EXPERIMENTAL STUDY ON THE VIBRATIONAL CHARACTERISTICS OF GROUND

by

S. Yoshikawa,[I] M. Shima[II] and R. Irikura[III]

ABSTRACT

The vibrational characteristics of the ground were investigated through the observation of natural earthquakes and by means of an vibrator, seismographs being set on the ground surface and in the bore holes. Fourier spectra were obtained in the case of seismograms of natural earthquakes, and the amplitude-frequency relations for a constant vibrational force in the case of vibrational test. It was shown that the vibrational characteristics of the layered structure is obtained by calculating the ratios and differences of both amplitude and phase spectra between the observation points at the surface and underground, and are fairly good agreement with those of the ground due to SH waves generated by the vibrator.

INTRODUCTION

It has long been recognized that the seismograms of an earthquake recorded at many observation points differ according to the geological structure at each point. Many seismologists have attributed this to the fact that the ground has its own predominant period corresponding to the geological structure. From the Fourier analyses of micro-tremors and natural earthquakes the frequency characteristics of the earthquake motion may be obtained and this is theoretically calculated as the multiple reflection of infinite S waves coming upwards perpendicularly to the surface layer (1). It has also been proved that the ground has its own spectrum, which is related to the thicknesses of surface layers and their physical properties (propagating velocity of S wave, density, rigidity and viscosity) (2, 3).

When the frequency characteristics of the seismogram on the surface of the ground $U(\omega)$, and $O(\omega)$, $Q(\omega)$ and $G(\omega)$ are the frequency characteristics of the origin, the path through the crust and the geological structure of the observation point respectively, we get

---

[I] Professor, Disaster Prevention Research Institute, Kyoto University

[II] Associate Professor, Disaster Prevention Research Institute, Kyoto University

[III] Research Assistant, Disaster Prevention Research Institute, Kyoto University

$$U(\omega) = O(\omega)\cdot Q(\omega)\cdot G(\omega) \qquad (1)$$

To obtain $G(\omega)$ from $U(\omega)$, the forms of $O(\omega)\cdot Q(\omega)$, which are irrespective of the sub-surface geological structure of the observation point have to be clarified. For this purpose observation at a point more than 100 m below ground is required, but in practice there are certain difficulties. It was reported that at the Hitachi mine several hundred meters below ground the velocity spectrum for observing earthquakes was constant (4). However, the frequency range in this case was limited and hence the spectrum of the earthquake at base rock level was still unclear, because it would be affected by the origin and the path of the seismic ray.

The spectrum of an earthquake motion incident to the base rock, which is calculated from the seismogram at the presumed bed, will be the interference wave between the incident and reflected wave at the surface and hence the incident wave itself is very difficult to observe directly. It can be calculated from the seismogram recorded at a very deep observation point where the effect of the surface layer is negligible, or by theoretical calculations taking into consideration all the parameters such as elastic constants and viscosity coefficients of surface layers , which enable us to separate the incident and reflected wave. Thus the frequency characteristic of the ground at an arbitrary point is unobtainable from the surface observation of earthquake motion, since the frequency characteristic of the incident wave at the base is unclear. At present, the displacement (velocity, acceleration) spectrum is generally obtained and the free oscillation period of the ground is assumed from the relative positions of peaks and dips on the graph.

The spectra of earthquake motions at the surface and underground are related to the function $H(\omega)\cdot\exp(i\varphi(\omega))$ when the wave type is the same and the function form is determined only by the parameters, such as the thickness between two observation points and elastic constants, as will be explained in the next chapter. Therefore, the free oscillation period corresponding to geological structure may be found among the peaks of the amplitude spectrum on the ground surface by calculating the ratio and the difference of the obtained amplitude and phase spectra on the surface and underground. However, only the relation between the distribution of the amplitude spectrum and geological structure has been discussed and the distribution of the phase spectrum should also be taken into consideration.

In this paper $H(\omega)$ and $\varphi(\omega)$ are obtained from the spectra of earthquake motions observed on the surface and underground simultaneously and compared with the theoretical values calculated from the geological structure which is obtained by S wave prospecting. When the geological structure near the surface is simple, for instance when there are just a few layers overlaying the base rock, the observed spectrum is well explained by the multiple reflection theory in the parallel layers. However, when the geology is complicated i.e., the surface and the boundary layer are inclined or the layer boundary is uncertain, the theoretical curve corresponding to the geological structure is very difficult to obtain and hence the complicated layer structure is replaced by the equivalent parallel layers and then the theoretical curve is obtained. For this purpose, the phase spectrum at the underground observation point is first obtained, the main discontinuous layers are assumed and then proper equivalent layers will be obtained.

Thus, the frequency characteristics of the ground can be obtained even when the spectrum incident to the base is unclear, but they can also be obtained by observing the corresponding ground motion excited by a vibrator whose force characteristics are known. In order to investigate the vibration characteristic corresponding to S wave motion, the vibrator was set on the ground surface and the SH wave was generated by forcing in the horizontal direction. The result of the analysis of the oscillogram taken by underground observation was compared with that for a natural earthquake and it was shown that this method is very effective. Since the vibrator is pretty powerful at low frequency, a Love wave of fundamental mode is also generated in this case, and from the dispersion curve the geological structure regarding S wave is also discussed.

METHOD OF EXPLORATION OF THE GROUND

The vibrational characteristics of the ground are determined by the mechanical structure of the ground, in particular, the thickness of the layer and the elastic constants, but does not directly depend on the strength of the ground. Therefore, the method of seismic prospecting, especially the use of S waves, is most suitable for obtaining the dynamic structure of the ground.

The method using SH waves by striking a plate on the ground surface is effective for the determination of the upper structure of the ground. That is, long wooden plates, 1-3 meters in length, 30 cm in width, 5-10 cm in thickness, were pressed down or stuck on the ground surface. The SH waves were generated by striking the plate with a hammer parallel to the plate surface. The profiles were taken perpendicular to the direction of the plate and pickups were set on the surface at intervals of 5 meters and in the bore hole, if we could utilize it. The waves recorded were direct, refracted and reflected ones and we determined the ground structure by the method of refraction and reflection.

DISTRIBUTION OF THE AMPLITUDE AND PHASE SPECTRA

Let $f_0(\omega)$ be the spectrum of waves at the surface and $f(\omega)$ that in the ground. Then between $f_0(\omega)$ and $f(\omega)$, there is the following relation

$$f(\omega) = H(\omega)\exp(i\varphi(\omega))f_0(\omega) \tag{2}$$

When waves are perpendicularly incident on boundaries and are not converted into other kinds of waves, $f_0(\omega)$ and $f(\omega)$ are functions of frequency ω depending only on the ground structure.

We shall now investigate the case where there are two layers on a semi-infinite solid and take a visco-elastic body of the Voigt type as the model of the solid (5). Let $f_1(\omega)$ be the spectrum of earthquake motion in the first layer, $f_2(\omega)$ that in the second and $f_3(\omega)$ that in the third, that is, $f_3(\omega)$ is the spectrum observed by a seismometer in a bore hole in the semi-infinite medium; it is not the spectrum of waves itself incident on the basin floor, but contains waves reflected from the surface layers.

We shall now resolve the wave equation for the case where plane harmonic SH waves are incident vertically upwards. Then,

$$\begin{aligned} f_1(\omega) &= \sqrt{h_1^2 + h_1'^2}\exp(i\varphi_1)f_0(\omega) \\ f_2(\omega) &= \sqrt{h_2^2 + h_2'^2}\exp(i\varphi_2)f_0(\omega) \\ f_3(\omega) &= \sqrt{h_3^2 + h_3'^2}\exp(i\varphi_3)f_0(\omega) \end{aligned} \tag{3}$$

$$\begin{aligned}
h_1 &= \cos\left(\frac{z}{v_1}\omega\right) \\
h_1' &= \frac{\xi_1 z}{2\rho_1 v_1^3}\omega^2 \sin\left(\frac{z}{v_1}\omega\right) \\
h_2 &= \cos\left(\frac{H_1}{v_1}\omega\right)\cos\left(\frac{z-H_1}{v_2}\omega\right) - \alpha\sin\left(\frac{H_1}{v_1}\omega\right)\sin\left(\frac{z-H_1}{v_2}\omega\right) \\
h_2' &= \left\{\frac{\xi_1 H_1}{2\rho_1 v_1^3} + \frac{\alpha\xi_2(z-H_1)}{2\rho_2 v_2^3}\right\}\omega^2\sin\left(\frac{H_1}{v_1}\omega\right)\cos\left(\frac{z-H_1}{v_2}\omega\right) \\
&\quad + \left\{\frac{\xi_2(z-H_1)}{2\rho_2 v_2^3} + \frac{\alpha\xi_1 H_1}{2\rho_1 v_1^3}\right\}\omega^2\cos\left(\frac{H_1}{v_1}\omega\right)\sin\left(\frac{z-H_1}{v_2}\omega\right) \\
h_3 &= \left\{\cos\left(\frac{H_1}{v_1}\omega\right)\cos\left(\frac{H_2}{v_2}\omega\right) - \alpha\sin\left(\frac{H_1}{v_1}\omega\right)\sin\left(\frac{H_2}{v_2}\omega\right)\right\}\cos\left(\frac{z-H_1-H_2}{v_3}\omega\right) \\
&\quad -\beta\left\{\cos\left(\frac{H_1}{v_1}\omega\right)\sin\left(\frac{H_2}{v_2}\omega\right) + \alpha\sin\left(\frac{H_1}{v_1}\omega\right)\cos\left(\frac{H_2}{v_2}\omega\right)\right\}\sin\left(\frac{z-H_1-H_2}{v_2}\omega\right) \\
h_3' &= \left\{\left(\frac{\alpha_1\xi_1 H_1}{2\rho_1 v_1^3} + \frac{\xi_2 H_2}{2\rho_2 v_2^3}\right)\cos\left(\frac{H_1}{v_1}\omega\right)\sin\left(\frac{H_2}{v_2}\omega\right)\right. \\
&\qquad \left. + \left(\frac{\xi_1 H_1}{2\rho_1 v_1^3} + \frac{\alpha\xi_2 H_2}{2\rho_2 v_2^3}\right)\sin\left(\frac{H_1}{v_1}\omega\right)\cos\left(\frac{H_2}{v_2}\omega\right)\right\}\cos\left(\frac{z-H_1-H_2}{v_3}\omega\right) \\
&\quad +\beta\left\{\left(\frac{\alpha\xi_1 H_1}{2\rho_1 v_1^3} + \frac{\xi_2 H_2}{2\rho_2 v_2^3}\right)\cos\left(\frac{H_1}{v_1}\omega\right)\cos\left(\frac{H_2}{v_2}\omega\right)\right. \\
&\qquad \left. - \left(\frac{\xi_1 H_1}{2\rho_1 v_1^3} + \frac{\alpha\xi_2 H_2}{2\rho_2 v_2^3}\right)\sin\left(\frac{H_1}{v_1}\omega\right)\sin\left(\frac{H_2}{v_2}\omega\right)\right\}\sin\left(\frac{z-H_1-H_2}{v_3}\omega\right)
\end{aligned} \tag{4}$$

$$\varphi_1 = \tan^{-1}\left(\frac{h_1'}{h_1}\right)$$

$$\varphi_2 = \tan^{-1}\left(\frac{h_2'}{h_2}\right)$$

$$\varphi_3 = \tan^{-1}\left(\frac{h_3'}{h_3}\right)$$

$$\alpha = \rho_1 v_1 / \rho_2 v_2$$

$$\beta = \rho_2 v_2 / \rho_3 v_3$$

where ξ is solid viscosity, ρ density, v velocity of S waves, H thickness of layer. And we assumed

$$\frac{\xi_1 \omega}{2\rho_1 v_1^2} \ll 1 \qquad \frac{\xi_2 \omega}{2\rho_2 v_2^2} \ll 1 \qquad \frac{\xi_3 \omega}{2\rho_3 v_3^2} \doteqdot 0 \tag{5}$$

Taking the solid viscosity into consideration, $H(\omega)$ has not any zero points, and $\varphi(\omega)$ varies continuously from -180° to 180°. As it is obvious from (3) (4), since $H(\omega)\exp(\varphi)$ is a function of ω depending only on the ground structure, that is, thickness of layer and dynamic constant of medium, we can know the natural frequency due to the ground structure by calculating $f_0(\omega)/f(\omega)$, from the results of the observations made at several points in the ground.

RESULTS OF ANALYSES FROM NATURAL EARTHQUAKES

We shall now show the results of the analysis of earthquake motion in the case of the Matsushiro earthquake, as an illustration of the observed spectra. The observations were carried out on a sandbank in the River Chikuma. The geological structure of the site determined by the method of seismic exploration and examinations of the bore hole is shown in Fig. 2. The points of observation were set at the same time on the surface and in the bore hole at a depth of 10 m.

The natural period of the seismographs of velocity type used for the observation was 3 cps and they were connected to the data recorder.

The greater part, about 2 seconds, of the S wave were used to obtain the spectrum of the earthquake motion. The results are shown in Fig. 3, 4. In detail, some differences will be apparent in the mechanism, path and incident circumstance with respect to the basin floor and hence the statistical mean is taken to eliminate these effects to some degree. The geometrical mean of the amplitude spectra of velocity of four earthquakes at each depth is shown in Fig. 5. It can be understood from the examination of the spectrum in Fig. 5 that the curves show peaks at 6 cps and

9.5 cps on the surface and at 6 cps and 11-12 cps at 10 m below the ground. The ratios of amplitude spectra and the difference of phase spectra were calculated from $f_0(\omega)/f(\omega)$ to obtain the correspondence of these spectra to the ground structure, and are shown in Fig. 6. These figures illustrate that the reversion of the phase occurs and that the amplification of amplitude on the surface to that of below the ground amounts to the maximum, at 9 cps in both EW and NS components. It can be inferred from this fact that 9 cps, the frequency of the peak of the mean amplitude spectrum on the surface results from the boundary between 0 m and 10 m in depth. The theoretical curves of $f_0(\omega)/f(\omega)$ corresponding to the ground structure are shown by solid curves in the figure. We could not measure the solid viscosity in this observation, and so we used $\xi = 10^6$ C.G.S. which was reasonable as the viscosity of soundy clay. These spectrum curves agree fairly well with the observed value at 9 cps corresponding to the boundary 2.5 m in depth. The emphasis has been placed on the inquiry into the characteristics of waves incident on the basin floor, in investigating the characteristic frequency of the ground. But it may be said that the characteristic frequency of the ground can be obtained from the ratios of spectra, without any knowledge of the characteristics of the incident waves.

RESULTS FROM ANALYSES OF VIBRATION TESTS

The vibrator was set as shown in Fig. 7. The frequency can be controlled in the range 1-20 cps, the force is about 2 ton·g in the range 4-20 cps and the maximum of the moment 3000 kg·cm. The center of weight of the vibrator was lowered so that the rocking motion might be suppressed. With respect to the settlement of the vibrator, the ground was dug down to 0.5 m (16 m^2) and the base of the vibrator was made by casting concrete in the place that had been dug. The foundation bolts were placed in the base, and after the concrete had set, the vibrator was fixed in the center of the base with nuts. Regarding the layout of the measurements, the observation points were chosen perpendicularly to the direction of vibration, and the seismometers with three components were set at the same time on the surface and in the base of each bore hole, which had been dug separately several meters from the base of the vibrator.

The transverse components of the waves generated were fairly good harmonic waves and considerably larger than the other components, on the surface and in the bore hole in the vertical plane perpendicular to the direction of vibration. Therefore, we regarded those waves which were generated from the vibrator as SH-type. Waves of a frequency twice as large as those of the vibrator were recorded sometimes in the vertical component. These waves may result from the rocking motion of the base. In order that the ground around the base may be vibrated as elastically as possible and not be injured, we vibrated the ground as small as possible, considering the noise level.

The ground structures around the observation points are shown in Fig. 8, and the spectra of the amplitude of the components of the direction of vibration are shown in Fig. 9. As the curves of the velocity spectrum are flat in some measure and become smaller with the decrease of frequency at each point, we could regard these waves as being sent out directly from the vibrator and not influenced by the ground structure. According to the theoretical calculation, the amplitude of displacement generated by excitation on the surface of a semi-infinite body is nearly constant, except near the resonance point, and the fall with the decrease in frequency as shown in Fig.9 does not appear in the theoretical result. During vibration of the base, the ground near it may exceed the limit of elasticity and yield. As a result, it seems that the ground behaves more fluidly at the low frequency. Therefore, we would have done better to enlarge still more the area of the base, from the viewpoint of the effectiveness of the transmission of low frequency vibration into the ground.

Regarding the effect of a resonance point of the dynamic ground compliance which results from the dimension and weight of the base, the curve of the amplitude of vibration is comparatively flat in the range 1-20 cps and tends to fall gradually at the low frequency, according to the calculation of the vibration of the base of a rectangle on a semi-infinite medium, and hence the base may hardly affect the spectral curve corresponding to the ground structure.

At each observation point the peaks and dips of spectra become clearer with the increase in depth. This fact corresponds to the appearance of the mode of vibration depending on the ground structure characterized by the velocity of S waves. Viewed in this light, we investigated the correspondence between the characteristics of the ground influenced by vibrator and those of the ground influenced by earthquakes. Though the position of the origin of the vibration is opposite to that in an earthquake, according to the reciprocity theorem in elastic dynamics, the recept in the ground for the excitation of the surface is equivalent to that on the surface for the excitation underground. Therefore, having no regard to the fact that the waves generated from the vibration are spherical and not plane waves, as is the case of earthquake waves, we can make and analogy between the spectra obtained from surface vibration and those from earthquakes and could estimate the characteristics of the ground in earthquakes from the results of the vibrator. The mean amplitude spectra of the S waves of two earthquakes at this place 4 m in depth are shown in Fig. 10. In the figure, the solid line shows the theoretical spectra 4 m in depth calculated on the base of the model in Fig. 8. As shown in Fig. 9 the spectra underground, 17 m and 30 m in depth, have weak peaks at 5 cps and 15 cps. It may be considered for the above reason that the peak at 5 cps corresponds to that of the two earthquakes.

PHASE VELOCITY OF LOVE WAVES

In order to detect the transverse component of waves, surface geophones were set horizontally at intervals of 5 meters along a line perpendicular to the direction of vibration by the vibrator. The phase velocity of Love waves was measured between 120m-220m from the above vibrator in the region of the above observation point. As stationary harmonic waves of a single frequency are generated by the vibrator, the wave form does not vary during propagation and then the correspondence of the phases between each observation point can be easily detected, if the interval between the points is less than the wave length and the ground structure does not vary between a distance several times as large as the interval.

We intended to generate only waves of SH type from the vibrator, to utilize the small number of kinds of wave mode, but surface waves of Rayleigh type were sometimes generated by the rocking motion of the base. As the waves generated from the vibrator are continuous ones, the different modes overlap in the region where there are waves of more than two modes for a single frequency. But, as the amplitude of the fundamental mode is much larger than others in surface waves, only this mode can almost be recorded near a frequency corresponding to the Airy phase. The frequency of waves recorded in this case was in this range.

The observed valued and dispersion curve calculated on the base of the ground structure obtained by the seismic exploration of SH wave are shown in Fig. 11. There was fairly good agreement within the range 6-20 cps.

CONCLUSIONS

It is shown that $H(\omega)\exp(i\varphi(\omega))$ theoretically represents the vibrational characteristics of the layered structure of the ground, which is obtained by calculating the ratios and differences of both amplitude and phase spectra between the observation points at the ground surface and underground in order to investigate the vibrational characteristics of the ground due to natural earthquake. The analysis of the spectrum of the observed earthquake motion based on the above facts has been studied and the free oscillation period of layered ground was obtained from the given spectral ratios. This value showed fairly good agreement with the theoretical spectral curve which is calculated from the geological structure clarified by S wave prospecting and the spectral form is shown to be theoretically due to the layered structure. The method of investigating the vibrational characteristics of the ground considering both amplitude and phase spectra is conceived to be necessary when the geological structure is complicated.

By vibrating the ground in a horizontal direction artificially by a vibrator set on the ground surface the vibrational characteristics of the ground due to SH wave generated in this case were observed and it showed behavior similar to that of a natural earthquake. This method will provide a more accurate clue to the vibrational characteristics of the ground when it is combined with the method of analysis concerning natural earthquakes.

BIBLIOGRAPHY

1. Kanai, K., "Relation between the Nature of Surface Layer and the

Amplitude of Earthquake Motions," Bull. Earthq. Res. Inst., 30 (1952), pp. 31-37

2. Kanai, K., T. Tanaka and K. Nagata, "Measurement of the Micro-Tremor," Bull. Earthq. Res. Inst., 32 (1954), 35 (1957)

3. Omote, S , S. Konaka and N. Kobayashi, "Earthquake Observation in Kawasaki and Tsurumi Area and the Seismic Qualities of the Ground," Bull. Earthq. Res. Inst., 34 (1956), pp. 335-364

4. Kanai, K., T. Tanaka and S. Yoshizawa, "Comparative Studies of Earthquake Motions on the Ground and Underground," Bull. Earthq. Res. Inst., 37 (1959), pp. 53-87

5. Kanai, K., "Relation between the Amplitude of Earthquake Motions and the Nature of Surface Layers," Bull. Earthq. Res. Inst., 34 (1956), pp. 167-184

6. Kobori, T., "Dynamical Ground Compliance of Rectangular Foundation on a Semi-infinite Elastic Medium," Annuals, Disaster Prevention Research Institute, No. 10A (1967), pp. 283-314

7. Knopoff, L. and A. F. Gangi, "Seismic Reciprocity," Geophysics, 24 (1959), pp. 681-691

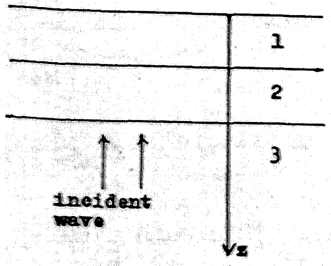

Fig. 1

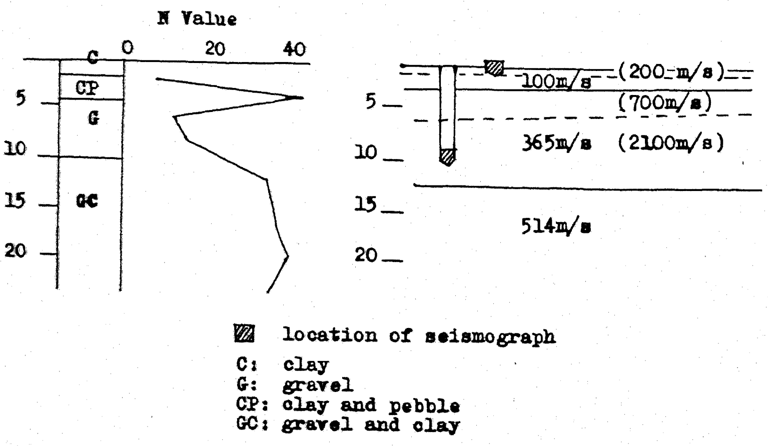

Fig. 2 Geological structure and location of seismographs at observation points

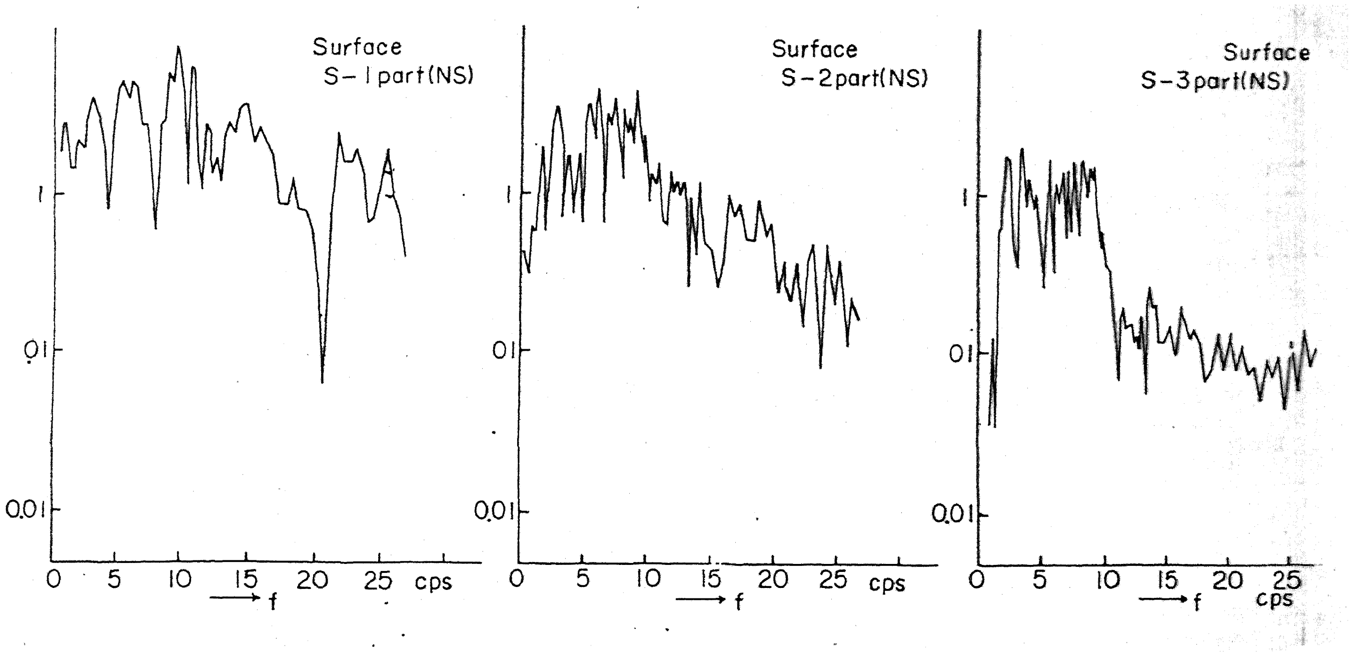

Fig. 3 Amplitude spectra of velocity of each S wave part divided into three (Surface, S-1, S-2, S-3) in arbitrary units

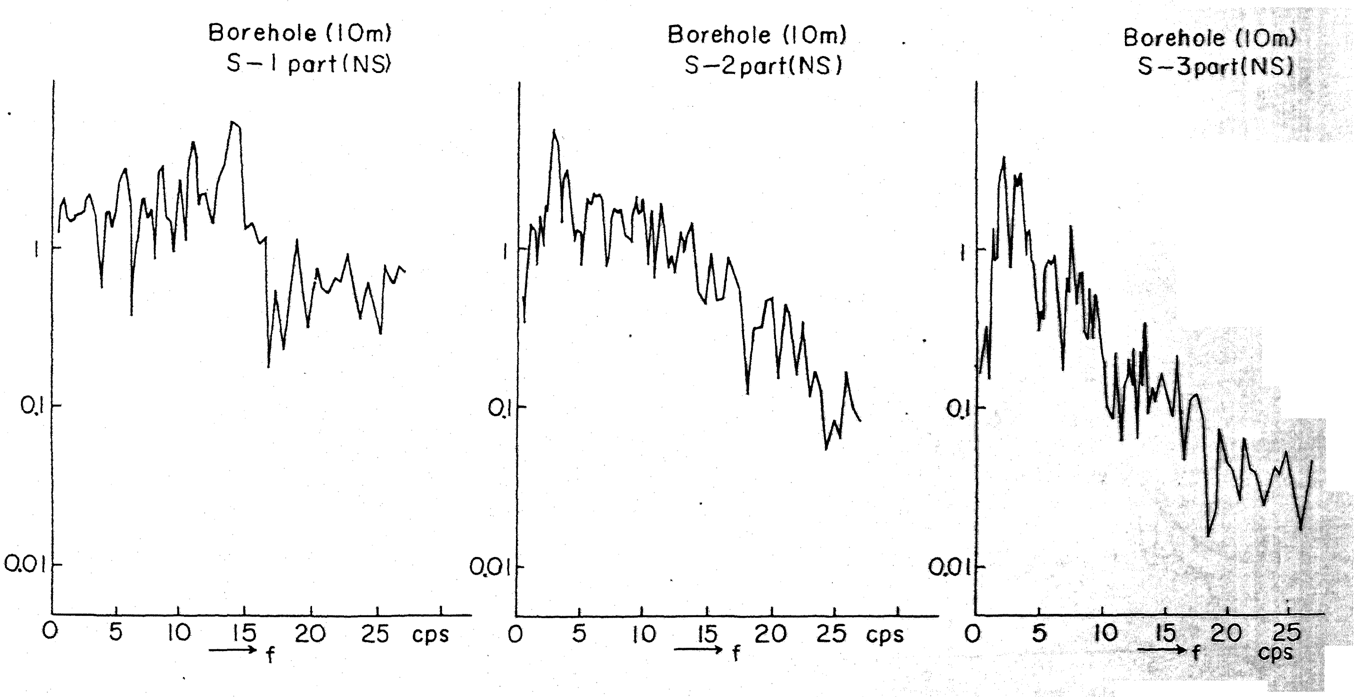

Fig. 4 Amplitude spectra of velocity of each S wave part divided into three (Borehole, S-1, S-2, S-3) in arbitrary units

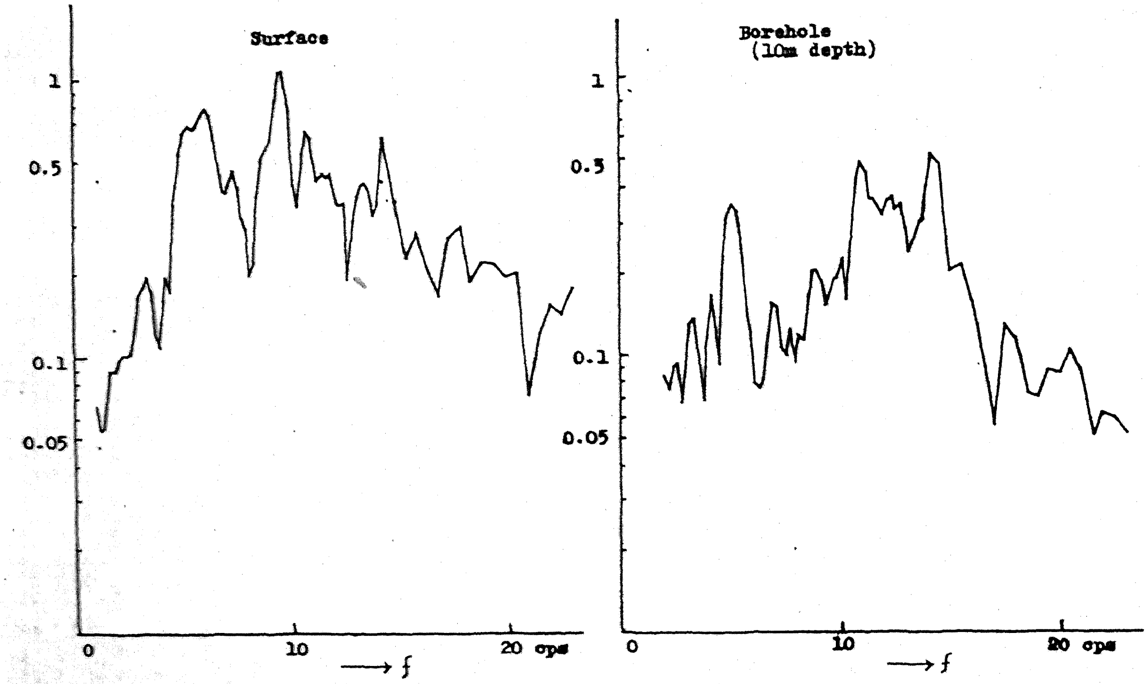

Fig. 5 Average amplitude spectrum of velocity (NS-component of S waves) in arbitrary units

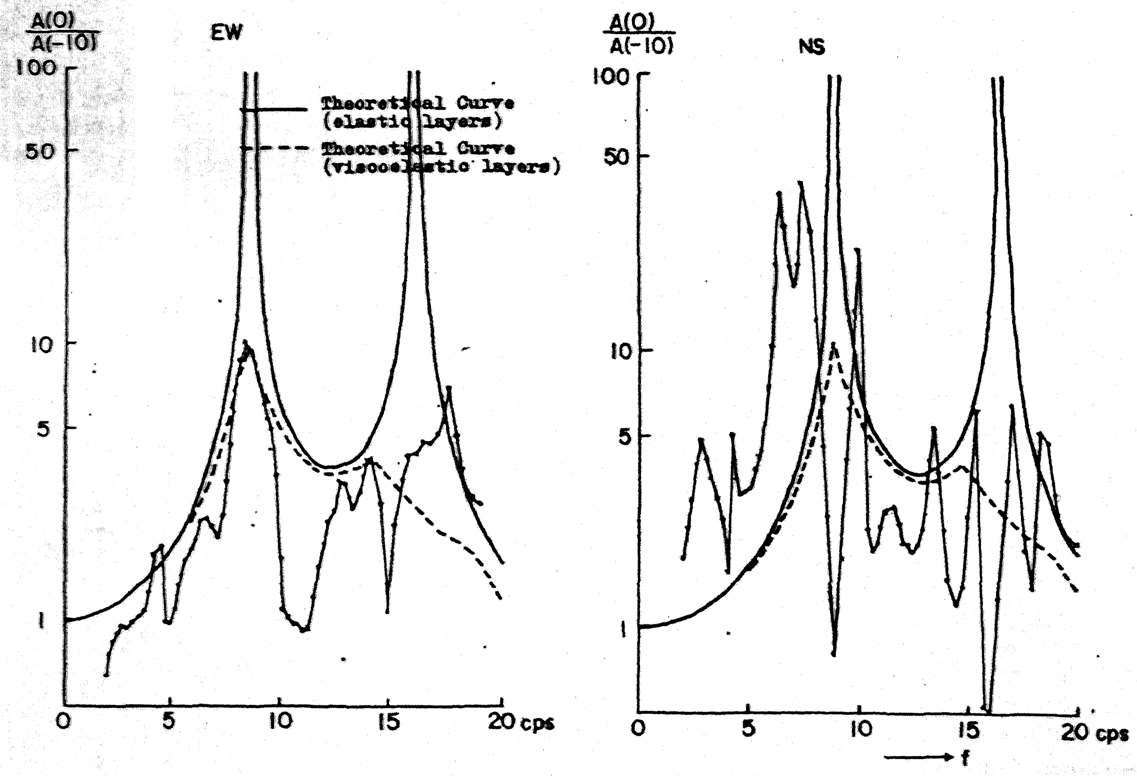

Fig. 6a Amplitude ratio surface to 10m depth

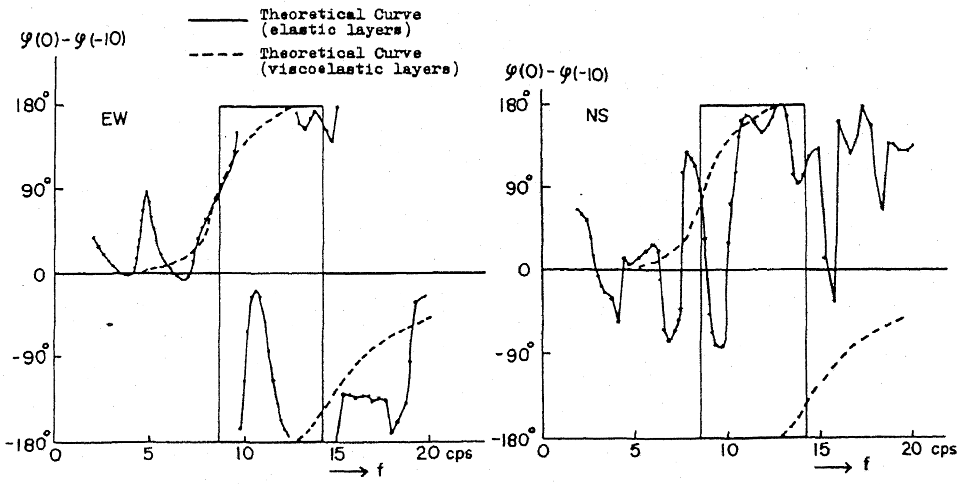

Fig. 6b Difference of phase between surface and 10m depth

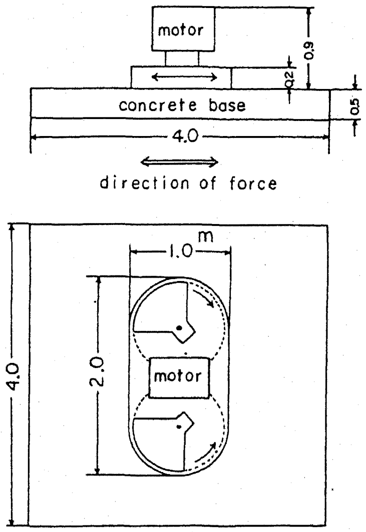

Fig.7 Structure of vibrating machine

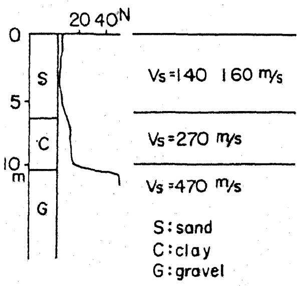

Fig.8 Geological structure

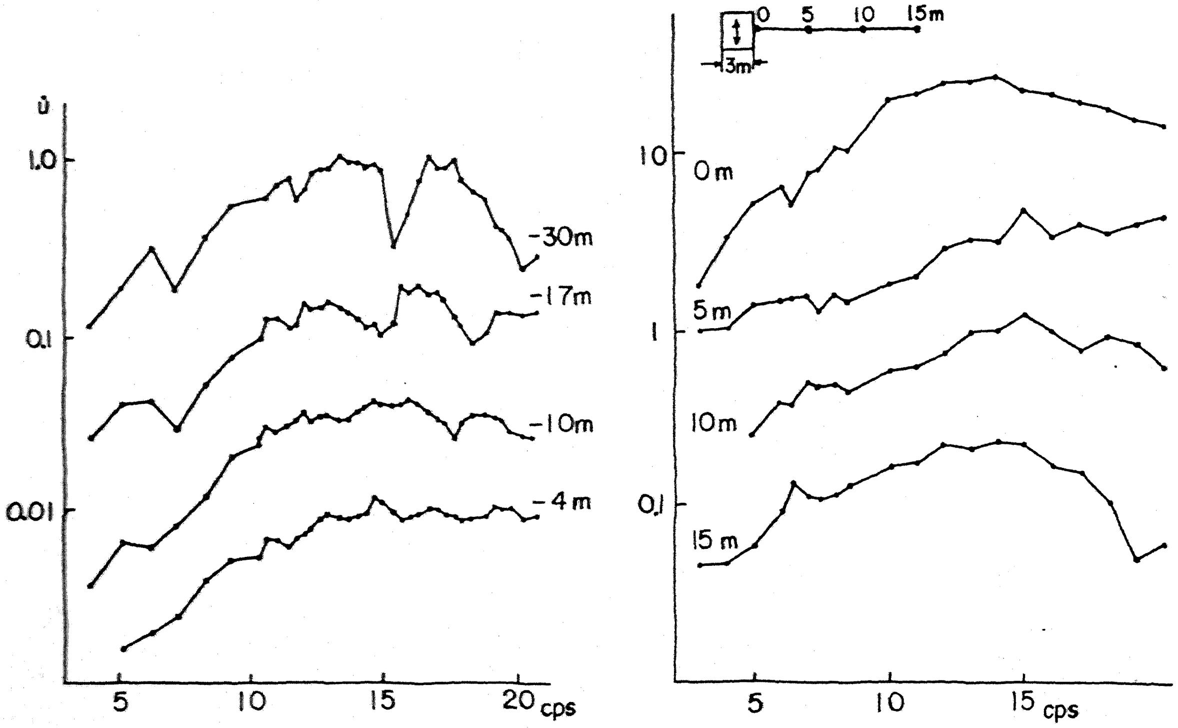

Fig.9 Velocity spectra for constant vibrational force, left: observed underground, right: observed on ground surface

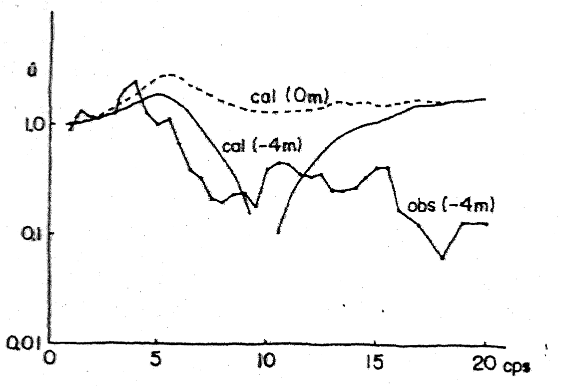

Fig.10 Average amplitude velocity spectrum of two earthquakes underground (4m depth), full line: theoretical velocity spectrum of 4m depth, dotted line: theoretical velocity spectrum of ground surface

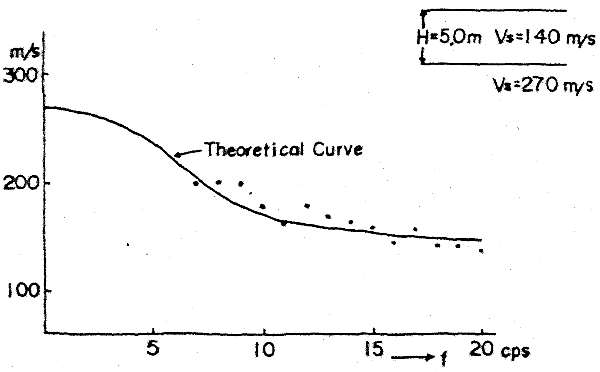

Fig.11 Observed phase velocity and theoretical dispersion curve