A METHOD OF DYNAMIC MODEL TEST OF ARCH DAMS

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Synopsis : A method of the dynamic model test of arch dams considering the hydrodynamic effect of water is described. Model tests heretofore in use for this purpose have been carried out on the vibration table. But as it needs a very large vibration table, it has much technical and economical difficulties. The method described in this paper makes possible to carry out model tests without using a large vibration table and will open the way to investigate problems concerning influences of storage water or foundation conditions on behaviors of dams during earthquakes.

1. Introduction

Earthquake loading is one of the main loadings to which dams might be subjected. When the dam is an arched type, the topographycal and geological conditions of the site have great influences on stress within dams. Therefore, these conditions must be taken into account in the calculation of seismic stresses as correctly as possible.

There are two ways of stress analysis, namely, the numerical and the experimental analysis. Since they have their several merits and fill up defects of the other way, experimental analyses are still useful in spite of the recent remarkable progress of numerical analyses.(1,2,3)

The usual method of the dynamical model test of structures is to rock the model on a vibration table. This method is available to models of buildings, bridges and gravity or fill-type dams or the like. However, if this method is used for the test of arch dams, as the abutment rock of the dam must be reproduced in this case, the vibration table must support very heavy loading. As a result, the vibration table becomes large whereas the model dam is obliged to be small and its natural period becomes very short. It is evident that it is difficult to rock the large table with very high frequency.(4)

2. A new method of dynamic model test of arch dams

The method of the dynamic model test of arch dams, developed in our laboratory is composed of two series of tests. In the 1st test, a small and rigid model of the reservor as well as the upstream face of the dam is used. This model is called the rigid model. The rigid model is mounted on the vibration table and rocked and the hydrodynamic pressure on the face of the model dam is measured by pressure gauges (Fig.1). However, it is practically impossible to build the model of whole area of the reservor on the vibration table. But it is not necessary because a part of the reservor which is located very far from the dam has practically no influence on hydrodynamic pressure on the dam. The period of the vibration of the table can be properly assigned to make the measurement of water pressure easy and to check the growth of the seiche in the pool. It is evident that when

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the vibration frequency is not so high, the hydrodynamic pressure can be measured easily by pressure gauges and that it is proper to separate the problem of the seiche from the problem of the behavior of dams during earthquakes.

In the 2nd test, a large scale model of the dam as well as a part of the reservor is made on the fixed floor. This model is called the flexible model. The shape of the dam and the topography and the geology of the foundation are correctly reproduced in the model with appropriate similarity. The inertia force of the mass of the model dam times the acceleration of the ground motion and the hydrodynamic pressure which has been determined by the 1st test are both applied to the model dam by electromagnetic exciters (Fig.2). Subjecting to these vibrating forces, the model dam and storage water vibrate developing strain within the model dam. Strains are measured by wire strain gauges and converted into strains of the prototype by the law of similitude.

The flexible model is better to be as large as possible. If the model is large, it becomes easy to satisfy the law of similitude and to reproduce complicated conditions of the foundation rock, for instances, faults or local weak zones. The model dam which has been built in our laboratory is 100 cm in height.

Sometimes, it is necessary to test the dam under the condition of reservor empty. In this case, the 1st test can be omitted. Furthermore, the specific gravity of material of the flexible model can be determined arbitrarily. This makes the test easier.(5)

3. Principles of the new method of model tests

The new method of model tests is based on following principles.

a) Stresses within the structure which is standing on the vibrating ground is equal to stresses within the structure, which is locating on the fixed floor and being subjected to the distributed load whose intensity is the mass of the structure times the acceleration of ground motion. This is the D Alembert s principle of mechanics.

b) The storage water behaves as if it was incompressible fluid. Prof. Westergaad considered that the storage water was compressible and introduced the result that the hydrodynamic pressure increased considerably when a certain relation was held between the water depth and the frequency of the ground vibration. However, such a considerable increase of water pressure has never been observed during past earthquakes or the dynamic tests of dams. Recently, Dr. Hatano pointed out that storage water could be regarded as incompressible because reservor bottoms were generally covered with mud and absorbed sound waves in water.(6) This is an important finding and one of the basic priciples of our model test.

c) When the differential equations and the boudary conditions which assign a physical phenomenon are all linear, the physical phenomenon can be separated into two phenomena. In case of the seismic vibration of the dam, the differential equations of motion of the dam and the storage water are both linear and the boundary conditions are also linear if the slight chage of water level of the reservor is ignored. Therefore, our method of the model test can be separated into two series of tests as previously mentioned.

d) Damping of the vibration of arch dams is mainly caused by the energy dissipation through the ground and the reservor. The reproduction of such damping mechanism in models correctly is difficult because models are a closed system whereas actual dams are systems without boundaries. Systems which has no definite boundary have no normal vibration and the modal method can not be available to the dynamic analysis of the system in rigorous meaning. This is a fundamental difference between model dams and prototype dams. However, at present, it is assumed that for prototype dams, there are approximately normal vibrations and the modal method is practically available to their dynamic analysis.

4. Illustrative example of the model test

Let us consider a dam of cantilever beam type and its two dimentional vibration. In Fig.3 OO' be the position of the beam under the static .. water load and AB its position during earthquakes. The equation of motion of the beam is

$$B\frac{\partial^4 u}{\partial y^4} + m\frac{\partial^2 (U+u)}{\partial t^2} = -p$$

where

B : bending rigidity of the beam,
m : mass of the unit length of the beam,
p : hydrodynamic pressure of storage water,
U : displacement of ground vibration,
u : relative displacement of the beam to the ground.

$$B\frac{\partial^4 u}{\partial y^4} + m\frac{\partial^2 u}{\partial t^2} = -m\frac{\partial^2 U}{\partial t^2} - p \quad \text{------------(1)}$$

Assuming the ground motion is synsoidal

$$U = U_0 \cos \omega t = \frac{\alpha}{\omega^2}\cos \omega t \quad \text{-----------(2)}$$

where

U_0: displacement amplitude of the ground motion,
ω: circular frequency of the ground motion,
α : acceleration amplitude of the ground motion.

Then (1) becomes

$$B\frac{\partial^4 u}{\partial y^4} + m\frac{\partial^2 u}{\partial t^2} = m\alpha \cos \omega t - p \quad \text{----------(3)}$$

This is the equation of motion of the dam.

The motion of water in the reservor is considered to be irrotational. Then its velocity has a potential.

$$\xi = \frac{\partial \phi}{\partial x}, \quad \eta = \frac{\partial \phi}{\partial y} \quad \text{--------------(4)}$$

where

ξ : velocity component in the cordinate x,
η : velocity component in the cordinate y,
ϕ : velocity potential.

Assuming water is incompressible, ϕ satisfies the Laplace's equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad \text{-------------------(5)}$$

The equations of motion are

$$\frac{\gamma}{g}\frac{\partial \xi}{\partial t} = -\frac{\partial p}{\partial x}, \quad \frac{\gamma}{g}\frac{\partial \eta}{\partial t} = -\frac{\partial p}{\partial y} \qquad (6)$$

where
γ : weight of unit volume of water.

$$p = -\frac{\gamma}{g}\frac{\partial \phi}{\partial t} \qquad (7)$$

The boundary conditions are approximately as follows,

$$\text{at } y = 0, \quad \frac{\partial \phi}{\partial t} = 0, \qquad (8)$$

$$\text{at } y = H, \quad \frac{\partial \phi}{\partial y} = 0, \qquad (9)$$

$$\text{at } x = \infty, \quad \phi = 0, \qquad (10)$$

$$\text{at } x = 0, \quad \frac{\partial \phi}{\partial x} = \frac{\partial U}{\partial t} + \frac{\partial u}{\partial t}. \qquad (11)$$

As the equation of motion and boudary conditios are both linear, ϕ can be separated into two potentials ϕ_1 and ϕ_2,

$$\phi = \phi_1 + \phi_2 \qquad (12)$$

Where ϕ_1 and ϕ_2 both satisfy the Laplace's equation and ϕ_1 satisfies the boundary conditions (8), (9), (10) as well as

$$\text{at } x = 0 \quad \frac{\partial \phi_1}{\partial x} = \frac{\partial U}{\partial t} \qquad (13)$$

and ϕ_2 satifies the boudary conditions (8), (9), (10) as well as

$$\text{at } x = 0 \quad \frac{\partial \phi_2}{\partial x} = \frac{\partial u}{\partial t}. \qquad (14)$$

The 1st test satifies the condition (13) and the 2nd test the condition (14).

After determination of hydrodynamic pressure due to ϕ_1 experimentally by the 1st test, (3) becomes

$$B\frac{\partial^4 u}{\partial y^4} + m\frac{\partial^2 u}{\partial t^2} = (m\alpha\cos\omega t - p_1) - p_2 \qquad (15)$$

where
p_1 : hydrodynamic pressure due to ϕ_1,
p_2 : hydrodynamic pressure due to ϕ_2.

This equation can be solved experimentally by measuring the deflection of the beam which is subjected to the vibrating external load $m\alpha\cos\omega t - p_1$.

5. Procedures of the test

a) The 1st test

A small wooden model of the upstream face of the dam and a part of the reservor nearby the dam is made and mounted on a vibration table. The table is vibrated with fairly long period and large amplitude. Acceleration of the vibration table, the scale of the model and hydrodynamic pressure on the face of the dam are to be recorded. The model dam which has been made in our laboratory is 23 cm in height.

b) The second test

A model of the dam and its foundation is made of deformable but elastic material and placed on a floor. Faults and weak zones in the foundation rock are reproduced in the model. Liquid is stored in the model reservor. Several exciters are attached to the down stream face of the model dam. Electro-magnetic forces corresponding to the sum of the inertia force which is equal to the mass times the acceleration of ground motion and hydrodynamic pressure which has been determined by the 1st test are applied to the model dam by means of the exciters. It is convenient to set up exciters on the face of the dam in order that each exciter will share equally in mass of the dam. Dynamic strain and displacement of the dam are measured and they are converted in those of the prototype by the law of similitude. The fetch length of the pool is not necessary to be more than three times of the height of the model dam, because hydrodynamic pressure decreases rapidly with the distance from the dam.

The ratio of the specific gravity of liquid stored in the reservor and that of the body of the dam must be identical for the model and the prototype. Practically, liquid other than water can not be available for the test. Therefore, the specific gravity of the model dam must be equal to that of concrete. However, since concrete is too rigid, concrete is not suitable material as that of the model dam. Therefore, a proper method must be considered to increase density of the model dam without increasing its rigidity. In our laboratory, the model dam was made of plaster and diatomearth and its specific gravity was about 0.8. Lead plates were bolted to the down stream face of the dam at their center so as to increase density of the dam body. For the test of model dams under the condition of reservor empty, the above mentioned similitude law can be ignored and the binding of lead plates becomes unnecessary.

6. Law of similitude

In this method of model tests, following four kinds of loadings are considered separately.

1) Inertia force of the dam due to ground acceleration where the dam is assumed to be rigid (rigid inertia force),
2) Hydrodynamic pressure where the dam is assumed to be rigid (rigid water pressure)
3) Inertia force of the dam due to motion of the dam relative to the ground (deformation inertia force)
4) Hydrodynamic pressure due to motion of the dam relative to the ground. (deformation water pressure)

In the 1st test, rigid water pressure is proportional to the acceleration of ground motion, specific gravity of storage water and the depth of water then

$$\frac{p'_{1m}}{p_{1p}} = \frac{k'_m \gamma'_m H'_m}{k_p \gamma_p H_p}$$

where

k'_m : acceleration of ground motion at the 1st test,
k_p : acceleration of ground motion of actual earthquakes,
γ'_m : specific gravity of storage liquid at the 1st test,
γ_p : specific gravity of storage water in prototype reservor,

H_m' : height of the model dam at the 1st test,
H_p : height of the prototype dam,
P_{1m}' : rigid water pressure measured by the 1st test,
P_{1p} : rigid water pressure on the prototype dam.

$$\therefore \quad P_{1p} = P_{1m}' \frac{k_p \gamma_p H_p}{k_m' \gamma_m' H_m} \quad \text{-------------} (16)$$

When storage liquid is water, (16) becomes

$$P_{1p} = P_{1m}' \frac{k_p H_p}{k_m' H_m'} \quad \text{-------------} (17)$$

The 2nd test is designed so as to strain of the model dam be identical to that of the prototype. Let the scale of the flexible model be λ

$$\lambda = \frac{H_m}{H_p} \quad \text{-------------} (18)$$

As relative displacement of the dam is an integlation of strain

$$\delta_m = \lambda \delta_p \quad \text{-------------} (19)$$

where
δ_m : displacement of the model dam at the 2nd test,
δ_p : displacement of the prototype dam.

Assuming that the Poisson's ratio of the material of the model and that of the prototype is identical, strain is proportional to stress and inversely proporional to Young's modulous, then

$$\frac{\sigma_m}{E_m} = \frac{\sigma_p}{E_p}$$

where
E_m : Young's modulous of the model dam at the 2nd test,
E_p : Young's modulous of the prototype dam,
σ_m : stress within the model dam at the 2nd test,
σ_p : stress within the prototype dam.

$$\therefore \quad \frac{\sigma_m}{\sigma_p} = \frac{E_m}{E_p} \quad \text{-------------} (20)$$

In order that stresses within the model and the prototype satisfy (20), intensity of four loads mentioned above must also satisfy (20). As the deformation inertia force is $\frac{wD\delta}{T^2}$, the law of similitude becomes to

$$\frac{w_m D_m \frac{\delta_m}{T_m^2}}{w_p D_p \frac{\delta_p}{T_p^2}} = \frac{E_m}{E_p}$$

where
w_m : specific gravity of the model dam at the 2nd test,
w_p : specific gravity of the prototype dam,
D_m : thickness of the model dam at the 2nd test,
D_p : thickness of the prototype dam,
T_m : natural period of the model dam at the 2nd test,
T_p : natural period of the prototype dam.

$$\frac{T_m}{T_p} = \frac{H_m}{H_p}\sqrt{\frac{w_m E_p}{w_p E_m}} = \tau \quad \text{-------------} (21)$$

If the time scale is assigned to be

$$\frac{t_m}{t_p} = \tau \quad \text{-------------} (22)$$

the similarity of the natural period can be held because the natural period of the dam is proportional to $H\sqrt{\frac{w}{E}}$.

As the deformation water pressure is proportional to $\frac{\gamma H \delta}{T^2}$, then

$$\frac{P_{2m}}{P_{2p}} = \frac{\gamma_m H_m \frac{\delta_m}{T_m^2}}{\gamma_p H_p \frac{\delta_p}{T_p^2}}$$

where

P_{2m} : deformation water pressure to the model dam,

P_{2p} : deformation water pressure to the prototype dam.

In order to hold the relation

$$\frac{P_{2m}}{P_{2p}} = \frac{E_m}{E_p}$$

following relation must be held

$$\frac{\gamma_m}{\gamma_p} \doteq \frac{w_m}{w_p} \qquad \text{-----------------(23)}$$

Rigid inertia force is applied to the model dam electro-magnetically by the exciters. Rigid inertia force per unit area of the model dam is

$$F_m = k_m w_m D_m \qquad \text{-----------------(24)}$$

where

F_m : rigid inertia force per uni area of the model dam,

k_m : acceleration of ground motion considered at the 2nd test.

Therefore, the application of Fm is equivalent to the acceleration of groun motion

$$k_m = \frac{F_m}{w_m D_m}$$

The corresponding acceleration of ground motion of the actual earthquakes is determined by

$$\frac{F_m}{k_p w_p D_p} = \frac{E_m}{E_p}$$

$$\therefore \quad k_p = \frac{E_p}{E_m}\frac{F_m}{w_p D_p} = \frac{E_p}{E_m}\frac{w_m D_m}{w_p D_p}\frac{F_m}{w_m D_m} = \frac{\tau^2}{\lambda}\frac{F_m}{w_m D_m} \qquad \text{----(25)}$$

When exciters are setup on the downstream face of the model dam so as each exciter to share equelly in mass of the dam, $\frac{F_m}{w_m D_m}$ is easily obtained by dividing total external load by total weight of the model dam.

Rigid water pressure is also applied to the dam by electro-magnetic exciters. Electro-magnetic force which is equivalent to the rigid water pressure is

$$P_{1m} = \frac{E_m}{E_p} P_{1p} \qquad \text{-----------------(26)}$$

where P_{1m} : rigid water pressure of the model dam at the 2nd test.

Strain within the model dam gives the strain within the prototype and displacement of the prototype is $\frac{1}{\lambda}$ times of the displacement of the model dam at the 2nd test.

7. An example of a model test

A test was carried out for the model dam, 100 cm in height, 320 cm in crest length. The center line of the crest arch was a parabola and the thickness of the crown cantilever was 5 cm at the top and 20 cm at the bottom. The section of the valley was slightly asymmetric. Furthermore, before testing, the model dam was cracked vertically from the top to the bottom at the section 120 cm distant from the right abutment due to over drying. It was immediately repaired but the repair work was considered to increase more the asymmetric property of the model. Dynamic modulous of elasticity of material of the model dam was 7850 kg/cm² and its specific gravity was 0.776. Equivalent specific gravity of the body of the model dam came to 2.3 after attaching lead plates to the body.

Amplitude of the vibration of the model increased considerably when it was excited by electromagnetic force with frequency 41 cps. This predominant vibration could be regarded practically as a 1st order natural vibration of the model. When the reservor was empty, the predominance of the 2 nd order normal vibration was recognized, but it was little when the reservor was full.

If the prototype dam is assumed to be 100 m in high and has Young's modulous of 300000 kg/cm², (21) becomes to

$$\tau = \frac{1}{100}\sqrt{\frac{2.3\times 300000}{2.3\times 7850}} = 0.0617$$

$$T_p = \frac{T_m}{\tau} = \frac{1}{0.0617\times 41} = 0.40$$

Accordingly, the natural period of the prototype dam is estimated to be 0.4 sec. The estimated period seems to be appropriate one compared with natural periods so far observed at existing arch dams.

Displacement and strain of the natural vibration of the model are shown in Fig. 5∼7. The asymmetric distribution of strain seems to be caused by the asymmetric shape and property of the model. Furthermore, the observed values are scattered in the neibourhood of the vertical section $V_r \sim V_5$. The local repair work of the cracked section may cause this scattering.

Comparing the distribution of the seismic deflection of the model dam with that of the deflection by static water load, the former is comparatively large at the upper part of the dam. As to the strain, it is noticed that the menbrane strain is remarkable, especially at the upper part of the dam. The dynamic arch strain is large at the upper part of the dam, whereas the static arch strain is large at the mid height as well as at the upper part of the dam. It is also noticed that large dynamic cantilever strain developes at the foot of the up stream face of the dam in the same way as the static strain. These results show the seismic intensity at the upper part of the dam is fairy larger than those at the lower part of the dam.

According to this test, it is confirmed that the new method mentioned above is practically available to study dynamic properties of arch dams,

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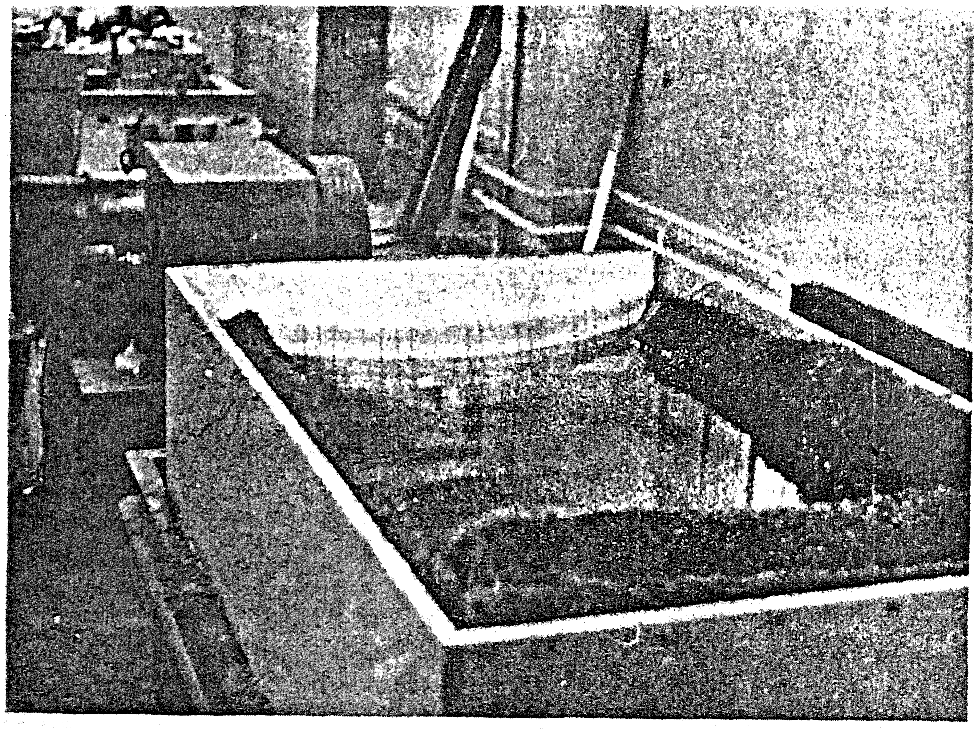

Fig. 1 The 1st Test of Arch Dam Model

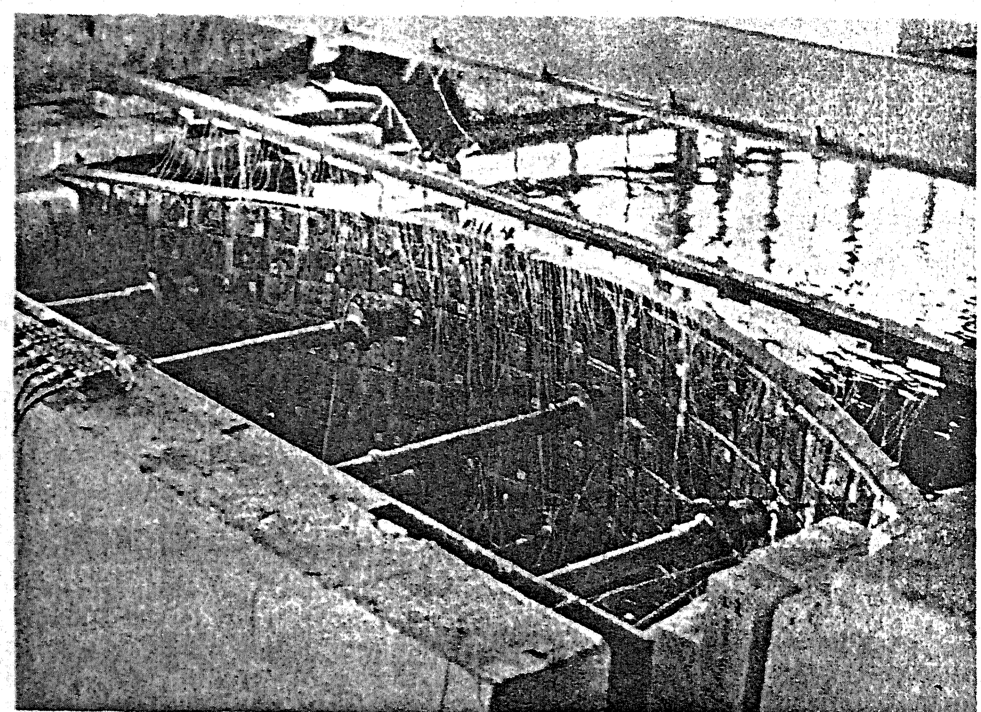

Fig. 2 The 2nd Test of Arch Dam Model

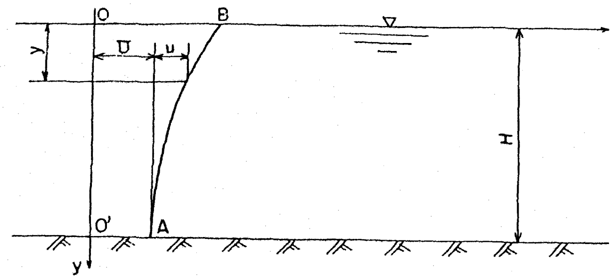

Fig. 3 Dam is rigid and ground moves

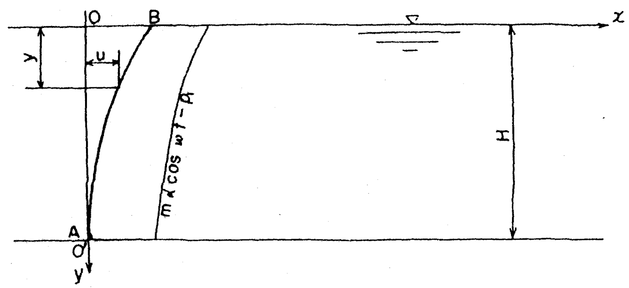

Fig. 4 Dam is flexible and ground does not move

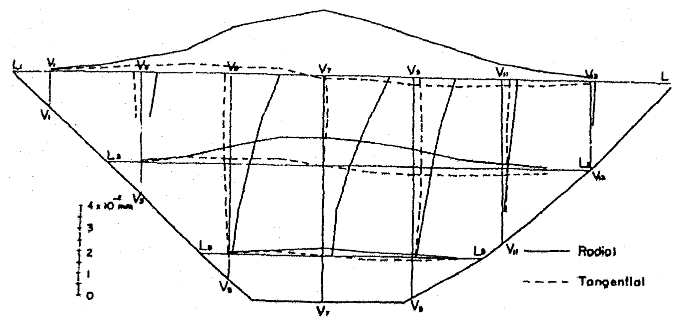

Fig. 5 Tangential and Radial Relative Displacements

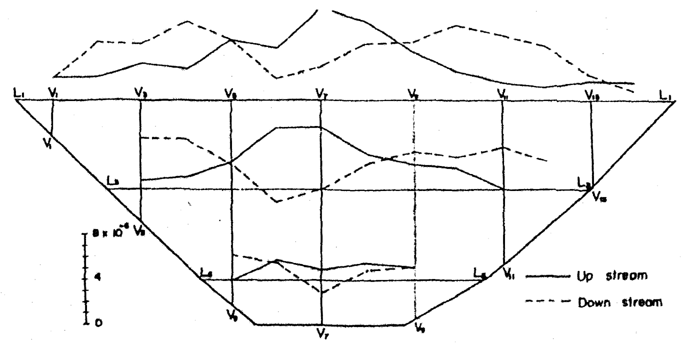

Fig. 6 Arch Strain

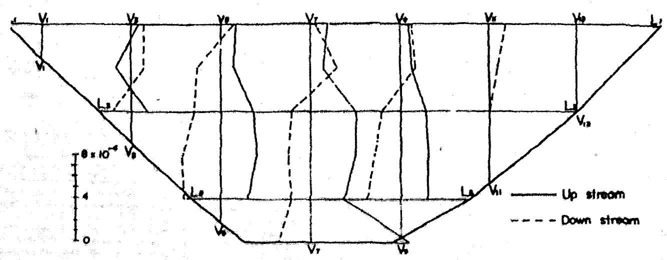

Fig. 7 Cantilever Strain