iuf}} \beta_i(t) \qquad (21)$$

Hence, to determine the maximum amplitude of vibrator force it is necessary to know the ordinates of the modes of natural vibrations at the model points we are concerned with. However, we are faced with the difficulty of designing a vibrator of preset guidance.

In order to simplify further the solution of task in modelling investigation of earthquake resistance of an arch dam we assume that the vibrator is capable of generating only sinusoidal vibrations, with a possibility of making a graded change of amplitude of the force P_{uf} and of a corresponding frequency of vibrations. Then, after a properly transforming and satisfying the maximum value of seismic force, we obtain:

$$P_{iuf} = K_c g \frac{\delta_i}{\pi} \frac{\sum_{j=1}^{n} m_i \Phi_{iuj} \cos(\widehat{U_j, W})}{\Phi_{iuf}} \beta_i \qquad (22)$$

Thus to determine the pulsating force it is necessary to have the value of the logarithmic decrement of the natural vibrations of the model δ_i, whereas the dynamic coefficient will be determined by the modelling standard curve β_i.

It is seen from (22) that to obtain a moderate force from the vibrator the latter must be fixed at the point where the ordinate of the self-induced function Φ_{iuf} reaches a maximum.

Realization of the proposed method of modelling investigations in earthquake resistance of structures by a single vibrator of oriented action is envisaged in the following form.

By attaching the vibrator to any point of the model, resonance vibrations are axcited, with an arbitrary exciting force of the vobrator, and the modes of model vibrations are fixed with the accuracy of a constant factor. Further, the modelled amplitude of the vibrator force is estimated by the formula (22) and with such a force we cause the model to produce resonance vibrations according to a preset normal mode. Incidentally, accelerations and dynamic stresses we are concerned with are registered by instruments. Having found these values in separate natural modes, as indicated above, the design values of seismic forces and stresses may be ascertained, with due allowance for all the vibration modes under consideration.

Application of the method described may be restricted to a determination of only natural modes and vibration frequencies of the model with a view to using them in an analytical solution of the problem of determining the seismic load by the formula (17).

It should be noted that the procedure of modelling investigations based on the use of a single vibrator is somewhat inferior to modelling investigations on a seismic platform, for it is connected with rather laborious computation by the formula (22). Incidentally, such computation prove to be roughly approximate ones inasmuch as the division of the whole mass of the model into separate concentrated masses m_i will be conditional. In spite of this, under certain conditions, with the aim of rapidly obtaining approximate data on the earthquake resistance of structures such investigatons may prove valuable.

The results are cited below of tests of one large model of an arch dam on the seismic plaform, and of another model of smaller size tested with the aid of a single vibrator.

One of the variants of the concrete arch dam 270 meters height of the Inguri Hydroelectric Plant was adopted as the prototype of the large model and for the small model - - another variant of the same dam, 300 meters in height.

The scale of length of the large model was equal to 1 : 200, i.e., the height of the model was 135 cm.

A low-modulus material obtained on the basis of cement--sand mortar with addition of rubber crumbs, concrete stone clay, and lead shot was used as material for this model.

The principal physical and mechanical characteristics of this material are: modulus of elastisity - 5200 kg/sq cm, volume weight - 3200 kg/cu m; Poisson's ratio - 0.25; logarithmic decrement of vibrations - 0.32.

The seismic platform on which the large model was tested involves a rigid metal-construction resting on flexible supports (Fig.1) The working surface of the platform equals 25 sq m and its lifting power is 35 tons. The platform permits testing models separately on the horizontal and vertical dynamic action according to sinusoidal law. The platform vibrations are excited by powerful mechanical vibrators with maximum exciting force of 25 tons and frequency range of 6 to 50 cps.

The dam model (Fig. 2) was designed and made with due account of the heterogeneity of the rocks of canyon edges and according to their deformation characteristics.

The test data of the model state of stress for the seismic action of 8 degree along and across the canyon, and their values translated for the prototype, are shown in Figs. 3 and 4.

The small model of 1 : 300 scale was tested an electric dynamic vibrator in the frequency range of 20 to 200 cps, in order to reveal stresses and deformed states.

A low-modulus material - also on the cement-sand basis, with addition of rubber crumbs and cement-stone clay, with the modulus of elasticity of 8000 kg/sq cm and volume weight of 1200 kg/ cu m was used as the material for this model.

The results of measurement of the model vibration modes are shown in Fig.5.

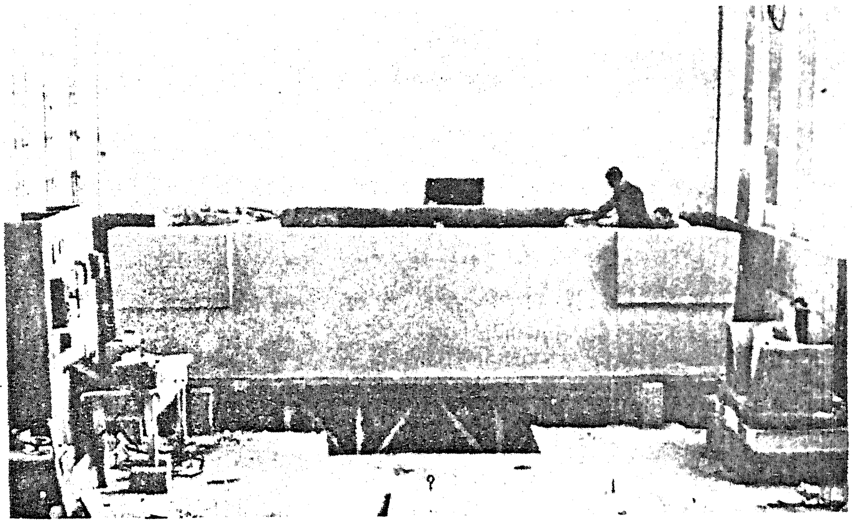

Fig. 1. General View of the Seismic Platform

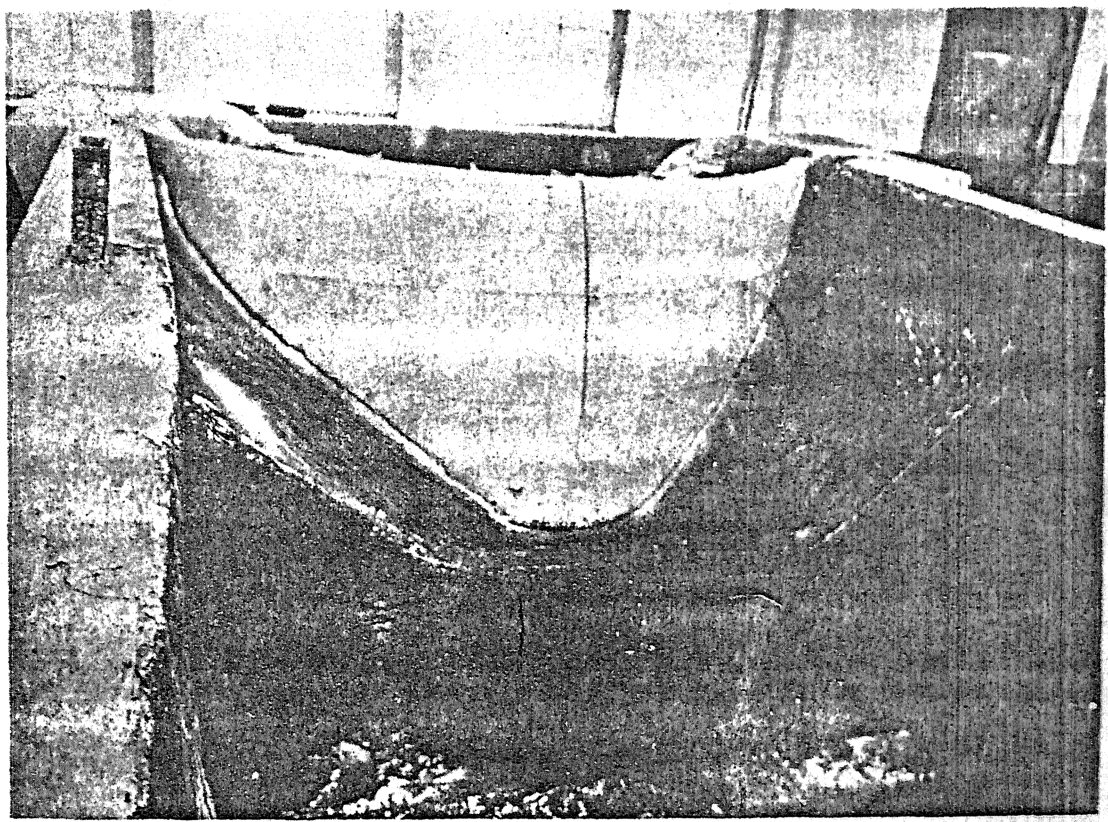

Fig. 2. A View of the Model of the Dam

Stress Diagram (kg/sq cm) of Arch Dam
During Longitudinal Seismic Action

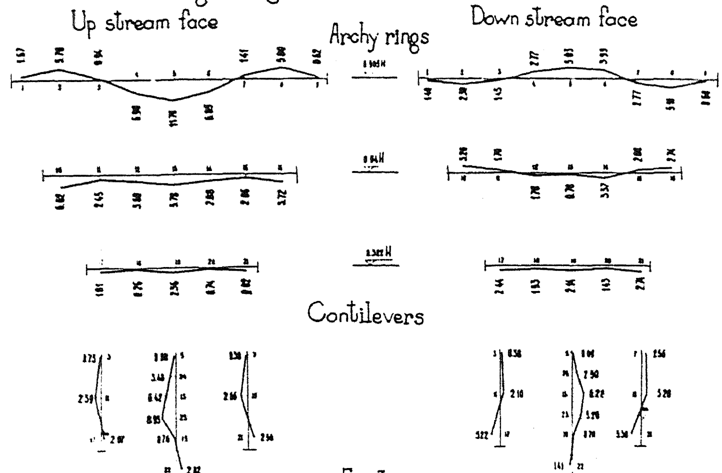

Fig. 3.

Stress Diagram (kg/sq cm) of Arch Dam
During Transverse Seismic Action

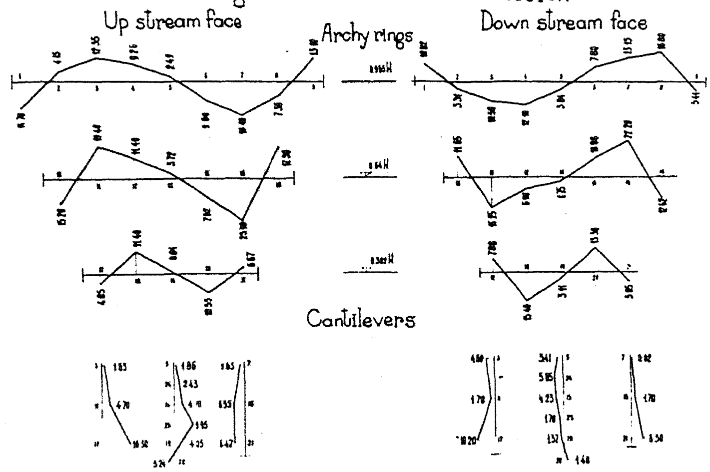

Fig. 4.

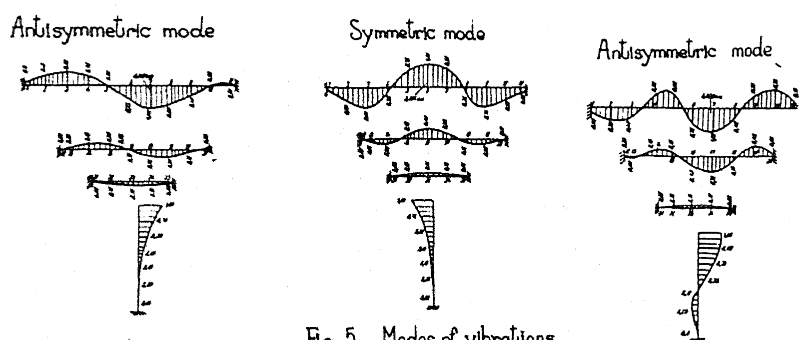

Fig. 5. Modes of vibrations