RESONANCE TESTING OF MULTISTOREY INFILLED FRAMES

by D.V. Mallick Ph.D

Introduction

For defining a tall building, the dividing line should be drawn where the design of structures moves from the field of statics into the field of structural dynamics. It becomes more important to ensure adequate lateral stiffness of these buildings to resist loads which may arise due to wind, earthquake or blast. As a result of the effect of earthquake tremors, forces of inertia are developed in the structural elements of a building. If the individual members are insufficiently strong they suffer residual displacements and cracks, and the structure as a whole becomes unfit for further use. In many structures it is desirable, if not essential, that the amplitude of near-resonant vibrations encountered in service be decreased. This can often be done by ensuring that the natural frequencies of the structure do not coincide with the frequency of the exciting force; which means that the natural frequency of either system should be known. In the case of an earthquake it is impossible to predict the magnitude and the frequency of the exciting force as both these factors are beyond human control. It is also not always possible to design a structure with a particular time period because the shape of the structure is largely determined by factors not connected with vibration. However, in such cases the magnitude of near-resonant vibrations will be determined by the damping of the structure. It is essential therefore to investigate methods which provide not only knowledge of the natural frequencies and mode shapes of a structure, but also give some idea of the damping which will be present. Therefore, a realistic study of a structure subjected to dynamic loads must include the determination of natural frequencies and mode shapes of the structure and, some quantitative idea of the damping which will be available. In this paper, resonance tests will be described to determine these three properties of multi-storey infilled frames or, in general any structure.

There is not much literature available on the damping capacity of infilled frames subjected to vibrations. There are a few references available on the damping capacity of composite structures, with a special reference to nailed plywood shear wall panels. Jacobsen and Kaneta(1) have shown that for small plywood models, alternating load deflection tests yield results that include the deterioration effects of nails and plywood by the very nature of loading. Medearis and Young(2) have used the same technique of cyclic loading for the nailed shear panels subjected to a state of combined stress. Their approach is not directly applicable to present study of infilled frames because, in their case, the shear wall panels are connected through the nails. The author carried out cyclic load tests(3) on infilled frames to determine their damping capacity. All the references described above deal with the techniques of cyclic loading for finding the damping capacity of a composite structure. Very little information is available on the dynamic testing of composite structures. To study the actual dynamic behaviour of a structure, it becomes necessary to assess the amount of damping available when the structure is in motion. Keeping this in mind, the author has carried out resonance tests on infilled frames.

Resonance test

In this method the infilled frame is exciting harmonically at a certain point whilst the amplitude and phase angle (relative to the exciting force) of the response at other points is recorded. If the exciting frequency is varied in a controlled manner the recorded data may be analysed to determine the natural frequencies and mode shapes of the infilled frame, together with the coefficient of equivalent viscous damping. Depending upon the physical quantities which are measured, and the way in which the experimental data is plotted, there are various techniques available by use of which the plots may be analysed. Before describing these techniques it will be necessary first of all to study the theoretical background to the problem.

Normal mode theory

In this theory, a real structure having an infinity of degrees of freedom is approximated by a finite degree of freedom system, say n, provided only that n is large enough. With each degree of freedom of a system is associated a principal or normal mode of oscillation of the system. Every principal mode is a pure harmonic motion. In general, the response of a conservative system having n degrees of freedom, subjected to forced vibration, consists of the superposition of the responses in each normal mode of vibration. The response of a structure in each normal mode, when referred to normal coordinates, can be determined as if it was a single degree of freedom system vibrating with the natural frequency of that mode. In this theory the effect of damping can be accounted for by introducing a single damping factor for each normal mode. In matrix form, when referred to normal coordinates, the damping matrix is then a diagonal. Thus, the equation of motion of a damped system in any one normal mode, can be written as

$$M_r \ddot{Z}_r + K_r (1 + ig) Z_r = F_r e^{iwt} \quad (1)$$

where M_r, K_r and F_r are the consistent generalised mass, stiffness and force vectors associated with the rth mode specified by the generalised normal coordinate $_r$, and

$$g = 2b \text{ for } w = p \quad \text{where}$$

b is the ratio of the coefficient of equivalent viscous damping, c, to the coefficient of critical damping, c_r, the latter being defined as the value which c would have if the free motion of the system were just non-oscillatory. The motion in any one mode, dropping mode subscripts, is given by

$$M \ddot{Z} + K (1 + ig) Z = F e^{iwt} \quad (2)$$

for a simple harmonic motion.

Characteristic Phase Lag Theory

Fraeijs de veubeke(4,5) presented the characteristic phase lag theory of damped motion of a system with finite degree of freedom system. Unlike the normal mode theory, the damping matrix considered by veubeke was a square, symmetric and real matrix. The main features of this theory are summarised below.

According to this theory, if an n degree of freedom system with hysteretic or viscous damping is made to vibrate by a harmonic force vector of some frequency, w, then for each given value of frequency, there are n modes of vibrations quite different from principal modes, which are called forced modes of vibration for a given frequency w. Rather than attempt a complete and rigorous mathematical definition, only the pertinent properties of forced modes of vibration will be given here.

1. Each forced mode of vibration is associated with a phase angle, θ, and a harmonic force vector of given frequency, Σ, also called 'Forced mode of vibration'.

2. The displacements q_1, q_2, ---------------, q_n in any mode, say the rth, are all in phase, having the phase angle θ_r.

3. Unlike the principal or normal mode, the phase angle and shape of any one of these modes varies with the frequency.

4. These modes depend only on the shape of damping and not on its density.

5. When w, the exciting frequency, equals p_s, the natural frequency of the sth principal mode of the system, then one of the forced modes, say $K^{(s)}$ which corresponds to 90^o phase angle, may be identified with the sth principal mode $\psi^{(s)}$, that is,

$$\theta_s = \pi/2, \; K^{(s)} = \psi^{(s)} \quad \text{when } w = p_s$$

For this value of w, there will also be $(n - 1)$ other forced modes corresponding to the remaining $(n - 1)$ root values of the phase angle θ_r.

Based on the phase lag theory of damped motion of a system the following conclusion, as stated by Bishop and Gladwell(6), is rewritten here to understand the dynamic behaviour of damped system in general.

The difference between heavily damped and lightly damped system lies mainly in the behaviour of phase angle θ_r. In a lightly damped system the angles θ_r are either small or near π at off-resonant frequencies so that, in a sense, the modes cluster together and the response is either in phase or anti-phase with the excitation. At resonance frequency one mode detaches itself, so to speak, and, over a very small range of frequency sweeps from zero to near π. In a heavily hysteretically damped system, the angles $\theta_r(w)$ have greater initial values and sweep through the natural frequencies more gradually. However in a heavily damped system the angles θ_r approach π quite soon after passing through the natural frequency.

It can be inferred from the above statement that the normal mode theory is quite adequate for lightly damped systems, amongst which frames with brick or concrete infilling can be included. For heavily damped systems it is virtually impossible to find the response in a practical case by using veubeke theory, because neither the diagonal nor the off-diagonal terms are known at the start. All that can usually be done is to assume that the damping matrix is diagonal, and use the normal mode theory, mentioned before, to get some results.

Response Analysis

The motion in any one normal mode, using normal mode theory, is given by equation (2). Substitution of the trial solution $Z=Re^{iwt}$ into this equation leads to

$$R = (F/K)/ \{(1-(w/p^2)^2 + ig\} \quad (3)$$

where $p^2= K/m$. Non-dimensionalizing the displacements in terms of the static deflection, $\delta st = F/K$, the following expression is obtained

$$R = \delta st/ \{(1-(w/p)^2)^2 + ig\} \quad (4)$$

or in polar form

$$R = \delta st/\sqrt{(1-(w/p)^2)^2+ g^2} \cdot e^{-i \tan^{-1}(g/(1-(w/p)^2)} \quad (5)$$

The modulus of R and the argument, η, of the vector are given by

$$|R| = \delta st / \sqrt{(1-(w/p)^2)^2 + g^2} \quad (6)$$

and

$$\eta = \tan^{-1}(g/(1-(w/p)^2)^2 \quad (7)$$

that is

$$R = \delta st . \frac{1 - (w/p)^2}{\{(1-(w/p)^2)^2+g^2\}} + i\delta st. \frac{- g}{\{(1-(w/p)^2)^2+g^2\}} \quad (8)$$

Equations (6), (7) and (8) describe the theoretical background of the three techniques generally used in resonance testing, and each will now be discussed.

The Peak Amplitude Method

This method has derived its name from the plot of equation (6) as shown in Fig.1, for a given value of damping. It can be seen from equation (6), that the maximum value of R occurs when $w \simeq p$, which defines the natural frequency of the system. The system is then said to be in resonance. It can be shown[7] that if Δw is now chosen such that $|R| \; max^m / |R| = \sqrt{2}$ (see Fig. 1), then the co-efficient of equivalent viscous damping will be given by

$$b = g/2 = \frac{\Delta w}{2b} \quad (9)$$

Thus, the required information, like natural frequency and damping is extracted from the plot. In this method, the resonant vibration is assumed to take place in the corresponding principal mode. In the light of Veubeke's phase lag theory, the results obtained by using this method are affected by (a) the effect of damping which couples the modes and, (b) the contribution from other modes at that frequency. But if the system is lightly damped then the analytical errors due to the above factors are unlikely to be large.

If the natural frequencies of a system are close together, which is very likely in real structures, then it is quite possible with the peak-amplitude method that modes will be missed altogether. Whereas the effect of heavy damping is that at the resonant frequency of a heavily damped mode, extraneous vibration from other modes may be comparable in magnitude to the vibration in the resonant mode. This may eventually result in large error in the relevant damping coefficient.

Phase angle plot

The phase lag of the response vector with respect to the applied force can be obtained from equation (7). Fig (2) represents the plot of phase angle, η , against the frequency for a given value of g. At resonance, $w \simeq p$ and $\eta = 90^o$. The natural frequency of a system, when vibrating in one of its principal mode, can be determined by the intersection of the phase angle plot with the line $\eta = 90^o$. In order to find the damping, substitution of $w = p \pm \Delta w/2$ into equation (7) shows that $\tan \eta = \pm 1$ so that the value of b can be found by measuring the width of frequency band corresponding to $\tan \eta = \pm 1$.

Pendered and Bishop[(8)] gave a modified technique for extracting the natural frequency and damping from the phase angle plot. Noting that $g = 2bw/p$ and using equation (7), we get

$$\eta = \tan^{-1}\left[\frac{2\,bw/p}{1-(w/p)^2}\right] . \qquad (10)$$

Now if η is differentiated with respect to w and resonance condition $w = p$ is substituted, an expression for equivalent viscous damping is obtained as given below.

$$b = \frac{1}{p(\,d\eta/dw)} = \frac{1}{p\ (\text{slope at } p)} \qquad (11)$$

Thus, the natural frequency is obtained from the intersection of the phase angle plot with the line $\eta = 90^o$, as before, the coefficient of equivalent viscous damping is obtained from the slope of the curve at this point. The advantage claimed for this method of determining b is that the phase angle plot need only cover a small range of frequency in the vicinity of the resonant frequency. However, the author considers this advantage to be offset by the difficulty of actually measuring the slope from the phase angle plot as shown in Fig. (7).

The reason for this method of phase-angle plot not being popular in the past may be due to the indirect role played by phase angle during resonance testing. In the peak-amplitude method, the change in the amplitude of vibration with the change in frequency of the exciting force can be visualised physically, whereas the change in phase angle cannot be visualised.

The author found that phase angle plot gives better estimate of natural frequency than the peak amplitude plot.

Vector Plot Method

For a complex structure, the peak amplitude and the phase angle plots can be very misleading as far as natural frequency and modal damping are concerned, because of the off-resonant contributions of other modes. These shortcomings were recognized by Kennedy and Pancu who proposed an alternative method of plotting and analysing the results of a resonance test.

When the damping matrix for a multi-degree of freedom system is diagonal, so that a single damping coefficient is attached to each mode, the pure response in any one mode is given by equation (8). If the real and imaginary components of R relative to F are plotted on an Argand diagram, the result is as shown in Fig (3a), which is a circle with its diameter passing through the resonant frequency point. If equal frequency intervals are chosen the circumferential distance between points can be shown (7,9) to be maximum at the natural frequency. The natural frequency can therefore easily be determined. Kennedy and Pancu have shown that this method of vector plot in which amplitude and phase angle, both are plotted, is more reliable than either the peak amplitude or the phase angle method in exhibiting the existance of modes, and that the accuracy with which the natural frequencies can be determined seems to be less affected by the presence of other modes. In a real structure, there will be some coupling between the modes, and the vector plot will not be as simple as shown in Fig (3). It will rather consist of a large number of circular loops, each offset from the origin by the amount of motion in the other modes. The true peak amplitude free from off-resonant contributions in the resonant mode, can be determined with good accuracy, first by plotting the curve and locating the natural frequency from the maximum spacing technique; then the best circle is fitted to the loop, placing particular emphasis on the part of it in the immediate vicinity of the natural frequency as shown in Fig. (3b). The peak amplitude in that mode will then be given by the diameter HJ of the circle. It is unlikely that these circles will pass through the origin, O. The point J is called the displaced origin for the mode. The vector OJ represents off-resonant contribution of other modes at the resonant frequency.

The modal damping can be found by noting that the diameter BE (Fig.3b) which is parallel to the real axis, corresponds to $\tan \eta = \pm 1$, so that again

$$b = \frac{\Delta w}{2p} = (w_B - w_E)/2p. \qquad (12)$$

where p is the modal natural frequency corresponding to the point H.

The vector plots obtained from the resonance testing of multi-storey infilled frames showed that it is not possible to read accurately the values of natural frequency w_B and w_E corresponding to B and E for a particular mode of vibration of a structure. For such cases the damping can be calculated by using the following procedure.

Choose any two points, P and Q in the immediate vicinity of the natural frequency point H such that the vector line JP and JQ make equal angles with the resonant diameter JH. If $w_Q - w_P = \Delta w$, and $w = p \pm \Delta w/2$ is substituted into equation (7), the expression for $\tan \eta$ becomes

$$\tan \eta = g/(1-(p \pm \Delta w/2)^2/p^2) \quad (13)$$

Because Δw^2 is small, equation (13) can be simplified to

$$\tan \quad = g/(\Delta w/p) .$$

For small values of Δw, $w \simeq p$ and $g = 2b$. Therefore

$$b = (\Delta w/2p) \tan \eta . \quad (14)$$

For frequency interval, Δw, corresponding to points P and Q as shown in Fig. (3b), $\tan \eta = \pm \cot \theta$.

Therefore

$$h = (\Delta w/2p) \cot \theta$$

or

$$b = ((w_Q - w_E)/2p) \cot \theta . \quad (15)$$

θ being measured from the resonant diameter as shown.

The normal mode shape can be determined quite accurately from the ratios of the peak amplitudes determined by vector plots at various points when the structure is being driven at a natural frequency.

Resonance Tests in Infilled Frames

The forced vibration tests were carried out on four-storey, three-storey, two-storey and single-storey, single bay square infilled frame as shown in Fig. (4), to determine the natural frequency, mode shape and to assess the amount of damping associated with each normal mode of vibration. The infilled frames were excited by a Goodman electrodynamic vibrator, model 790, driven through a Leak 50-watt amplifier by Muirhead, two phase L.F. decade oscillator. It was assumed that the force exerted by the vibrator was proportional to the input current. The amplitude of the forced vibration was maintained constant by keeping the input current constant throughout the experiment. The input current was also used as a reference signal for the response of the frame.

The vibrator was suspended by means of high tensile steel bowden wire as shown in Fig. (4). This type of suspension was designed so that the fundamental frequency of the suspended vibrator is very low as compared to that of the infilled frame.

The response signal at any point of the frame was picked up by an inductance type transducer called the proximity meter. The advantage of using this pick up was that it required no physical contact with the vibrating structure whose response is being measured and its sensitivity is independent offrequency. The output from the proximity meter was fed into the Muirhead low frequency phase meter through the tunable filter, the purpose of the tunable filter circuit being to select the required frequency from the wave-form input. The amplitude of the response, which is proportional to the output voltage from the pickup system, could be read directly from the multi-range voltmeter fitted in the phase meter. The oscilliscope was introduced into the circuit for monitoring the input signals before they were fed into the phase meter. The block diagram of the equipment is shown in fig. (5).

Test Procedure

The following systematic procedure was followed for recording the response of the frame to a sinusoidal exciting force.

1. The input current to the vibrator was generally adjusted to a level of 1.0 amp., and this signal was also used as a reference signal for finding the phase difference between the displacement and the exciting force.

2. The estimate of natural frequencies was first obtained by feeding the response and the reference signal on to the oscilliscope until a 90° phase difference was obtained. The frequencies corresponding to 90° phase angle difference were defined as the natural frequencies of the frame.

3. After the natural frequencies of the infilled frame had been estimated, the test was repeated, but this time by directly feeding the response and the reference signal to the phase meter. The amplitude and the phase angle were measured for a range of frequencies of excitation. Initially the readings were taken at a frequency interval ranging from 5 to 2 cycles, but this interval was reduced to 0.2 cycles, in the vicinity of the natural frequency. In the vicinity of the natural frequency the phase angle and the amplitude variation becomes very sensitive to frequency.

Analysis of Experimental Results

Having measured the amplitude of the response and the phase angle between the response and the reference signals at a point on the vibrating structure for a range of frequencies, the results can be plotted in various ways. For illustration, the curve A in Fig. (6) shows the peak amplitude plot against the exciting frequency of a single storey, single bay square infilled frame. The maximum amplitude occur at a frequency $w = 317.8$ c/s. The damping can be determined by finding Δw for which $(R_{max}/R) = \sqrt{2}$ as shown in Fig. (6). The coefficient of viscous damping is given by

$$b = \Delta w/2p$$

Fig. (7) represents the phase angle plot against the exciting frequency of the above system. The natural frequency, p, can be obtained by the intersection of the phase angle plot with the line $\eta = 90^{\circ}$, and is found to be 314.4 c/s. The coefficient of damping can be determined from this plot by using equation (11); if the slope of the curve at the point of intersection is accurately measured.

Fig. (8) shows the vector plot around the first natural frequency of the above system. The natural frequency of the system is located by finding the point having maximum frequency spacing around the curve.

As described earlier, the method of vector plot provides a simple mean of finding the true peak amplitude with good accuracy, free from off-resonant vibration, in the resonant mode at a natural frequency. This is done quite easily by fitting a best circle to the arc, placing particular emphasis on the part of it around the natural frequency, as shown in Fig. (8). The point O denoted the new origin for the mode. Strictly speaking, the maximum spacing can only be determined by measuring the lengths corresponding to small frequency interval around the natural frequency. For the present case a frequency interval of 0.2 c/s was chosen and the arc lengths corresponding to this interval measured. The damping coefficient has been determined by using the expression (15). The curve B in Fig.(6) has been plotted taking O as the new origin of the pure mode. OH represents the true peak amplitude of the fundamental mode, and y gives the off-resonant contribution of other modes to this true peak amplitude.

It will be seen that values of natural frequencies as determined from vector plot and phase angle plot were same. Hence, as discussed before the phase angle plot gives a better estimate of natural frequency than the peak amplitude plot.

Tests were carried out on two storey, three storey and four storey infilled frames. The vector plots for the resonance tests in three and four storey infilled frames are shown in Fig. (9) and (10) respectively. The curves have been plotted around first, second, third and fourth natural frequency of a four storey infilled frame. The curves have been plotted to different scales of amplitudes. Best circles have been fitted to the arcs in the immediate vicinity of the natural frequencies and resonant diameter marked to indicate the points corresponding to modal natural frequencies.

Table 1 shows the results of three and four storey infilled frames. The experimental value of natural frequencies of four storey infilled frames are compared with the theoretically calculated values[10], assuming the structure to be a bending type structure in which axial deformation of frame members are not neglected.

It will be worthwhile to discuss the limitations of the technique employed for locating the natural frequency - that of finding the point having maximum frequency spacing around the curve in the vector plot method. Since the arc length can only be measured between two distinct frequencies rather than at a particular frequency, the maximum spacing can only be determined by measuring the lengths corresponding to known frequency intervals around the natural frequency. Another difficulty lies in the fact that any errors in plotting the phase or the amplitude in the Argand diagram gives rise to a greater error in measuring the spacing. It should also be noted that the change in spacing around the natural frequency is due mainly to the change in the increment of phase angle corresponding to a fixed frequency interval. As such it becomes essential to measure phase angle quite accurately.

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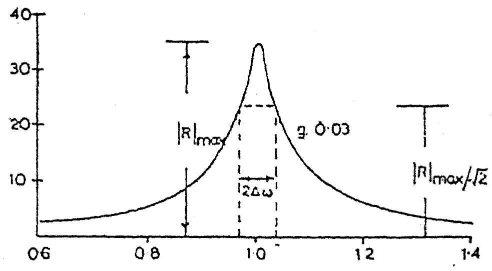
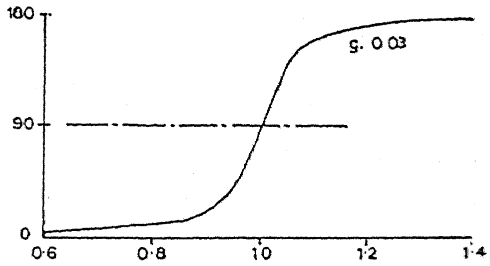
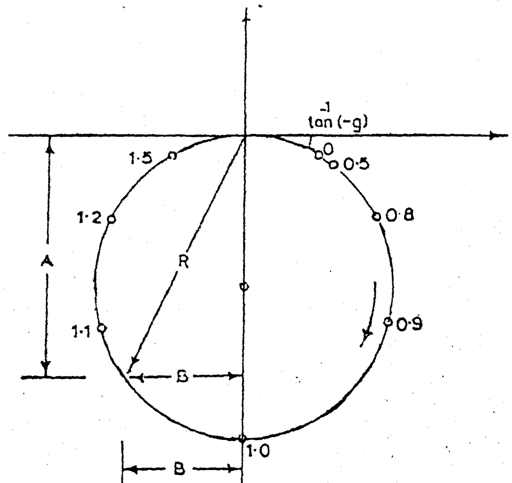
Fig. 1 (left). Peak amplitude plot

Fig. 2 (right). Phase angle plot







$$A = \frac{-\delta_{st} \cdot g}{\left[1-(\omega/p)^2\right]^2 + g^2}$$

$$B = \frac{\delta_{st} \cdot \left[1-(\omega/p)^2\right]}{\left[1-(\omega/p)^2\right]^2 + g^2}$$

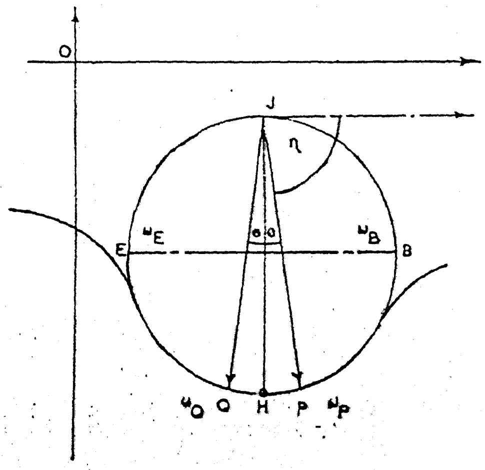


Fig. 3a (left). Displacement response for constant force amplitude by Kennedy and Pancu method

Fig. 3b (right)

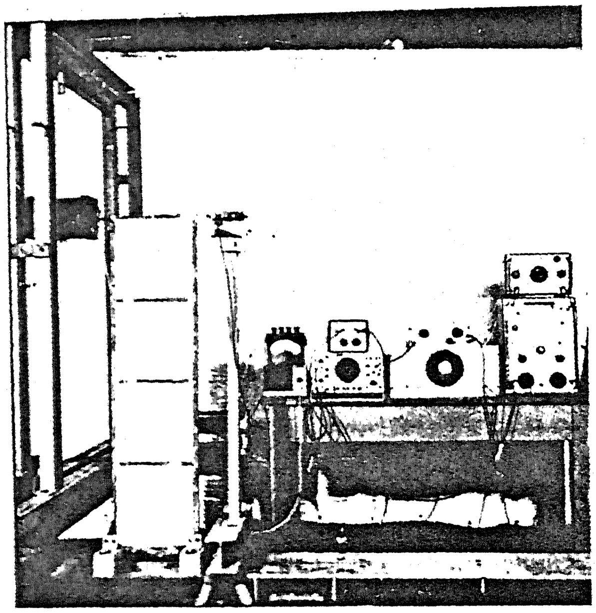


FIG 4

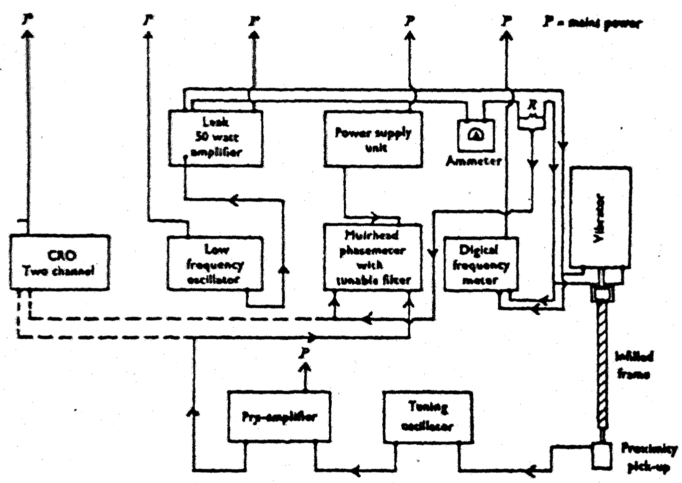
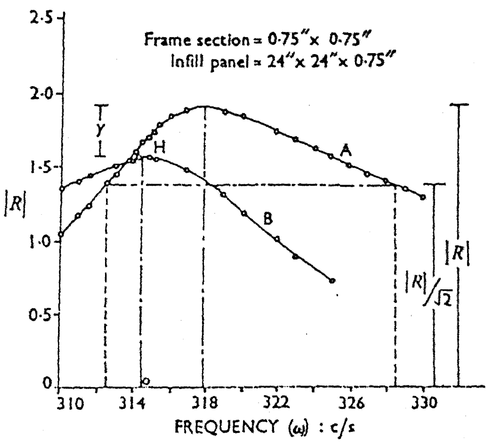


Fig. 5 Block diagram of vibration equipment



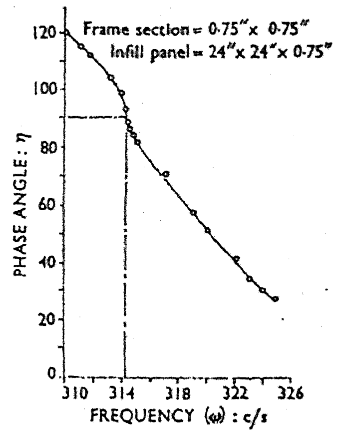


Fig. 6 (left). Peak amplitude plot of single-storey square infilled frame around first natural frequency

Fig. 7 (right). Phase angle plot of square infilled frame around first natural frequency

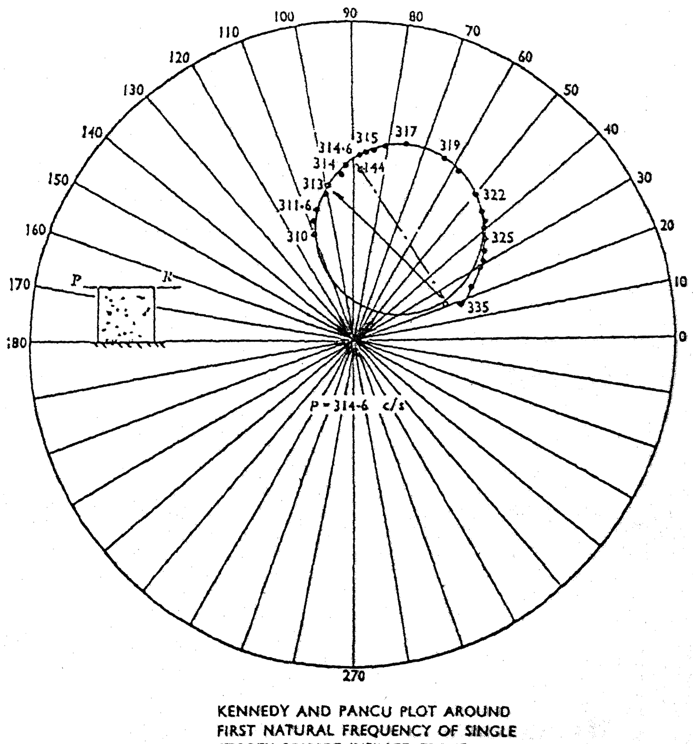


Fig.8

KENNEDY AND PANCU PLOT AROUND FIRST NATURAL FREQUENCY OF SINGLE STOREY SQUARE INFILLED FRAME

FRAME 0·75″ x 0·75″ INFILL 24″ x 24″ x 0·75″

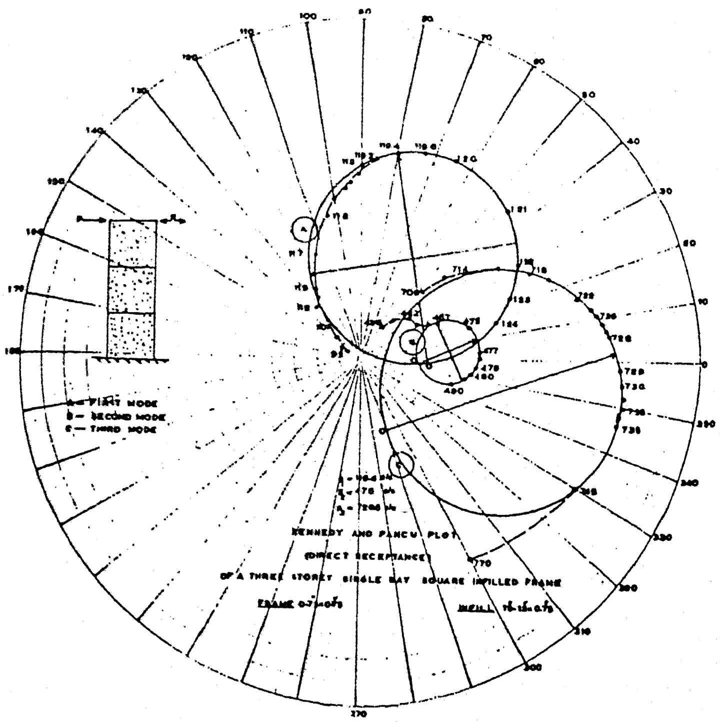


Fig. 9 Vector Plot of Three Storey Single Bay Square Infilled Frame

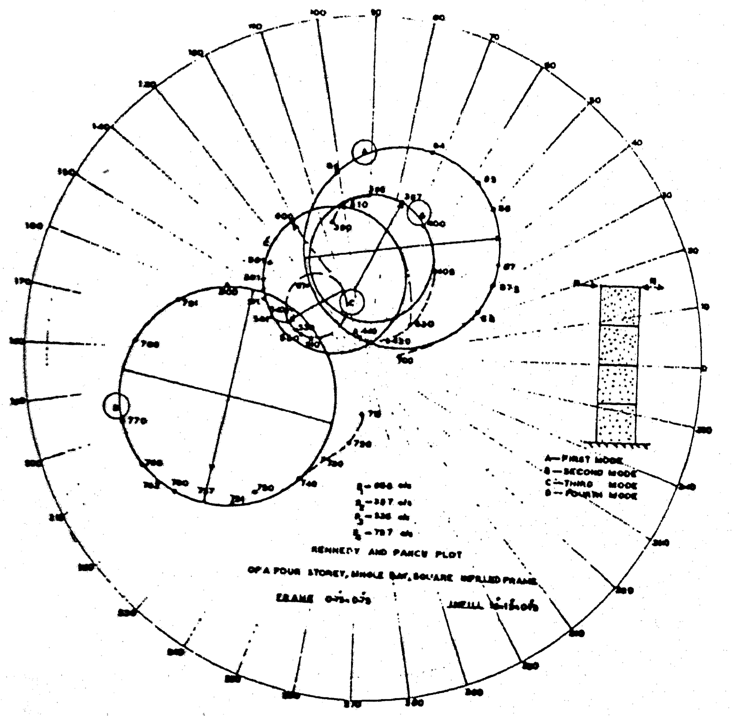


Fig. 10 Vector Plot of Four Storey, Single Bay Square Infilled Frame.

		FIRST MODE	SECOND MODE	THIRD MODE	FOURTH MODE
THREE STOREY SINGLE BAY SQUARE INFILLED FRAME	A	119.4 c/s	476 c/s	728.5 c/s	
	B	2.35%	1.98%	1.0%	
FOUR STOREY SINGLE BAY SQUARE INFILLED FRAME	A	86.5 c/s	397 c/s	535 c/s	757 c/s
	B	3.6 %	2.77 %	4.0%	2.65 %
	C	105 c/s	405 c/s	548 c/s	820 c/s

TABLE I Experimental values of natural frequency and modal damping of three storey and four storey, single bay square infilled frames.

Row A gives the experimental natural frequency

Row B gives the experimental modal damping as $\frac{C}{C_r}$ %

Row C gives the theoretical natural frequency