SIGNIFICANCE OF NONSTATIONARITY OF EARTHQUAKE MOTIONS

by

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SYNOPSIS

The effects of nonstationarity of strong-motion earthquakes are studied. In terms of the probability of first passage during an earthquake, it is shown that for lightly damped linear-single-degree-of-freedom systems nonstationarity does not appear to be too important. However, for nonlinear-inelastic systems the effects of nonstationarity may not be negligible. The differences in the elastoplastic response spectra obtained with a stationary and a comparable nonstationary pseudo-earthquake appear to increase with the degree of damage as measured by the ductility factor.

INTRODUCTION

It is well known that because of irregular transmission the accelerograms recorded on firm ground during destructive earthquakes have the appearance of random time functions. For this reason the applications of random vibration and random process theory in aseismic design has received increased attention.

It is not difficult to explain that the history of earthquake ground acceleration is nonstationary with time [1,2](IV), i.e. the statistics of the motion vary with time measured from the beginning of the record. Furthermore, relatively simple nonstationary processes can be constructed that produce ground motions very much like earthquake records and cause effects similar to those of earthquakes on linear structures [2]. Essentially this procedure has recently been used to model earthquakes of various types [3].

While there appears to be no difficulty in generating nonstationary earthquake-type disturbances using computers, the parameters describing the nonstationarity of the resulting pseudo-motions are

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(IV) Bracketed numbers refer to the corresponding references in the Bibliography difficult to estimate. It is, therefore, of interest to examine the consequences of nonstationarity of earthquake motions from an engineering viewpoint in order to assess whether or not nonstationarity is important in the stochastic modeling of earthquakes for engineering purposes.

The purposes of the present paper are to briefly review a simple model for strong earthquake motions, and to show results which indicate the effects on the response of simple systems caused by nonstationarity observed in such motions.

The nonstationarity considered here is of the type that is observed in strong-motion records registered on firm ground and at moderate epicentral distances. Therefore, the results obtained showing the effects of nonstationarity are limited to the earthquakes of the type considered.

STOCHASTIC MODEL OF STRONG-MOTION RECORDS

A study made on the two horizontal components of four strong-motion accelerograms registered at the west coast of the U.S., namely: El Centro, California records of 30 December 1934 and 5 May 1940; Taft, California record of 7 July 1952, and Olympia, Washington record of 13 April 1949 indicates that correlation exists among the closely spaced ordinates of the accelerogram. The solid lines in Fig. 1 show the envelopes of the correlation coefficient computed by time averaging segments of the accelerograms which contain high intensity acceleration pulses. The use of time averaging here implies that within the strong phase duration, the records are assumed to be stationary. The record duration, usually considered for structural response calculations, is not limited to the strong part of the accelerogram; the starting phase as well as the decaying phase of the acceleration pulses are also considered.

A simple stochastic model, which can include the properties of earthquake ground accelerations described above, is the filtered shot noise process. A shot noise S(t) can be defined as having the following properties:

$$m_S(t) = 0 \qquad (1a)$$

and

$$CoV_S(t_1,t_2) = I(t_1)\delta(t_2-t_1), \quad t_2 \geq t_1 \qquad (1b)$$

in which m_S and CoV_S are respectively the mean and covariance functions, $\delta(t)$ = Dirac delta function and the intensity function

$$I(t) = \begin{cases} I_o\left(\frac{t}{x_1}\right)^2, & o \leq t \leq x_1 \\ I_o, & x_1 \leq t \leq x_2 \\ I_o e^{-c(t-x_2)}, & t \geq x_2 \end{cases} \qquad (2)$$

where x_1 and x_2 are times as shown in Fig. 2 and c is a constant. If I(t) is taken to be a constant then S(t) becomes a stationary process within the duration of an excitation. If the excitation is of long duration then a stationary process, commonly known as white noise results. The constant I(t) of finite duration is shown by dotted lines in Fig. 2. In the sequel, when I(t) is as described in Eq. 2, the excitation will be called Input 1, whereas with constant I(t) it will be called Input 2.

To incorporate the type of correlation observed in earthquake records, the uncorrelated process S(t) is next passed through a filter. Results obtained using three types of linear ordinary differential operators as filter have been presented elsewhere [2]. In general, satisfactory comparisons with earthquake records have been obtained for second order filters. Denoting the random earthquake acceleration by $\ddot{Y}(t)$, the excitation process can be generated by solving

$$\frac{d^2\ddot{Y}}{dt^2} + 2\gamma\omega\frac{d\ddot{Y}}{dt} + \omega^2\ddot{Y} = S(t) \qquad (3)$$

For the generation of pseudo-earthquakes from this model, S(t) is treated as a sequence of impulses of random magnitude. These random impulses occur at uniform and small time intervals. The random impulse magnitudes are generated as uncorrelated Gaussian random numbers whose variance is determined from the intensity function I(t) [2]. Input 1 with zero initial conditions, therefore, constitutes a nonstationary record. A stationary record is obtained with Input 2 and the initial conditions determined on the assumption that the solution starts sufficiently long in the past so that $\ddot{Y}(0)$ and $\frac{d}{dt}\ddot{Y}(0)$ have become stationary. Since S(t) is Gaussian so is $\ddot{Y}(t)$. A stationary and a nonstationary record thus generated will be comparable if the same random numbers are used in generating S(t) for both records. This has been done to generate a pair of comparable stationary and nonstationary pseudo-earthquakes the results of which will be presented later.

Because in the range $x_1 \leq t \leq x_2$, $\ddot{Y}(t)$ is approximately stationary, the filter frequency ($f = 2\pi\omega$) and damping γ can be selected so that the correlation coefficient of $\ddot{Y}$ agrees with the results obtained from the real earthquakes. This yields $0.5 \leq \gamma \leq 0.6$ and 4 cps $\leq f \leq$ 5 cps. The resulting correlation coefficient is shown by the dashed line in Fig. 1.

The structural responses calculated from the simulated motions using the above filter and x_1 = 1.5 sec., x_2 = 15 sec., and c = 0.18 to 0.23 sec. are in fair agreement with responses obtained from the earthquake records considered. In Fig. 4 a comparison is made between the deformation spectra of one simulated nonstationary motion and those obtained for the NS component of El Centro 5/18/40 and S 80°W component of Olympia 4/13/49. In preparing this figure the records were normalized to have the same peak ground velocity. Fig. 3 shows the comparison between the average deformation spectra of 8 real and pseudo-earthquakes.

Despite these comparisons, it should be emphasized that the estimation of the parameters x_1, x_2, and c defining the nonstationary model are subject to gross variations and also no clear-cut evidence exists that the earthquake records are stationary during the strong phase of the accelerogram. It is, therefore, of practical interest to investigate the effects of nonstationarity on the response of simple linear and nonlinear systems. To draw specific conclusions in this regard, it is desirable to evaluate the probability that the system response will exceed prescribed response levels for the first time during the duration of an earthquake. The computational means available for the evaluation of the above probability of failure are limited at this time to damped linear single-degree-of-freedom systems subjected to uncorrelated excitation processes. Consequently, it is necessary to indicate for what ranges of the system frequency it is reasonable to consider the excitation as unfiltered. For these systems the effect of nonstationarity is studied in terms of the probability of failure. The results from these cases are then extended to other linear and elastoplastic systems by comparing the response spectra produced when systems are subjected to member functions generated from stationary and comparable nonstationary models.

It is recalled that a stationary input to a lightly damped linear single-degree-of-freedom system may be approximated as a white noise if the power spectral density is essentially constant for several bandwidths in the neighborhood of the system frequency [4]. For the stationary pseudo-earthquakes generated from Eq. 3 the power spectral density is given by

$$\Phi(\Omega) = \frac{I_o}{2\pi\omega^4} \cdot \frac{1}{[1 - (\frac{\Omega}{\omega})^2]^2 + 4\gamma^2(\frac{\Omega}{\omega})^2} \qquad (4)$$

where Ω has units of $sec.^{-1}$. This equation is plotted in Fig. 5 vs. the frequency $f = \Omega/2\pi$. It is seen that for systems with natural periods $T_o = 1/f \geq 2$ sec. this curve is essentially flat.

EVALUATION OF PROBABILITY OF FAILURE

Preliminary Remarks

Consider a linear damped oscillator subjected to the base excitation S(t). The equation of motion is

$$\ddot{U} + 2\beta\omega_o\dot{U} + \omega_o^2 U = -S(t) \qquad (5)$$

where U = spring deformation, β = fraction of critical coefficient of damping, ω_o = undamped circular frequency, and S(t) is a Gaussian shot noise with prescribed intensity function. It is assumed that failure of the system occurs when U(t) crosses the barriers $\pm$ B for the first time during the duration of the excitation t_d. The probability of safety, therefore, may be defined as

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$$P_s(t_d) = \text{Prob.}\left\{ |U(t)| < B,\ o \leq t \leq t_d \right\} \tag{6}$$

and

$$P_f(t_d) = 1 - P_s(t_d) \tag{7}$$

is the probability of failure.

The variance functions of U(t) for Inputs 1 and 2 are shown in Fig. 6 for a specific system. For Input 2, if the excitation duration is sufficiently long the variance approaches the asymptote

$$\sigma_o^2 = \frac{I_o}{4\beta\omega_o^3} \tag{8}$$

The barrier level in Eq. (6) is measured by

$$B = \alpha\sigma_o \tag{9}$$

where α is a positive parameter.

In any dynamic system, the motion usually persists as free-vibration after the excitation terminates. The probability of safety defined in Eq. (6) refers to the forced vibration era only. As a matter of interest the values of P_s are tabulated below for two cases and two barrier levels. The following system parameters are used: T = 2 sec., β = 0.08. Duration of excitation is t_d = 25 sec.; I(t) is for Input 2 during the excitation and it is zero thereafter. In case 1, P_s is calculated for the duration of the pulse only whereas in case 2 one cycle of oscillation after 25 sec. was included.

Values of P_s

α	Case 1	Case 2
1.5	0.01287	0.01224
3.0	0.86888	0.86776

Method of Calculation

The numberical procedure used for computing probability of failure has been successfully used before for the case of white Gaussian excitation [5], and the results from it have been checked with those obtained from a simulation technique [6]. A slightly different presentation is given here for purposes of clarifying the procedure as applied to case of shot noise, and to give an indication that for long-period systems the probability of failure may not be severely affected by considering unfiltered motions.

The solution of Eq. (5) at time $t = t_o + \Delta t$ when $U(t_o) = U_o$ and $\dot{U}(t_o) = \dot{U}_o$ may be written as,

$$U(t_o + \Delta t) = U_o h_1(\Delta t) + \dot{U}_o h(\Delta t) + Y_1 \quad (10a)$$

$$\dot{U}(t_o + \Delta t) = U_o \dot{h}_1(\Delta t) + \dot{U}_o \dot{h}(\Delta t) + Y_2 \quad (10b)$$

where, a dot over a quantity implies differentiation with respect to time,

$$h(t) = \frac{e^{-\beta\omega_o t}}{\omega_d} \sin \omega_d t, \quad (11a)$$

$$h_1(t) = e^{-\beta\omega_o t} (\cos \omega_d t + \frac{\beta\omega_o}{\omega_d} \sin \omega_d t), \quad (11b)$$

$$Y_1 = -\int_0^{\Delta t} h(\xi) S(t_o + \Delta t - \xi) d\xi, \quad (11c)$$

$$Y_2 = -\int_0^{\Delta t} \dot{h}(\xi) S(t_o + \Delta t - \xi) d\xi, \quad (11d)$$

and

$$\omega_d = \omega_o \sqrt{1 - \beta^2} \quad (11e)$$

The Gaussian S(t) makes Y_1, Y_2 jointly Gaussian; however U_o and $\dot{U}_o$ are not Gaussian. These random variables have zero mean. Denoting the mathematical expectation by E the pertinent statistics are

$$E(Y_1^2) = \int_0^{\Delta t} \int_0^{\Delta t} h(\xi_1) h(\xi_2) E[S(t_o + \Delta t - \xi_1) S(t_o + \Delta t - \xi_2)] d\xi_2 d\xi_1 \quad (12)$$

Using Eq. (1b) this yields

$$E(Y_1^2) = \int_0^{\Delta t} h^2(\xi) I(t_o + \Delta t - \xi) d\xi \quad (13a)$$

Similarly

$$E(Y_2^2) = \int_0^{\Delta t} \dot{h}^2(\xi) I(t_o + \Delta t - \xi) d\xi \quad (13b)$$

$$E(Y_1 Y_2) = \int_0^{\Delta t} h(\xi) \dot{h}(\xi) I(t_o + \Delta t - \xi) d\xi \quad (13c)$$

The correlation coefficient

$$\rho = \frac{E(Y_1 Y_2)}{\sqrt{E(Y_1^2)E(Y_2^2)}} \tag{14}$$

and the joint density is

$$g_{Y_1,Y_2}(y_1,y_2) = \frac{1}{2\pi\sqrt{E(Y_1^2)E(Y_2^2)(1-\rho^2)}} e^{-L(y_1,y_2)} \tag{15}$$

where

$$L(y_1,y_2) = \frac{1}{2(1-\rho^2)}\left[\frac{y_1^2}{E(Y_1^2)} - 2\rho\left(\frac{y_1}{\sqrt{E(Y_1^2)}}\right)\left(\frac{y_2}{\sqrt{E(Y_2^2)}}\right) + \frac{y_2^2}{E(Y_2^2)}\right]$$

Because $S(t)$ for $t < t_o$ is independent of $S(t)$ in the interval $t_o < t < t_o + \Delta t$ the random vectors $\{U_o, \dot{U}_o\}$ and $\{Y_1, Y_2\}$ are also independent.

It is of interest to note here how any correlation in the excitation caused, for example, by filtering $S(t)$ will affect the foregoing analysis. There will be two effects. In the first place, the statistics of Y_1 and Y_2 as given by Eqs. (13a) through (13c) will be affected. Define τ as the value of τ at which the first zero of the correlation coefficient of the filtered model occurs, as shown in Fig. 1. It is seen that $\tau_c \approx 0.08$ sec. This quantity is a measure of the sharpness with which the correlation in the excitation drops. If $\Delta t > \tau_c$ and the natural period of the lightly damped system is long in comparison with τ_c, i.e. $T_o \gg \tau_c$, in the first integral of Eq. (12) the expectation will appear as a delta function in comparison to the function h and the statistics computed on the basis of an uncorrelated input will be close to those of a correlated input.

The second effect of correlation in the input is to make the vectors $\{U_o, \dot{U}_o\}$ and $\{Y_1, Y_2\}$ dependent. This dependence comes about because the excitation for times $t_o < t < t_o + \Delta t$ and $t < t_o$ are no longer independent. The dependence between these vectors will decrease the longer is Δt in comparison to τ_c. However, in order to avoid overlooking the possibility of system failure in the times $t_o < t < t_o + \Delta t$, the time step Δt should be small in comparison to the natural period of the system. An indication of the smallness required of Δt is given in Table 1. This table shows how P_f (25) varies with $\Delta t/T_o$ for specific system subjected to Input 2. These considerations therefore indicate that if $T_o \gg \tau_c$ the vectors $\{Y_1, Y_2\}$ and $\{U_o, \dot{U}_o\}$ will not be highly correlated.

On the basis of the above observations it is concluded that if $T_o \gg \tau_c$ the elements affecting the computation of the failure probability will not be seriously affected by considering the excitation as unfiltered. With one exception, only systems with periods of 2 sec. or longer have been considered.

Continuing with the description of the numerical procedure, let D denote the safe domain in the phase space. For uncorrelated S(t) the conditional density of $\{U, \dot{U}\}$ given that $\{U_o = u_o, \dot{U} = \dot{u}_o: (u_o, \dot{u}_o)$ are in D$\}$ is

$$g(u, \dot{u} | u_o, \dot{u}_o) = g_{Y_1, Y_2}(u - a, \dot{u} - b) \tag{16}$$

where

$$\begin{Bmatrix} a \\ b \end{Bmatrix} = \begin{bmatrix} h_1(\Delta t) & h(\Delta t) \\ \dot{h}_1(\Delta t) & \dot{h}(\Delta t) \end{bmatrix} \begin{Bmatrix} u_o \\ \dot{u}_o \end{Bmatrix} \tag{17}$$

In the numerical procedure the domain D in the phase space is discretized into neighboring rectangular regions $(\Delta u) \cdot (\Delta \dot{u})$ and each rectangle is identified by the coordinates at its center. If

$$p_{ij}(\ell, m) = \text{Prob.} \left\{ U = u_i, \dot{U} = \dot{u}_j \mid U_o = u_{o\ell}, \dot{U}_o = \dot{u}_{om} \right\} \tag{18}$$

this conditional probability is evaluated from

$$P_{ij}(\ell, m) = \iint g(u, \dot{u} \mid u_{o\ell}, \dot{u}_{om})\, du d\dot{u} \tag{19}$$

where the integration is from $\left\{ u_i - \frac{\Delta u}{2}, \dot{u}_j - \frac{\Delta \dot{u}}{2} \right\}$ to $\left\{ u_i + \frac{\Delta u}{2}, \dot{u}_j + \frac{\Delta \dot{u}}{2} \right\}$.

Now denoting the unconditional probabilities as,

$$P_{t_o + \Delta t}(i, j) = \text{Prob.} \left\{ U = u_i, \dot{U} = \dot{u}_j \right\}$$

and,

$$P_{t_o}(\ell, m) = \text{Prob.} \left\{ U_o = u_{o\ell}, \dot{U}_o = \dot{u}_{om} \right\}$$

the theorem of total probability gives

$$P_{t_o + \Delta t}(i,j) = \Sigma_{\ell,m}\Sigma p_{ij}(\ell,m)\, p_{t_o}(\ell,m) \tag{20}$$

The probability of safety at $t_o + \Delta t$ is evaluated from

$$P_s(t_o + \Delta t) = \Sigma_{i,j}\Sigma p_{t_o + \Delta t}(i,j) \tag{21}$$

For problems with zero initial conditions

$$P_o(0,0) = 1;\ p_o(\ell,m) = 0 \text{ for } \ell,m \neq 0 \text{ and } P_s(0) = 1$$

For problems with prescribed probabilistic initial conditions $p_o(\ell,m)$ and $P_s(0)$ can also be appropriately determined.

For the results to be presented the time step $\Delta t = T_o/8$ and on the basis of results in [5] the following values have been used for discretization of the phase space

$$\frac{\Delta u}{\sqrt{E(Y_1^2)}} = 0.653 \text{ to } 0.676$$

$$\frac{\Delta \dot{u}}{\sqrt{E(Y_2^2)}} = 0.521 \text{ to } 0.55$$

in which expectations are for Input 2. Also domain D in phase space is unbounded along the $\dot{u}/\omega_o$-axis. This problem was dealt with by considering a high barrier in that direction so that probability of exceedence is several orders of magnitude less than the corresponding probability in direction of u-axis.

PRESENTATION AND DISCUSSION OF RESULTS

Effects of Nonstationarity

In Fig. 7 are compared the variations of failure probability with barrier level for a system with natural period of 3 sec., β = 0.0: and the two intensity functions shown in Fig. 2. The duration of excitation in both cases is 25 sec. Clearly the nonstationarity in the input affects the failure probability. However, the differences in the failure probabilities corresponding to the two types of input are not significant.

For a linear system subjected to a Gaussian excitation a probability that can readily be computed from the knowledge of the variance function alone is the probability that the response U(t) will exceed a specified barrier level at time t. The maximum probability of instantaneous exceedence is always smaller than the probability of failure. This probability which occurs when the variance is highest has also been computed and shown in Fig. 7.

For two systems having the same damping and barrier level, α, but different frequencies and subjected to the Gaussian white noise, the probabilities of failure at the end of equal number of cycles of oscillation will be the same. This is so because the statistics are functions of the product $\omega_o t$. When white excitation is of finite duration the system with longer period will have smaller probability of failure, because it will have lesser number of cycles of oscillation during the excitation and, therefore, the oscillator will have a proportionately less chance of going into the unsafe domain of the phase space. Fig. 8a shows this effect for systems with β = .02 and two different periods subjected to Input 2. Similar information is shown in Fig. 8b for Input 1.

The practical significance of nonstationarity for earthquakes of the type considered here can also be examined by comparing the barrier levels required for a prescribed failure probability. This information is given in Table 2 for a failure probability of 5% for linear systems with β = 0.02. The maximum difference in this Table for the periods considered is 12%.

It is well known that the response of a linear system to a stationary input is nonstationary. The response approaches a stationary level after several cycles of oscillation. The time required to reach stationarity depends on the level of damping in the system; the higher the damping the earlier will the response become stationary. In view of the above observation that nonstationarity in the input does not affect significantly the failure probability of a linear system, it is of interest to know if the system response itself can be treated as stationary over the duration of an earthquake. This is of special interest in the case of Gaussian inputs and failures at high response levels, because of the availability of theoretical information relative to the extremes of stationary Gaussian processes [7,8]. In Table 3 are summarized the values of failure probabilities for Inputs 1 and 2 and a system having natural period of 2 sec. The barrier is $\pm 3\sigma_o$. The response for Input 1 starts from the rest position. For Input 2 in one case the system has zero initial conditions and in the other case called stationary start it is assumed that the response has reached a stationary level at t = 0 and therefore is set in motion with probabilistic initial conditions having stationary Gaussian statistics. For the two damping values considered, the failure probabilities obtained by assuming stationary response are on the safe side and are of the same order of magnitude as the other probabilities (for nonstationary response). This indicates that under some conditions the extremes of stationary Gaussian processes might be of relevance to the random response of systems to earthquakes of the type considered.

RESPONSE OF ELASTO-PLASTIC SYSTEMS

The deformation spectra for elasto-plastic systems subjected to a stationary and a comparable nonstationary pseudo-earthquake are presented in Fig. 9. In the resistance diagrams of these systems yield levels in the two directions of deformation are the same, and unloading from a point of maximum deformation takes place along a line parallel to the initial elastic portion of the diagram. The frequency f_o refers to that determined on the basis of the initial stiffness. The ductility factor is defined as

$$\mu = \frac{u_m}{u_y}$$

in which u_m = maximum deformation for a prescribed excitation and u_y = yield deformation. For an elastic system $\mu = 1$.

As it has been pointed out in [9], one convenient way of presenting results for elastoplastic systems which is useful in design, is to plot pseudo-velocity $\omega_o u_y$ vs. f_o for prescribed values of the ductility factor. In this manner the yield deformation which is necessary to limit the maximum deformation to a specified multiple of the yield deformation can be determined. Fig. 9 is such a plot.

It is seen from the spectra presented in Fig. 9 that for an elastic system the difference between the responses produced by a stationary and a comparable nonstationary pseudo-earthquake is negligible. For the elasto-plastic systems considered herein the differences are also small; however, the effect of nonstationarity appears to increase with increasing ductility factor. Therefore, for some nonlinear-inelastic systems the effects of nonstationarity may not be negligible.

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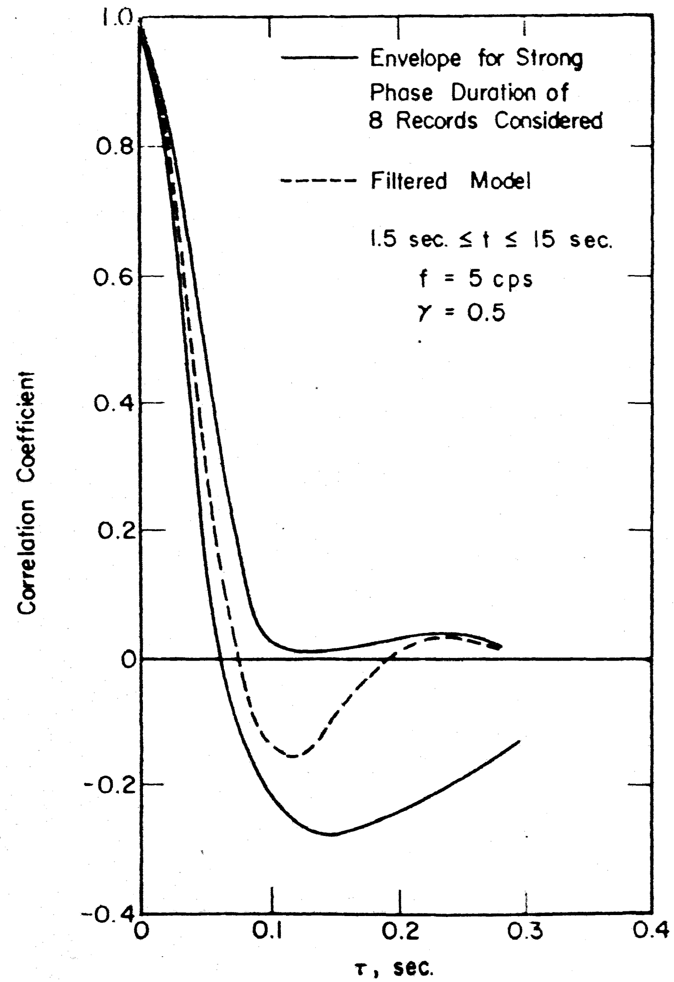

FIG. 1 CORRELATION COEFFICIENT OF GROUND ACCELERATION

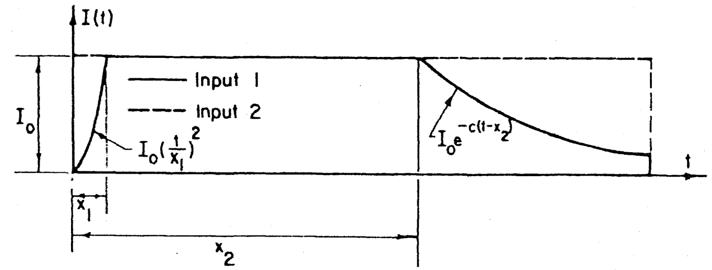

FIG. 2 INTENSITY FUNCTIONS CONSIDERED

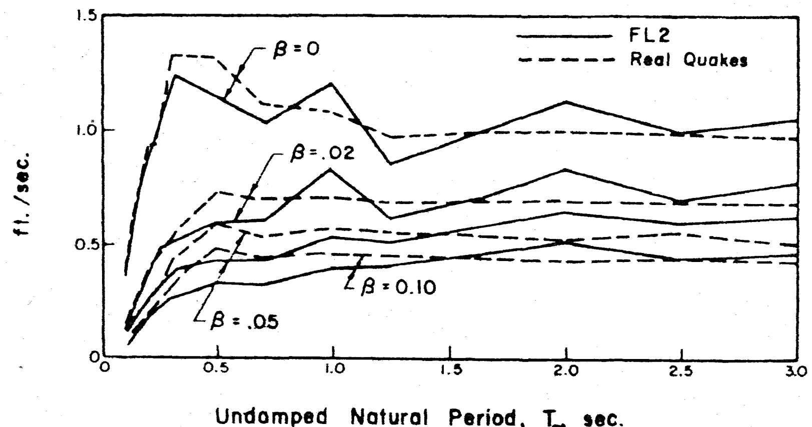

FIG. 3 COMPARISON OF AVERAGE DEFORMATION SPECTRA OF NONSTATIONARY FILTERED MODEL AND REAL EARTHQUAKES

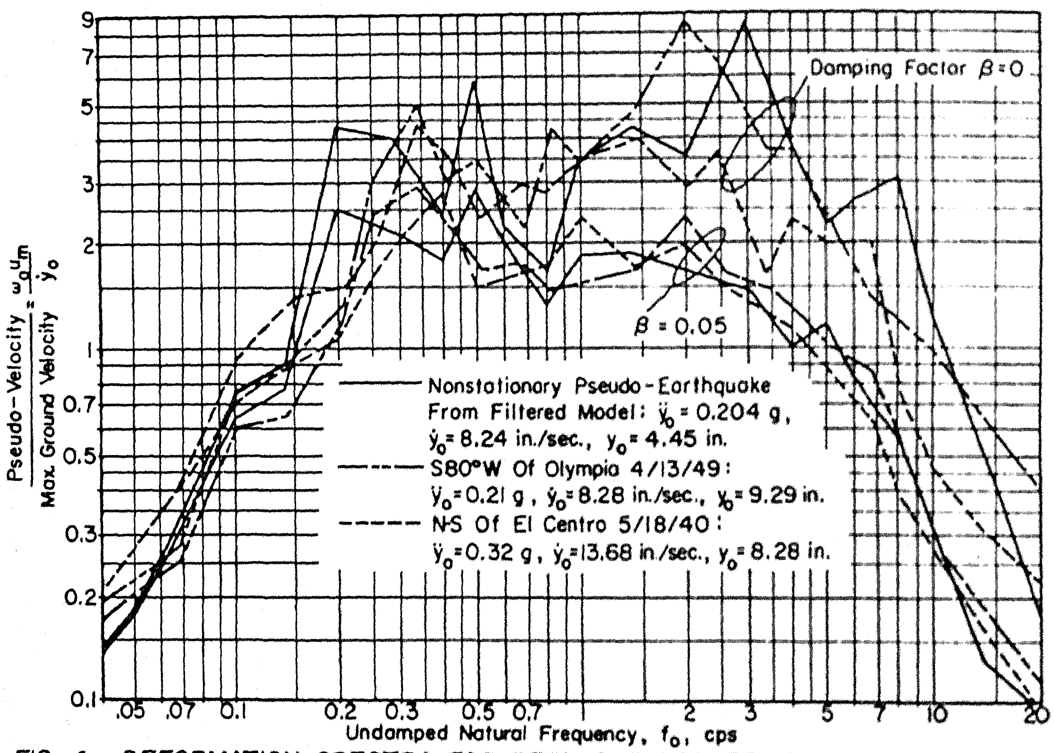

FIG. 4 DEFORMATION SPECTRA FOR REAL AND PSEUDO-EARTHQUAKES (LINEAR SYSTEMS)

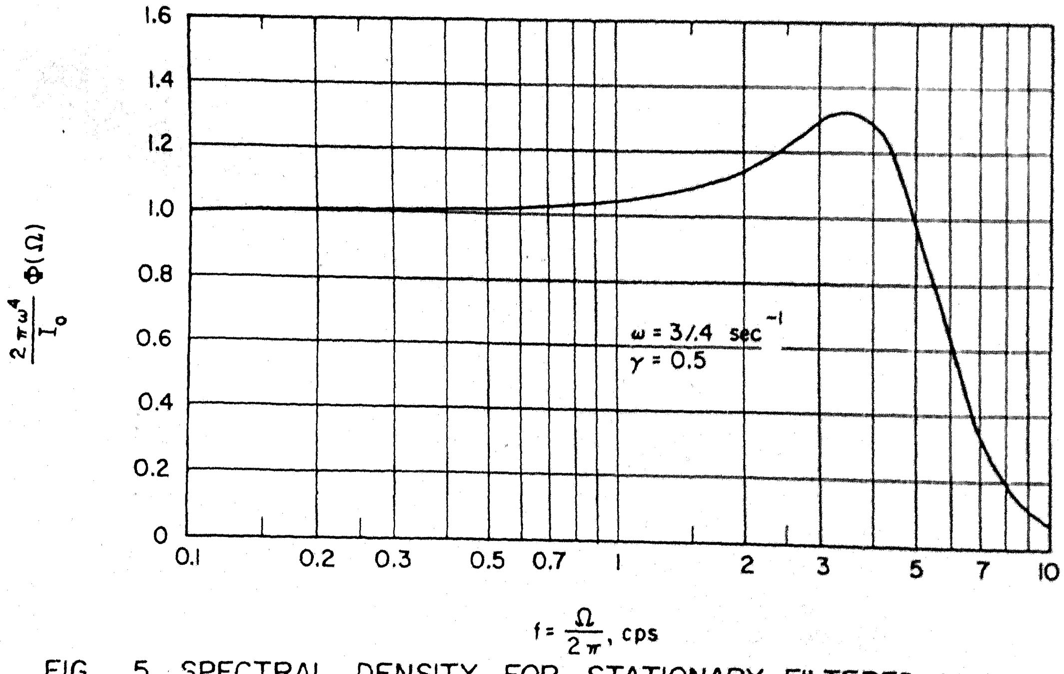

FIG. 5 SPECTRAL DENSITY FOR STATIONARY FILTERED MODEL

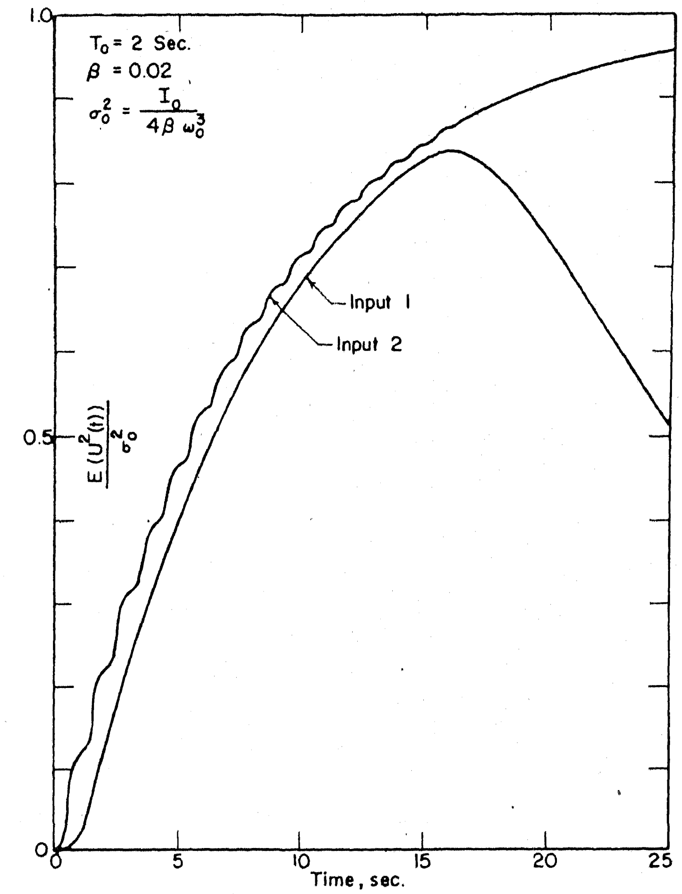

FIG. 6 VARIANCE FUNCTIONS OF SYSTEM RESPONSE TO UNCORRELATED INPUTS 1 AND 2

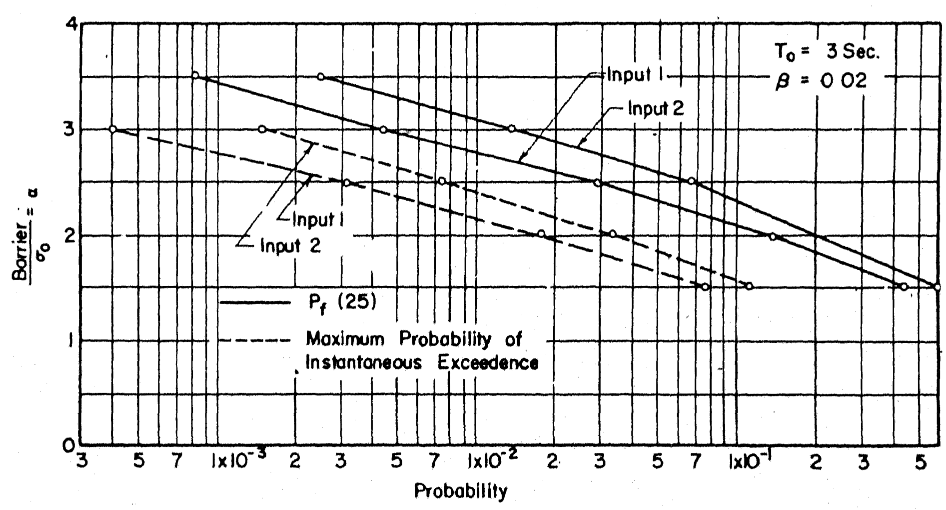

FIG. 7 FAILURE PROBABILITIES FOR INTENSITY FUNCTIONS CONSIDERED

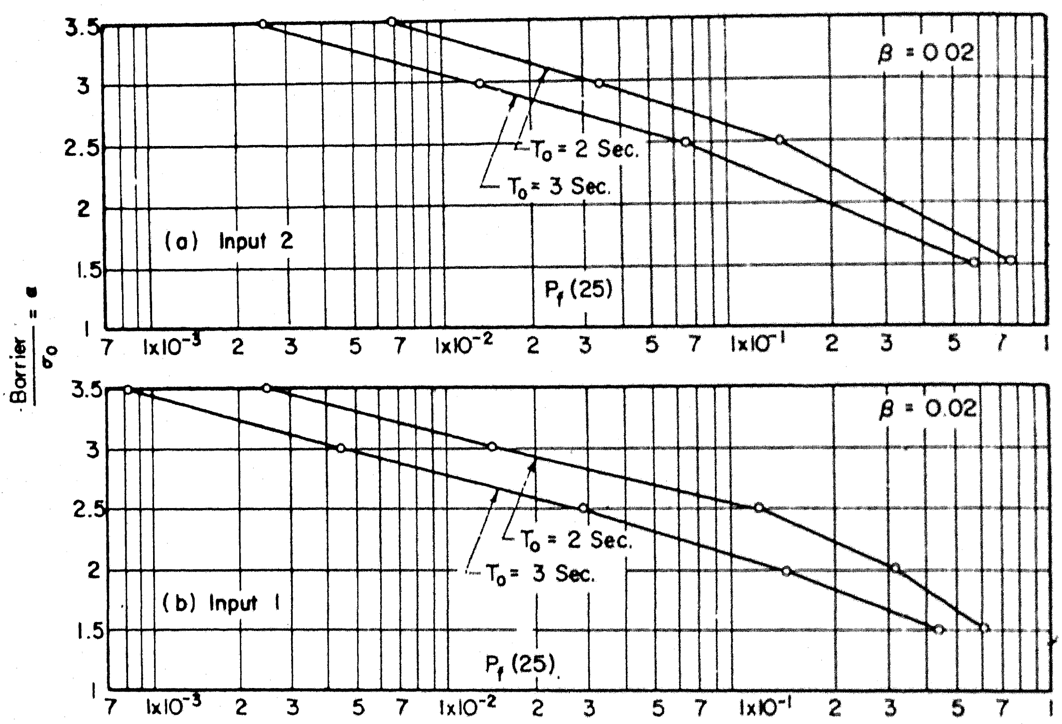

FIG. 8 FAILURE PROBABILITIES OF LINEAR SYSTEMS

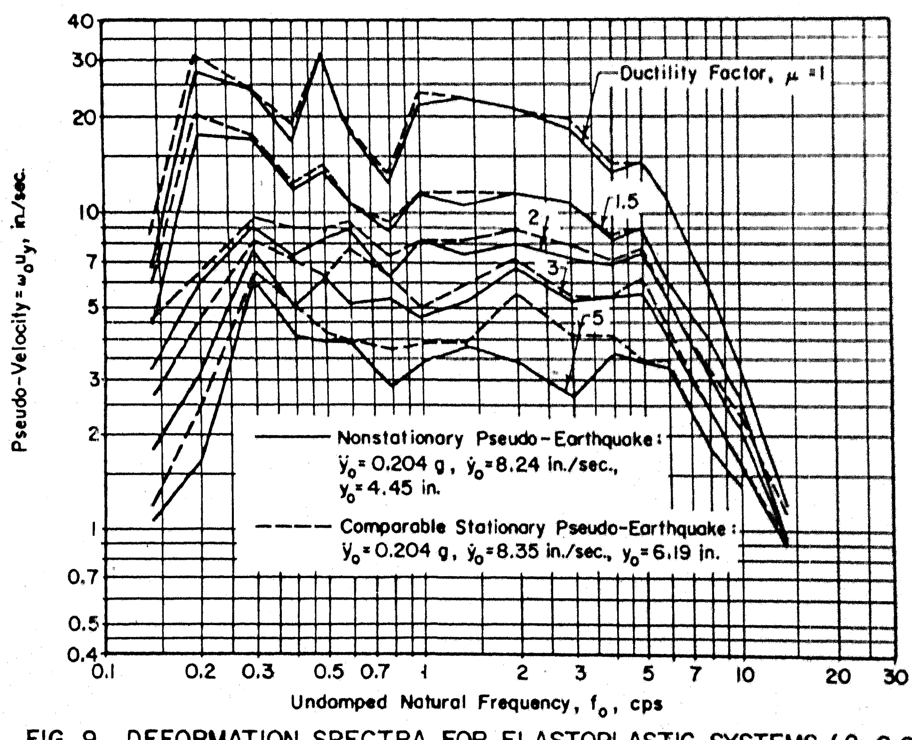

FIG. 9 DEFORMATION SPECTRA FOR ELASTOPLASTIC SYSTEMS (β=0.02)

TABLE 1 - EFFECT OF $\frac{\Delta t}{T_o}$ on $P_f(25)$

Elastic Systems with T_o = 2 sec., β = .08

Input 2, $\frac{B}{\sigma_o}$ = 3

$\frac{\Delta t}{T_o}$	$P_f(25)$
1/4	0.08376
1/8	0.12848
1/12	0.14363
1/16	0.14961

TABLE 2 - BARRIER LEVELS FOR $P_f(25)$ = 5%

Elastic Systems with β = .02

T_o Sec.	$\alpha_{.05} = \frac{B}{\sigma_o}$ Input 1	Input 2	Percent Difference
1	3.00	3.25	8.3
2	2.70	2.85	5.6
3	2.32	2.60	12.0

TABLE 3 - COMPARISON OF FAILURE PROBABILITIES

Linear Systems T_o = 2 Sec.

$\frac{B}{\sigma_o}$ = 3

Input	Initial Conditions	$P_f(25)$ β = .02	β = .08
1	0	0.0140	0.0755
2	0	0.0335	0.128
2	Stationary Start	0.0791	0.155