# Seismic Observation of Rigid Structure on Various Soils and Its Review

by

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Abstract

Three identical test structures were constructed on three sets of different soil conditions in suburbs of Tokyo. Several seismographs were installed not only in the model structures but also their surrounding ground, and actual informations about interactions of the earthquake-soil-structure system during earthquake motion were obtained.

As preliminary experiments, the physical prospecting tests and forced vibration tests were carried out on each coupled system of test model and soil so as to find the elastic properties of soil and the dynamic characteristics of the model structures.

The model structure was represented by a mathematical model having fewer degrees of freedom, and the calculation of its response was tried giving the observed earthquake records as the input. These theoretical responses have shown satisfactory agreement with the observed responses when the shear wave velocity of soil was properly estimated.

## 1. Introduction

For the design of an important structure such as a nuclear facility or a like, it is necessary to estimate the dynamic behavior of the structure during an earthquake even with a simplified assumption as a rigid body on the elastic soil.

The response of rigid structure would predominantly depend uopn the properties of soil, however, the interaction effects between a building and its supporting ground have not been presented for direct use in the practical design.

The main objects of this study are to investigate the influence of the subsoil medium on the response of a rigid building and to find a suitable dynamic model representation of the original structure which will yield satisfactory results for engineering purpose.

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Three identical test buildings were constructed on three sets of different subsoil layers in order to obtain informations about the actual interaction of those soil-structure systems during several earthquake motions. These informations were compared with theoretical results as obtained by conventional methods of analysis. In this way, it was hoped that some approximate method for analytical estimation of the dynamic behavior of a rigid structure during actual earthquake could be found.

## 2. Description of Test Model and Seismic Observation Method

Fig. 1 shows the locations of three test models in the western suburbs of Tokyo. The following three different soil conditions were selected to observe their contributions on seismic motion of the test structure;

(1) gravel (2) silty rock (3) loam

The test structures were the reinforced concrete with single story structure with a basement floor, having the plan of 3.5m x 3.5m and the overall height of 7.4m. Fig. 2 shows soil profiles and the arrangement of seismographs for each test.

Test model No. 2 was set on a silty rock, N value of which was more than 40 and the orientation of totally six accelerometers was the same as test model No. 1. The model was located on a hill and this area was a hilly district with many valleys and ridges, and subsoil conditions were complicatedly changed even within a narrow area.

Test model No. 3 was built on a Kanto loam layer which was comparatively homogeneous and uniform having about 5 of N value. Six accelerometers were set on the test model, and four accelerometers were installed in such a manner as one at the ground surface and other three vertically below at a distance of 20 m from the test model.

All of those accelerometers were schemed to move automatically as soon as an earthquake arrived and to record all accelerations on a single oscillograph chart during about 90 seconds.

## 3. Preliminary Experiments

As preliminary experiments, the physical prospecting investigations and forced vibration tests were carried out with each test model and its surrounding soil layer. The test results of wave propagation velocity by explosion are shown in Fig. 3.

Forced vibration test by horizontal excitation was carried out for each test model in order to find the dynamic characteristics of the structure such as its period of vibration and energy absorption. A vibration exciter was installed on the roof slab and applied sinusoidal force at very stage of frequencies to the structure. By conducting the test before and after back fill, effects due to the back fill soil were studied. The response curves for various conditions are shown in Fig. 4, and the natural frequency of each model can be read from these curves as seen in Table 1. The values of damping expressed as the ratio of critical damping can also be found from the resonance curves as listed in Table 1.

Even though these structures are exactly identical, the dynamic characteristics were fairly different with one another accroding to the conditions of subsoil medium. These experimental results prove that the elastic properties of soil are the definite factor to determine the frequency of a rigid structure.

No. 2 model, built on the hardest soil, has the highest frequency of 8.4 c/s and No. 3 model, built on the softest soil, has the lowest frequency of 6.7 c/s before back fill, and there is about 25 % difference between the above two cases. The effect of back fill or the hardening effect of back fill soil along with the elapse of time is found to cause notable change of raising the natural frequency.

## 4. Observation of Earthquake Motions

The observation of earthquake motions was started in 1965 for the model No. 1, in 1966 for No. 3 model and in 1967 for No. 2 respectively. Many earthquakes ranging 1 to 30 gals had been recorded on each model, and many informations to interrelate between ground motion and the response of structure were successfully obtained for each soil and earthquake condition. A sample record of earthquake motions is shown in Fig. 5. We have been studying in several methods of frequency analyses to inquire general characteristics of earthquake motion in connection with soil conditions, epicentral distance, magnitude and intensity of earthquake.

The analysis of recorded seismic waves themselves is not the objective of the present subject, therefore, it is not referred to detail in this paper. The major results which are relevant to the response problems of structures are pointed out in brief as follows:

(1) The predominant periods on the surface layers can be read from spectral analysis of surface records as about 0.175 sec. for No. 1, about 0.145 sec. for No. 2, and about 0.250 sec. for No. 3 model.

(2) It could be concluded that these predominant periods have grown in the surface layer, but another typical frequency components which also occasionally appeared on the surface layer may be caused by the difference of epicentral distances, course of arrival of seismic waves and mechanism of earthquake occurrence.

(3) When a seismic wave of long period arrives to the surface layer, the predominant period of the surface layer is excited but little and the form of waves is scarcely changes in the surface layer.

(4) The distribution of the maximum acceleration in the ground is shown in Fig. 6, which shows a tendency that the ground motion is suddenly amplified towards ground surface. In case of No. 3 model, the average amplification ratio is 5.7 between two points of 20 m depth and the surface, and the ratio tends to increase as the frequencies close to the predominant

period of the surface layer are contained in the original waves.

The change of frequency characteristic of seismic waves through the ground are easily understood by comparing the spectral analysis data of successive observation points. In this purpose, the autocorrelation function and power spectrum obtained from actual records will be useful, and the typical samples of them are shown in Figs. 7 and 8.

Each model structure can be defined to move just like a rigid body from the distribution of recorded responses on every part of the model. In general, the natural frequency of the structure is conspicuously predominant in the response records on the model, but there are some exceptions in which the natural frequency scarcely appears in the responses on model and the structure moves similar to ground motion, because of the vibration period of the ground motion being too long. Needless to say, movement of a structure is intensely amplified when the predominant frequency of the ground motion becomes close to the natural frequency of the structure.

## 5. Theoretical Study for Vibration of Rigid Structure

### (1) Spring Constant and Damping Coefficient of Soil Foundation

Up to present, the interaction effect of foundation-soil system has been investigated by several authors(1), and have been presented analytical solutions for the oscillation of a vibration body resting upon the surface of semi-infinite, isotropic, homogeneous elastic body. These past excellent studies might give more accurate solutions for idealized conditions, but the mathematics involved are a little too complex for a practical use.

A simplified method concerning the interrelationship of soil-structure system has developed by one of the authors of this paper in which wave reaction is satisfactorily introduced for the practical use(2). Some results obtained from the theory were compared with those introduced from the exact solutions and it was proved that there are no significant differences between them for a range of the practical application. The author sets the following two assumptions in order to simplify the mathematical procedures:

(1) The stress distribution in the soil under the static load will also hold for dynamic loading.

(2) The vibration energy is dispersed as a plane wave in the close vicinity beneath a foundation.

Thus spring constants and damping coefficients of soil-foundation system are introduced as follows, and it is convenient that they are given in the forms independent of frequencies.

Table 2

| | Spring Constant | Damping Coefficient |
|---|---|---|
| Vertical | $\frac{4a\rho V_s^2}{1-\nu}$ | $\rho V_p S$ |

| | | |
|---|---|---|
| Horizontal | $\frac{8 a \rho V_s^2}{2-\nu}$ | $\rho V_s S$ |
| Rocking | $\frac{8 a^3 \rho V_s^2}{3(1-\nu)}$ | $\rho V_p I$ |
| Torsional | $\frac{8 a^3 \rho V_s^2}{3(1-\nu)}$ | $\rho V_s I_p$ |

a : radius of circular foundation base
$\rho$ : mass density of foundation medium
$\nu$ : Poisson's ratio
$V_s$ : shear wave velocity
$V_p$ : compression wave velocity
S : contact area of foundation
I : moment of inertia of cross sectional area
$I_p$ : polar moment of inertia of cross sectional area

The spring constants and damping coefficients are given as the functior of the shear wave velocity $V_s$ . In truth, however, there are some questions in estimating the value of $V_s$ . The method of physical test is generally practised, but the following problems would be inevitable to come into questions.

1. Effect of overburden pressure
2. Change of wave velocity along with the depth
3. Variation of soil structure
4. Effect due to disturbance of soil during construction work

The values of shear wave velocity obtained from the physical test and that determined theoretically form resonant frequency of simple test foundation do not generally agree with each other. The main reasons is supposed, besides above four items, that both the vibration amplitude and energy are not at a same level in the physical test and forced vibration of the test foundation, and it is technically difficult to generate shear wave in its pure sense.

The estimation of the values $V_s$ as well as Poisson's ratio involved various questions in the present situation, therefore the values theoretically determined from preliminary experiment of the forced vibration tests are used for the calculations of earthquake response in this paper.

(2) Natural Frequencies of a Rigid Structure

A rigid structure will be represented by a simple model as shown in Fig. 9 which has two degree of freedom, i.e. rocking and swaying, and the elastic deformation of structure itself can be neglected because it is very small as compared with those of motions of rigid body.

The equations of motions of the body are

$$M\ddot{u}_x + C_x(\dot{u}_x - \dot{\phi}d) + K_x(u_x - \phi d) = 0$$
$$J\ddot{\phi} + C_\phi\dot{\phi} + K_\phi\phi - C_x(\dot{u}_x - \dot{\phi}d)d - K_x(u_x - \phi d)d = 0 \qquad (1)$$

in which
M : mass of the body, $u_x$ : horizontal displacement,
$c_x$ : horizontal damping of soil,
$K_x$ : horizontal spring constant of soil,
J : moment of inertia of the body, $\varphi$ : rocking angle,
$c_\varphi$ : rotational damping coefficient of soil,
$K_\varphi$ : spring constant for rocking,
d : distance from contact surface to the center of gravity of the body.

Using the following representations,

$$K_x/M=\omega_x^2 \qquad K_\varphi/J=\omega_\varphi^2, \qquad \omega_\varphi/\omega_x=\beta, \qquad M/J=m$$

$$h_x=C_x/2\sqrt{MK_x}, \qquad h_\varphi=C_\varphi/2\sqrt{JK_\varphi}$$

the frequency equation becomes as follows;

$$p^4+2(h_x+\beta h_\varphi+h_x md^2)p^3\omega_x+(\beta^2+md^2+1+4h_xh_\varphi\beta)p^2\omega_x^2$$
$$+2\beta(\beta h_x+h_\varphi)p\omega_x^3+p\omega_x^4=0 \tag{2}$$

The solutions of the above equation are given by the form of

$$p_1=(\mathcal{R}_1\pm i\mathcal{I}_1)\omega_x, \qquad p_2=(\mathcal{R}_2\pm i\mathcal{I}_2)\omega_x \tag{3}$$

in which $\mathcal{R}_1$, $\mathcal{R}_2$ mean the real parts and $\mathcal{I}_1$, $\mathcal{I}_2$ mean imaginary parts of $p_1$ and $p_2$.

Thus the general solutions of $u_x$ become

$$u_x=C_1\exp\{(\mathcal{R}_1+i\mathcal{I}_1)\omega_x t\}+C_2\exp\{(\mathcal{R}_1-i\mathcal{I}_1)\omega_x t\} \tag{4}$$

and

$$u_x=C_1'\exp\{(\mathcal{R}_2+i\mathcal{I}_2)\omega_x t\}+C_2'\exp\{(\mathcal{R}_2-i\mathcal{I}_2)\omega_x t\} \tag{5}$$

where $C_1$, $C_2$, $C_1'$ and $C_2'$ are determined by initial conditions. If $\mathcal{I}_1<\mathcal{I}_2$, Eq. (4) corresponds to the lower rolling mode, and Eq. (5) corresponds to the upper rolling mode. Consequently the natural frequencies of the system $f_1$ and $f_2$ are

$$f_1=\mathcal{I}_1\omega_x/2\pi, \qquad f_2=\mathcal{I}_2\omega_x/2\pi \tag{6}$$

in which $\mathcal{I}_1$ and $\mathcal{I}_2$ can be obtained by solving the frequency equation (2).

(Numerical Calculation for the Models)

The model can be replaced by a cylindrical body by the following formula which affords the solution of close agreement with the case having a square section.(3)

$$a_e=0.55\sqrt{S}$$

In which $a_e$ is an equivalent radius of the original section.

Using the following numbers,

| | |
|---|---|
| total weights of the model | 78.2 ton |
| unit weight of soil | 1.8 $t/m^3$ |
| equivalent radius of the model | 1.92 m |

and assuming Poisson's ratio to be 0.25, the solution of the frequency equation for the lower mode becomes as

$$p_1=(-0.0313\pm 0.367i)\omega_x$$

Thus the shear wave velocities are respectively decided as follows from the experimental data tested before back fill.

Table 3

| Model | Soil | $V_s$ |
|---|---|---|
| No. 1 | Gravel | 296 m/s |
| No. 2 | Silty Rock | 311 m/s |
| No. 3 | Loam | 248 m/s |

Compression wave velocity of the subsoil layer for No. 3 model was measured to be 420 m/s, from which the shear wave velocity was estimated to be 242 m/s assuming Poisson's ratio to be 0.25, and which was in close agreement with the value determined from the forced vibration test. In the cases of No. 1 and No. 2 models, $V_s$ values determined from forced vibration tests were extremely smaller than the values obtained from the physical tests. The main reason of that could be considered that the layer just beneath the base of model had been disturbed and loose sand and gravel were thinly laid on it during construction work. Such relatively rigid layers as the supporting layers of No. 1 and No. 2 models would be much affected by softer layers beneath the structures, and the stiffness of subsoil medium seems to appreciably decrease.

(3) Earthquake Response

To analyze the response of a structure, the test model may be also replaced by the mathematical model having two degrees of freedom as shown in Fig. 9. The equations of motion of the building are

$$M\ddot{u}_x + C_x(\dot{u}_x - \dot{\phi}d) + K_x(u_x - \phi d) = -M\ddot{x}$$

$$J\ddot{\phi} + C_\phi\dot{\phi} + K_\phi\phi - C_x(\dot{u}_x - \dot{\phi}d)d - K_x(u_x - \phi d)d = 0$$

in which $\ddot{x}$ is acceleration of the ground.

The linear acceleration method is generally utilized for calculating the above equations by high-speed computer.(4) By using the $V_s$ values obtained from the forced vibration tests, the spring constants and damping coefficients necessary for the calculation are theoretically determined.

Several earthquake motions recorded at surface layer were trially used for these calculations, and one illustrative result is shown in Fig. 10. For the convenience of comparison of the result thus obtained with experimental datum, actual response record on the model is shown parallel with the calculated result. The observed response shown in Fig. 10 is the record without back fill. The first curve is the record on ground, the second curve is the calculated response at the gravity center of the model and the third curve is the recorded response on the ground floor of the model.

These calculated results are in good agreement with the observed records. Such an agreement will owe to the favourable conditions that the model is simple in shape, small in scale and rigid enough to be represented by a perfect rigid body, moreover, the shear wave velocity were given from the forced vibration tests so as to agree with actual natural frequencies.

It has always been a problem what ground motion should be chosen as an input for the response calculation. As far as the present study concerns, the ground motion at the surface gave preferable results than any other motions at underground levels, in spite of the test model being set on a layer several meters deep from the surface.

The investigation of effects due to back fill soil was also included in this study. The change of spring constant and damping coefficient by back fill soil are not easily evaluated by an analytical means. The forced vibration test results indicated there were significant changes in the dynamic response of the test model before and after the back fill. In this case, the greater part of response was occupied by rocking motion because of a force acting at the roof, thus the model structure was inevitably exerted more reaction from surrounding soil on the side wall of basement.

On the other hand, when the structure was subjected to an earthquake loading, a relative displacement between soil and side wall would be very small as swaying motion is more predominant than rocking motion. Moreover, as the ground motions at the surface and at the bottom of basement were very similar to each other in such a scale as the model. There would be little coupling or interaction between the soil and wall. The difference of response modes for the forced vibration test and that for earthquake are graphically shown in Fig. 11.

The calculated response result, in which the effects of back fill soil were not taken into account were compared with the observed record obtained after the back fill, and a sample result is shown in Fig. 12. The interest is both two results show a good resemblance to each other, and it may be concluded that the effects of back fill soil can be disregarded for the earthquake response of such a small building as the test model.

## 6. Conclusion

Many experimental information about the ground motion and responses of structure during earthquakes were obtained for three test models constructed on different soil conditions.

To analyze the response of the test model to recorded ground motions, the model could be represented by a mathematical model having fewer degrees of freedom, holding the theoretical responses in good agreement with the observed records. Such an agreement would mainly owe to the use of shear wave velocity which were obtained form the forced vibration tests to keep a good agreement with actual natural frequencies. The key point on the response analysis of a rigid structure is how accurately spring constant and damping coefficient of soil are presumed. They are given as a function of shear wave velocity, and the value obtained from physical test does not always give a suitable result.

The formulas given in Table 2 are the solutions for idealized foundation medium, therefore, they do not always give proper result for actual soil even if the shear wave velocity is physically correct. The influence of disturbed and softer thin layer just beneath the base of structure is supposed to apparently decrease $V_s$ value. Consequently, the shear wave velocity obtained by physical methods should be treated with caution for the response study of actual buildings.

Needless to say that the equivalent value of $V_s$ obtained from vibration test of an actual structure gives more reliable result for the vibration response of that structure. Such a value has close relation to soil conditions i.e. soil classification and N value of standard penetration test, and the statistical study for a mutual relation between the equivalent value of $V_s$ and soil conditions examined from the natural periods of many actual rigid buildings will serve a good reference to the estimation of $V_s$ value for the practical purposes.[5]

It has often been discussed what ground motion should be chosen from those at various depth levels as an input for the response calculation. Not only this result but also full scale test using JPDR Plant recently carried out by Prof. H. Tajimi[6] have pointed out that the ground motions at the surface give the best preferable results out of any other motions at underground levels.

Though the back fill soil surrounding the basement semmed to have little influence on the earthquake response of the model in this test, it will not be generally concluded especially for a structure of larger scale.

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| | Natural Frequency (c/s) | | | Damping | | |
|---|---|---|---|---|---|---|
| Test Model | No. 1 | No. 2 | No. 3 | No. 1 | No. 2 | No. 3 |
| Before Back Fill | 8.0 | 8.4 | 6.7 | 0.075 | 0.048 | 0.052 |
| Soon After Back Fill | 10.5 | - | 7.4 | 0.143 | - | 0.054 |
| Ten Months After Back Fill | 11.8 | - | - | 0.095 | - | - |

Table 1. Natural Frequency and Damping of Test Models

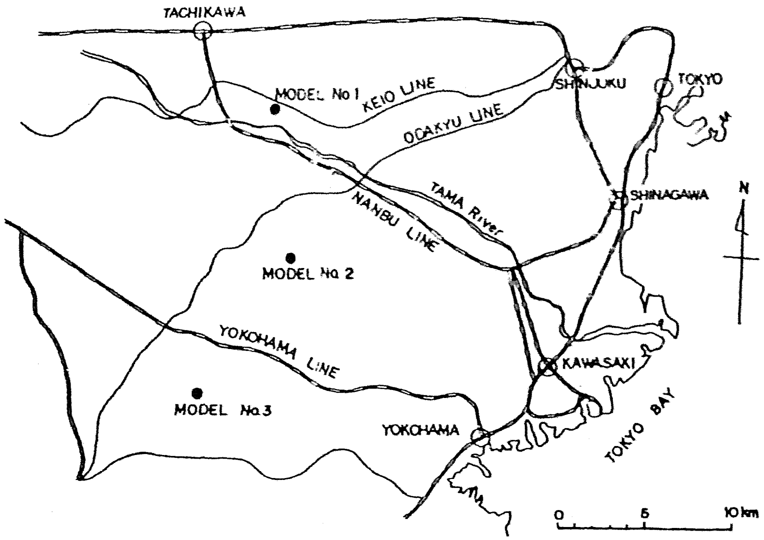


Fig.1 LOCATIONS OF TEST MODEL

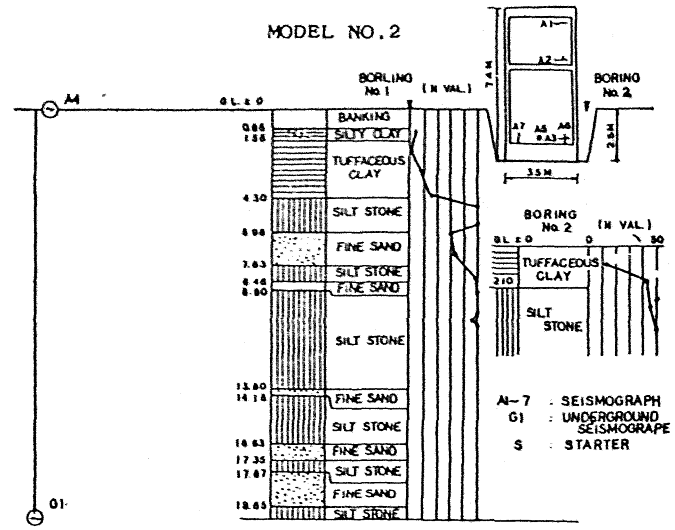


Fig.2-b ARRANGEMENT OF MEASURING INSTRUMENTS

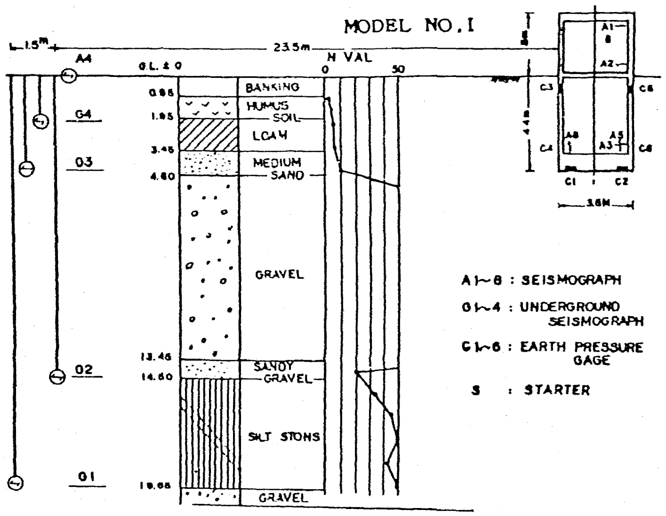


Fig.2-a ARRANGEMENT OF MEASURING INSTRUMENTS

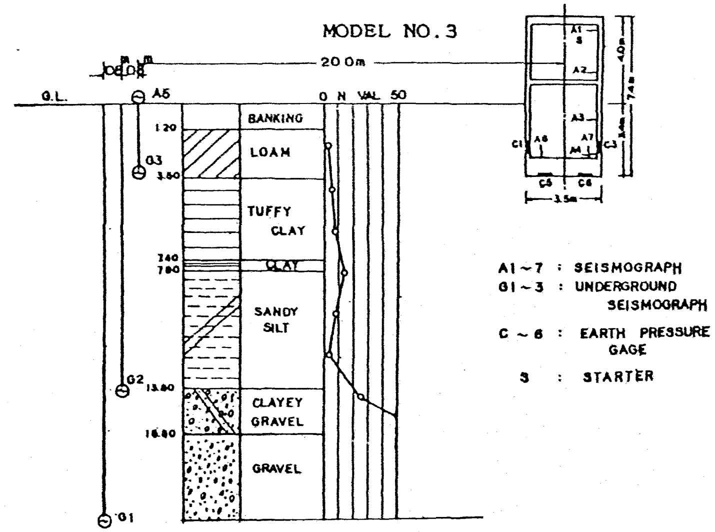


Fig.2-c ARRANGEMENT OF MEASURING INSTRUMENTS

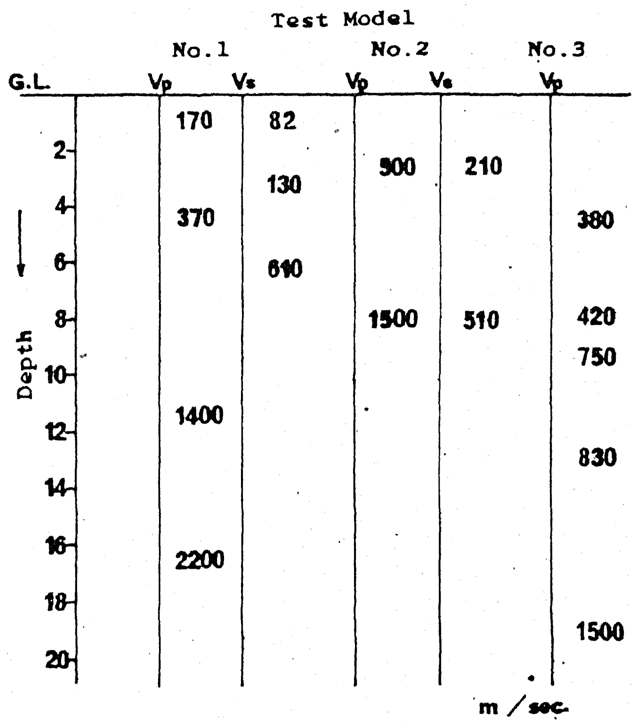


Fig.3 PROPAGATION VELOCITY

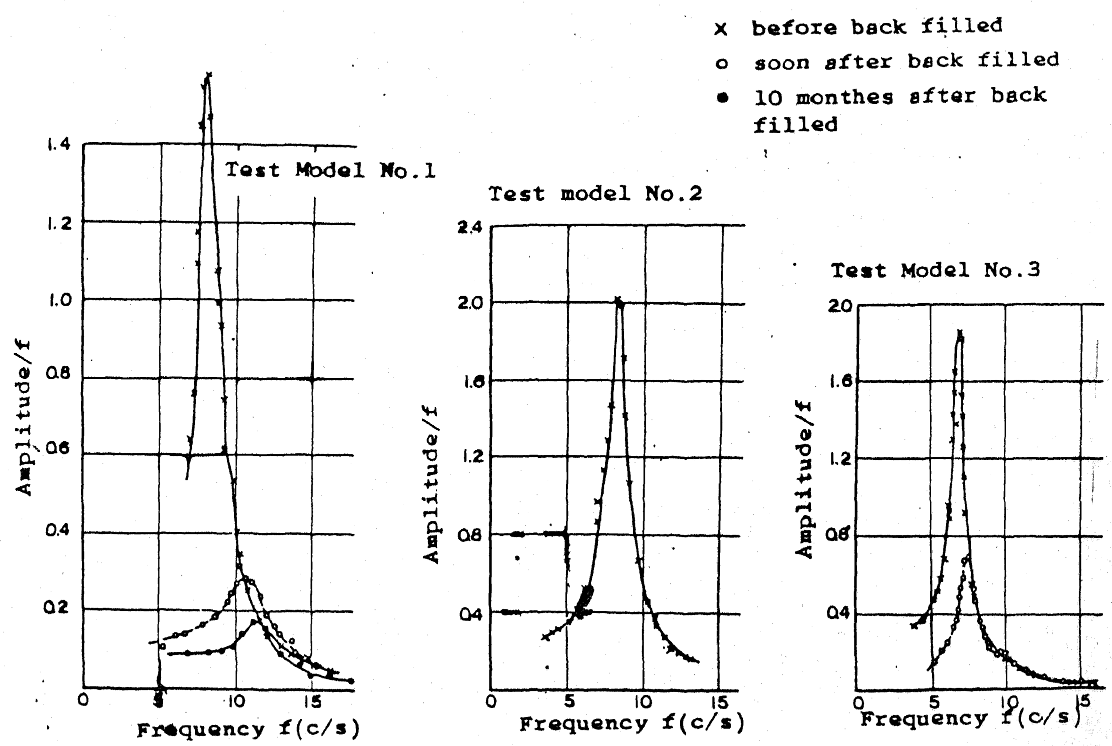


Fig.4 RESONANCE CURVE BY FORCED VIBRATION TESTS

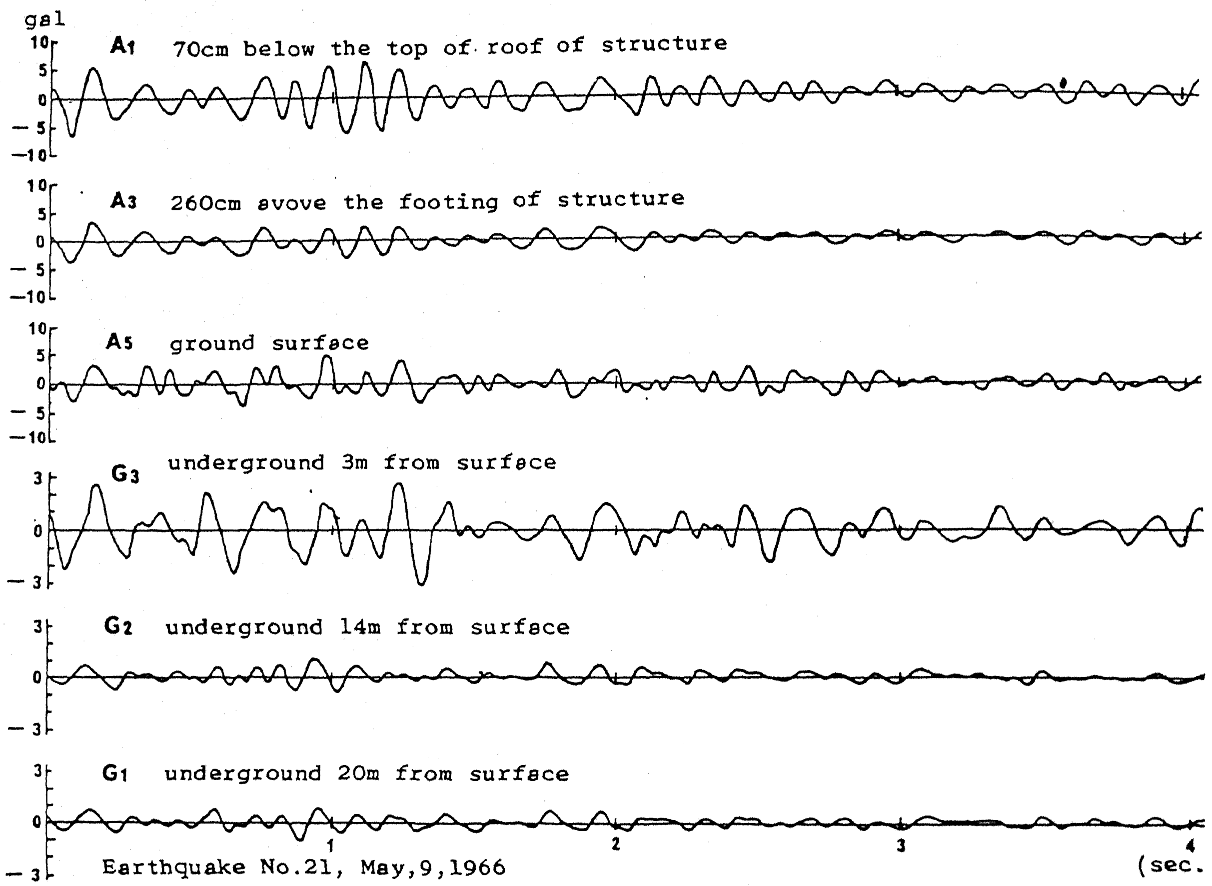


Fig.5 A SAMPLE RECORD OF EARTHQUAKE

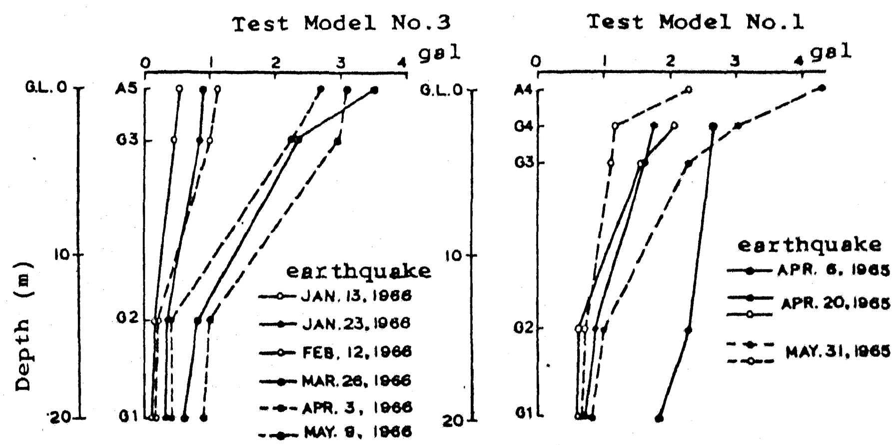


Fig.6 DISTRIBUTION OF ACCELERATION IN GROUND

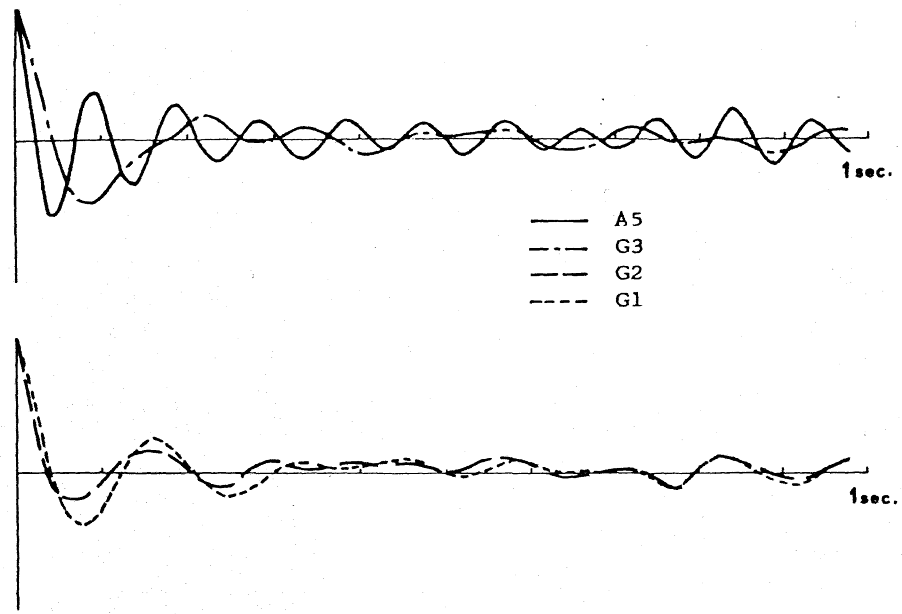


Fig.7 AUTOCORRELATION FUNCTION OF GROUND MOTION

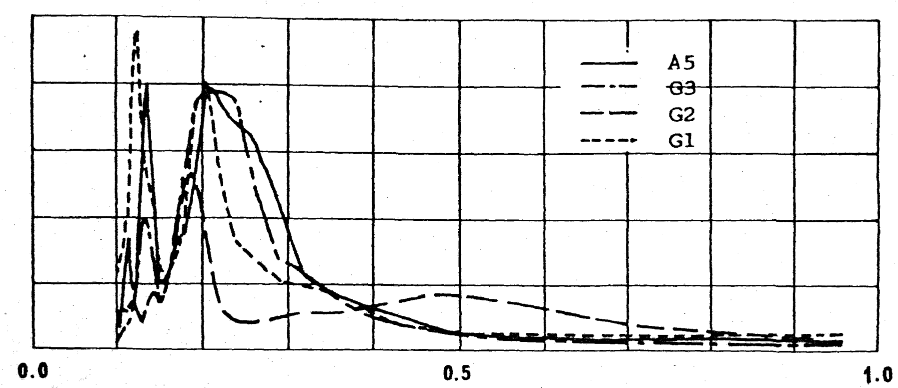


Fig.8 POWER SPECTRUM OF GROUND MOTION

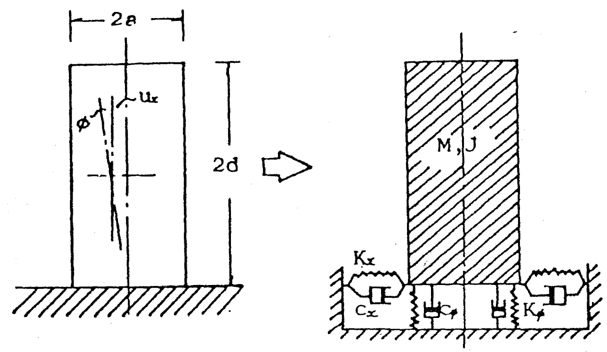


Fig.9 MATHEMETICAL REPRESENTATION OF TEST MODEL

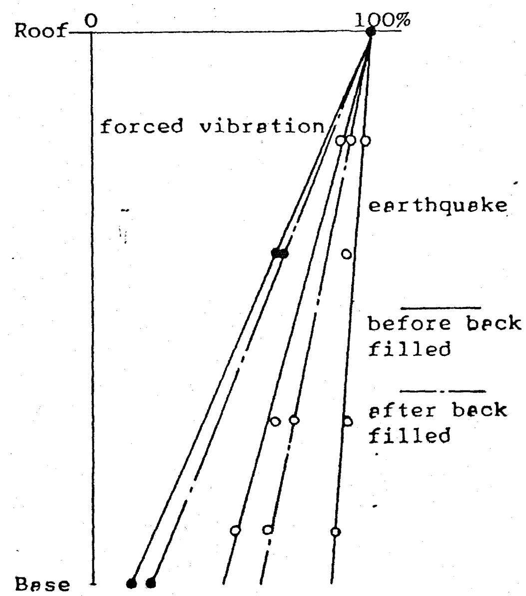


Fig.11 DIFFERENCE OF RESPONSE MODES OF TEST MODEL

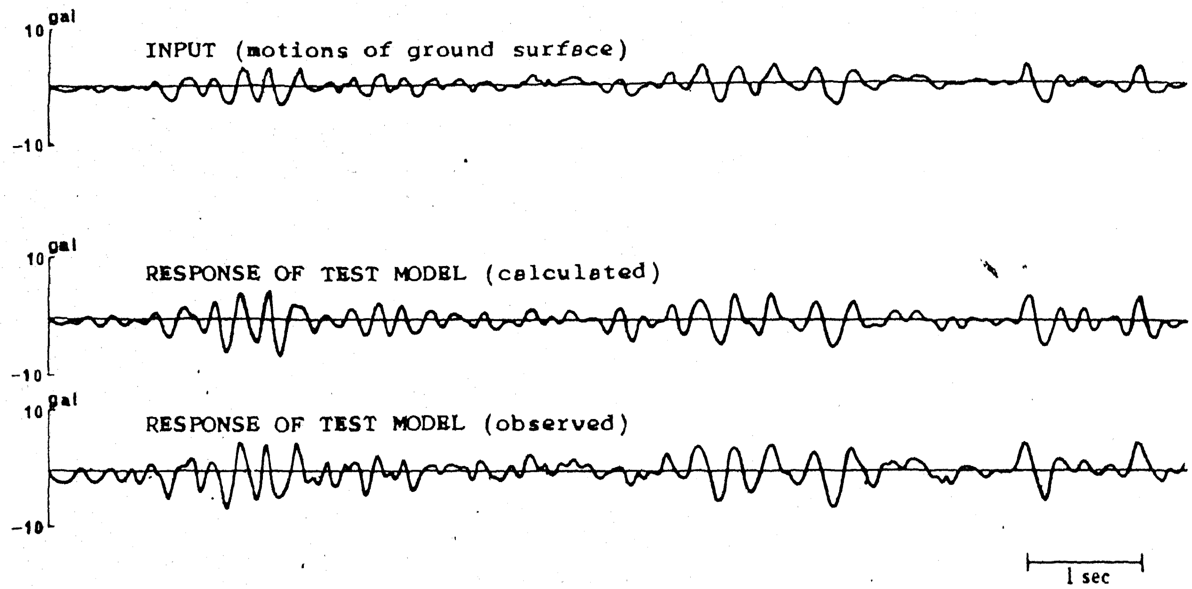


Fig.10 COMPARISON BETWEEN CALCULATED AND OBSERVED ACCELERATIONS

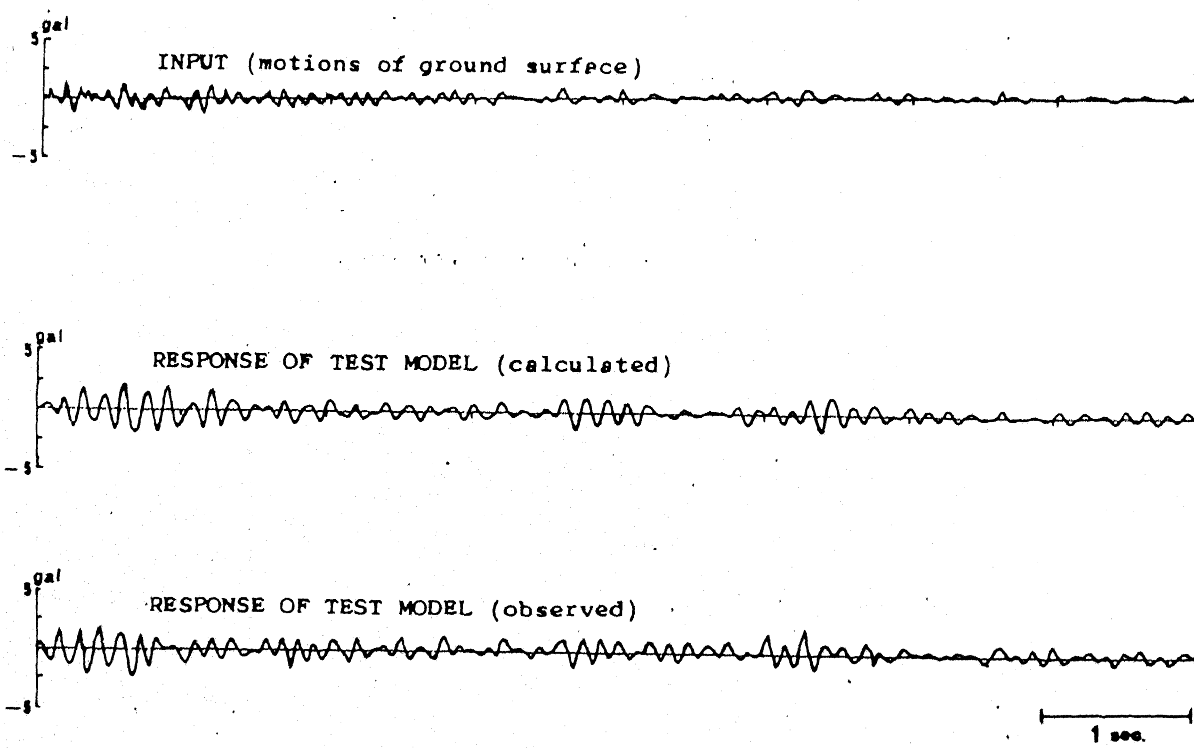


Fig.12 COMPARISON BETWEEN CALCULATED AND OBSERVED ACCELERATIONS