# AN EARTHQUAKE RISK MAP OF CHILE

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## ABSTRACT

The extrapolation of magnitude distributions beyond the time interval on which they are based leads to serious underestimation of the frequency of large earthquakes. An alternate method of estimating earthquake risk is proposed; this method makes use of the Bayesian approach. Given a prior distribution of earthquake risk, stepwise improvements of this distribution are obtained through the incorporation of seismic data as they become available. An initial earthquake risk estimate is derived from a model based on historical data. Application of this method to Chile yields a prior estimate of earthquake risk distribution as derived from a list of large historical earthquakes for the period 1535-1967. The model assumes the contribution of shocks less than magnitude 7.5 to the incidence of damaging accelerations to be negligible. The seismotectonic structure and seismicity features of Chile are discussed, and seven source areas of major historical shocks are defined. The map of earthquake risk obtained by this method may be used as base map in the procedure of Bayesian iteration, for the purpose of perfecting the estimate of earthquake risk distribution in Chile

## 1. Introduction.

Let

$$G_i(M \geqslant x) = 1 - e^{-b_i x} \qquad (x \geqslant 0) \qquad (1)$$

be the cumulative distribution function of earthquake magnitudes M at a location i. It is easy to show that the mean magnitude $\bar{M} = 1/b_i$. In other words, if the probability density function

$$g_i(M=x) = dG_i/dx = b_i\ e^{-b_i x} \qquad (2)$$

is represented as a straight line on semi-logarithmic paper the reciprocal slope equals the mean magnitude (considering positive magnitudes only). Up to this point we have used the traditional approach first given by Gutenberg and Richter(1), and later modified by other authors(2).

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The problem of estimating the exponent $b_i$ in equation (1) is to be equated to the problem of estimating the mean of an exponential distribution. But it is well known that the sample mean is a biased estimator of the true mean when the distribution is skewed. Since the exponential distribution always has a zero mode it follows that estimating the value of $b_i$ from eq. (2) by least-square regression will yield a biased result. Specifically, $b_i$ will be overestimated and the extrapolation of the seismicity thus estimated will lead to expected magnitudes which are consistently low and therefore unsafe. The smaller the sample used in the determination of $b_i$, the more serious is the error.

Consider now the magnitude distribution for the pooled data within a large geographic area, say Chile. If $p_i$ is the proportion or weight of the seismicity at location i we may write

$$F(M \geqslant x) = 1 - \sum_i p_i \, e^{-b_i x} \qquad (3)$$

The form of (3) depends on the unknown joint distribution of $(b_i, p_i)$. In general it is assumed, however, that one may fit the magnitude distribution at <u>any</u> level of geographical complexity to an exponential distribution:

$$F(M \geqslant x) = 1 - e^{-bx} \qquad (4)$$

It is also assumed that the value of b found from least-square fitting of such heterogeneous data is in some way representative of the region as a whole.

Epstein and Lomnitz(2) have proposed an alternate method of estimating b which largely avoids the bias by using extreme-value theory. But the effect of pooling data from different localities remains serious: the low values of $b_i$ are smoothed out, since they give rise to a smaller number of earthquakes per unit energy release.

Thus, no matter what data range is used the sample frequencies for the largest magnitudes invariably fall below the frequencies expected from eq.(4). The poor fit of this equation for large magnitudes has been noted by some authors(3).

## 2. The Bayesian approach.

The above discussion underscores the danger inherent in extrapolating the magnitude distribution beyond the time interval of the data on which it is based. While obvious to a statistician this has not always been evident to seismologists.

The only tenable alternative consists in finding a method for evaluating the full historical record of large earthquakes beyond the 50-odd years of instrumental record which are available. Until now this was not deemed possible, since data from historical earthquakes lack the precision of epicenter and magnitude estimation that one is accustomed to expect in statistical applications. This objection can now be overcome through the use of Bayesian statistics(3).

Let $R_i$ be the earthquake risk, i.e. the probability of at least one seismic event to occur in a unit time interval at a given locality i(4,5,6). If the basic process at each locality is assumed to be Poisson the expected number of events per unit time at the locality is

$$K_i = -\log(1 - R_i) \tag{5}$$

Let $X_i$ be the observed number of events in a sample unit time interval. We wish to utilize this observation to refine our estimate of $K_i$, and hence of $R_i$. Let $h_n(K_i)$ be the prior distribution of $K_i$, i.e. the probability assigned to the hypothesis that $K_i$ is the correct value of the mean number of events per unit time. Then the likelihood of observing $X_i$ events given the assumption that $h_n(K_i)$ is the correct hypothesis is

$$\mathrm{prob}(X_i \mid K_i) = K_i^{X_i} \exp(-K_i)/X_i! \tag{6}$$

on the Poisson assumption. Application of Bayes' Theorem now leads to the recurrence equation

$$h_{n+1}(K_i) = \frac{h_n(K_i)\ \mathrm{prob}(X_i \mid K_i)}{\int_0^\infty h_n(K_i)\ \mathrm{prob}(X_i \mid K_i)\ dK_i} \tag{7}$$

where $h_{n+1}(K_i)$ is the new distribution of $K_i$ after the nth iteration.

Equation (7) remains valid if we introduce the change of variable defined by eq.(5). If $f_n(R_i)$ and $f_{n+1}(R_i)$ are the prior and posterior distributions of the earthquake risk $R_i$ corresponding to $h_n$ and $h_{n+1}$, we obtain

$$f_{n+1}(R_i) = \frac{f_n(R_i)\ \mathrm{prob}(X_i \mid R_i)}{\int_0^\infty f_n(R_i)\ \mathrm{prob}(X_i \mid R_i)\ dR_i} \tag{8}$$

where

$$\mathrm{prob}(X_i \mid R_i) = -(1-R_i)'\left[\log(1-R_i)\right]^{X_i} / X_i! \tag{9}$$

according to eqs.(5) and (6).

At any given level of iteration the most likely estimate of $R_i$ may be obtained from

$$E\left[R_i\right] = \int_0^\infty R_i\ f(R_i)\ dR_i \tag{10}$$

Thus, an improved estimate of earthquake risk at a locality may be obtained by iterated application of Bayes' Theorem using the seismic data as they become available. This approach is particularly advantageous when the total interval of record is short. Of course, as the span of observations shrinks to the order of magnitude of the interoccurrence time between events a proper selection of the initial trial distribution $f_0(R_i)$ becomes of crucial importance. Only when this initial distribution (or at least its mean value) represent a close estimate of the long-term earthquake risk distribution will the Bayesian procedure converge rapidly toward a realistic value of the risk $R_i$.

For the purpose of this paper a "seismic event" will be defined as the occurrence of a threshold acceleration $A \geqslant 0.1g$ in the horizontal direction at a locality. By changing the value of A one may obtain maps of earthquake risk at various levels of acceleration. Such maps are in effect different sections through the three-dimensional field $R(\lambda, \varphi, A)$ where $\lambda$ is the longitude, $\varphi$ is the latitude, and A is the threshold acceleration.

The remainder of this paper is devoted to the estimation of $f_0(R_i)$, the trial distribution of earthquake risk, on the basis of the historical record of destructive earthquakes in Chile.

## 3. Seismicity of Chile.

In the early Seventeenth Century it was commonly believed that the seismic risk on the American Continent was minimal, "for earth-quakes are seldome in those Parts"(7). Two centuries later Perrey(8) informs us that "the Atacama Desert has a low seismicity". Today we know that Chile has, mile for mile, one of the highest rates of seismicity in the world. The relatively low estimate of earthquake risk in Chile is a well-known result of lack of information common in underpopulated areas, as Montessus de Ballore(9) has pointed out.

Seismic activity in Chile may be attributed to large-scale crustal movements. There is geophysical evidence that the South American continent is drifting westward against the oceanic crust, while the Southeast Pacific basin is simultaneously spreading against the Chilean coast at the rate of 5-6 cm/year (10,11). The Chile Rise, a probable transform fault, represents the southern boundary of this process of buckling and underthrusting of the continental border (fig.2).

At any given period in geologic time it may be assumed that the motion of these plates and platforms was fairly uniform. It seems questionable whether the attempt to establish shades and gradations of seismic risk within the territory of Chile has any fundamental justification. This matter was taken up in an earlier paper(4), where we concluded that seismic zoning as such may be "impractical because all of Chile north of the 42nd Parallel has been within the epicentral area of some destructive earthquake".

Yet the distribution of seismicity within Chile is far from uniform. While zoning as such may be impractical a number of other engineering applications require the estimation of earthquake risk at specific localities in Chile. Gajardo and Lomnitz (12) proposed four major seismic divisions of Chile; to some extent these divisions correspond to successive tectonic units from north to south. This classification has been adopted by Zeil(13) and other geologists. In extreme cases an entire seismic division may be activated and tectonically deformed, as in 1960(14), while the adjoining divisions remain quiescent.

## 4. Great Chilean earthquakes.

Table 1 represents a list of major Chilean shocks. This list was collated from a catalog of Chilean earthquakes which includes 15,000 separate entries over the period 1535-1967(15). Only the largest events have been included; several locally damaging shocks were omitted. Earthquakes of known or inferred magnitudes below 7.5 have not been listed.

Widespread damage in Chile begins at a relatively high magnitude level. This is perhaps due to the fact that many active epicenters are located offshore at distances of 40 to 150 km from the coast. Since the radius of large ground accelerations (0.1g+) is about 100 km for an earthquake of magnitude 7.5 it follows that the probability of a Chilean earthquake to cause damage decays rapidly as the magnitude falls below 7.5.

Isoseismal lines in Chilean earthquakes are elongated in the north-south direction. Along any east-west traverse the distribution of intensities tends to be fairly uniform from the shoreline to the foothills of the Andes. Intensities in the Andes decay rapidly, except for pockets of recent sediments in lakes and river valleys.

Some of the larger historical events are practically repetitions of each other. Some examples are the great Southern Chile earthquakes of 1575 and 1960, or the Arica earthquakes of 1604 and 1868. In general, it appears possible to classify the sources of major activity according to not more than seven regions (Table 2).

## 5. Computation of earthquake risk.

The above discussion of major Chilean earthquakes suggests a model of earthquake risk which may be useful to generate an initial estimate for a Bayesian iteration procedure. The proposed assumptions underlying such a model are the following:

(a) Magnitudes below 7.5 do not contribute significantly to the earthquake risk, and may be neglected;

(b) Each individual source is assumed to generate major shocks according to a simple Poisson process in time;

(c) The area of intensity 0.1g and above is assumed to be enclosed by isoseismal VI;

(d) Within this area the contribution to earthquake risk is uniform, as the risk depends only on the number of exceedances of 0.1g and not on the level of the exceedance. This is not a new assumption but rather a consequence of the definition of earthquake risk.

With respect to assumption (c) it should be noted that the

Chilean intensity ratings are consistently lower than ratings in California. A difference of a full step of the Mercalli intensity scale is not uncommon. This discrepancy in assigning intensities may be due to divergent interpretations of what intensity is characteristic of a locality. Whenever several reports are available from the same locality California practice tends to prefer the highest-ranking Mercalli level observed, whereas Chilean practice prefers some representative average level.

Under the model just outlined the earthquake risk is obtained from a simple count of the number of exceedances of intensity VI at each locality. If $N_i$ is the number of events at location i during a period of observation of D time units, the initial estimate of the mean number of events $K_i$ is obtained as

$$K_i = N_i/D \qquad (11)$$

and the estimate of earthquake risk is given by eq.(5):

$$R_i = 1 - e^{-N_i/D} \qquad (12)$$

It does not matter that the number of exceedances $N_i$ contains contributions from different sources. Since all sources are simple Poisson processes it may be assumed that the process at any locality i is also a Poisson process. None of these assumptions is likely to introduce an error as serious as the shortcomings of the historical record.

The actual estimation of intensities from historical descriptions was greatly facilitated by the use of unpublished isoseismal maps compiled by F. Greve between 1942-1958. Also, the published or otherwise available maps of the earthquakes of 1906, 1922, 1939, and 1960 were found useful. In cases of extreme lack of information the reported size of the meizoseismal area provided a scaling factor for inferring the extent of the Intensity VI isoseismal.

Example: The town of La Serena (latitude 30°) is potentially included in the range of influence of source areas 2, 3, and 4. The number of major shocks in source area 2 was five; of these, three events included La Serena in their Intensity VI isoseismal line. Similarly, one shock from region 3 and two shocks from region 4 also affected La Serena destructively. This gives a total of $N_i = 6$ events for a period of 432 years.

Now, in order to compute the earthquake risk for La Serena in a unit period of 30 years we have D = 432/30, and

$$R_i = 1\text{-}\exp(-6*30/432) = 0.34 \quad \text{or } 34\% \ .$$

## 6. Conclusions.

Figure 2 shows the completed map of earthquake risk as obtained by the above procedure. As might be expected, the earthquake risk contours are roughly parallel to the isoseismal lines. All risk figures are referred to a design period of 30 years;

this interval is of significance in connection with housing insurance and financing.

Some relevant differences between maps of earthquake risk and maps of seismicity are worth pointing out. It is known, for example, that the seismicity of La Serena is fairly low(12); yet the earthquake risk from figure 2 is high. As we saw from the preceding section, La Serena is within destructive range from three different source areas: Caldera-Huasco, Central-Offshore, and Aconcagua-Santiago. Hence the probability of exceedance of the base acceleration of 0.1g turns out to be higher than, say, for the town of Copiapó which has the highest incidence of felt earthquakes in Chile. On the other hand, Valparaíso has been subjected to much greater peak accelerations than has La Serena; but the risk of occurrence of a minimum destructive acceleration is not much higher. This example clarifies the use of earthquake risk in engineering design. In many applications it is required to estimate the likelihood that damaging accelerations will occur within the useful life of a structure. The actual value of the peak accelerations which have occurred in the past is relatively irrelevant in these cases, though it may be of importance in other problems (design of nuclear reactors, for example).

In order to use the information of fig. 2 as initial input for the iterative procedure outlined above, the form of the prior distribution $f_0$ needs to be assumed. In some processes the prior distribution of non-negative variables may be successfully approximated by lognormal or gamma distributions(16). The variance may be made to vary according to the amount of information available for each locality. The mean, of course, will be assumed to be equal to $R_i$, the initial earthquake risk from figure 2.

## 7. Acknowledgments.

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## Table 1

## Major Chilean Earthquakes, 1535-1967

| # | Date | Source area | Probable Magnitude | Observations |
|---|---|---|---|---|
| 1 | 1570 Feb 8 | 6 | 8 to 8 1/2 | Concepción.Great tsunami. |
| 2 | 1575 Mar 17 | 4 | 7 1/2 | About 100 Km from Santiago. |
| 3 | 1575 Dec 16 | 7 | 8 1/2 | Valdivia. Major tsunami. |
| 4 | 1604 Nov 24 | 1 | 8 1/2 | North of Arica. Major tsunami. |
| 5 | 1615 Sep 16 | 1 | 7 1/2 | Arica. |
| 6 | 1647 May 13 | 5 | 8 to 8 1/2 | Great Santiago Earthquake. |
| 7 | 1657 Mar 15 | 6 | 8 | Concepción, Tsunami. |
| 8 | 1730 Jul 8 | 3 | 8 1/2 | Great Valparaiso Earthquake. Major tsunami. |
| 9 | 1737 Dec 24 | 7 | 7 1/2 to 8 | Valdivia. |
| 10 | 1751 May 25 | 6 | 8 1/2 | Great Concepción, Earthquake. Major tsunami. |
| 11 | 1796 Mar 30 | 2 | 7 1/2 to 8 | Copiapó. |
| 12 | 1819 Apr 3 to 11 | 2 | 8 to 8 1/2 | Copiapó (3 earthquakes). Large tsunami. |
| 13 | 1822 Nov 19 | 3 | 8 to 8 1/2 | Valparaiso. Great tsunami. |
| 14 | 1835 Feb 20 | 6 | 8 to 8 1/2 | Concepción. Large geodetic movements. Major tsunami. |
| 15 | 1837 Nov 7 | 7 | 8 | Valdivia. Major tsunami. |
| 16 | 1859 Oct 5 | 2 | 7 1/2 to 7 3/4 | Copiapó. Tsunami. |
| 17 | 1868 Aug 13 | 1 | 8 1/2 | Great Arica Earthquake. Major tsunami. |
| 18 | 1877 May 9 | 1 | 8 | Iquique. Major tsunami. |
| 19 | 1880 Aug 15 | 4 | 7 1/2 to 8 | Illapel. |
| 20 | 1906 Aug 16 | 3 | 8.6 | Valparaiso. Tsunami. |
| 21 | 1918 Dec 18 | 2 | 7 1/2 | Copiapó. Tsunami. |
| 22 | 1922 Nov 10 | 2 | 8.4 | Great Vallenar Earthquake.Major tsunami. |
| 23 | 1928 Dec 1 | 5 | 8.4 | Talca. |
| 24 | 1939 Jan 24 | 5 | 8.3 | Great Chillán Earthquake. |
| 25 | 1943 Apr 6 | 4 | 8.3 | Illapel. |
| 26 | 1953 May 6 | 5 | 7 1/2 | Chillán. |
| 27 | 1960 May 21 | 6 | 7 1/2 | Concepción. |
| 28 | 1960 May 22 | 7 | 8 1/2 | Valdivia.Large geodetic movements.Major tsunami. |
| 29 | 1964 Mar 28 | 4 | 7 1/2 | La Ligua. |

## Table 2

## Source Areas for Chilean Destructive Earthquakes

| Area # | Description | Max. epicentral area (approx.) |
|---|---|---|
| 1 | North Chile - Offshore | Peru - 23° |
| 2 | Caldera - Huasco - Coastal | 26°- 30° |
| 3 | Central Chile - Offshore | 30°- 35° |
| 4 | Aconcagua - Santiago - Interior | 30°- 36° |
| 5 | Talca - Chillán - Central Valley | 34°- 38° |
| 6 | Concepción - Offshore | 34°- 39° |
| 7 | Valdivia - Chiloé - Offshore | 37°- 45° |

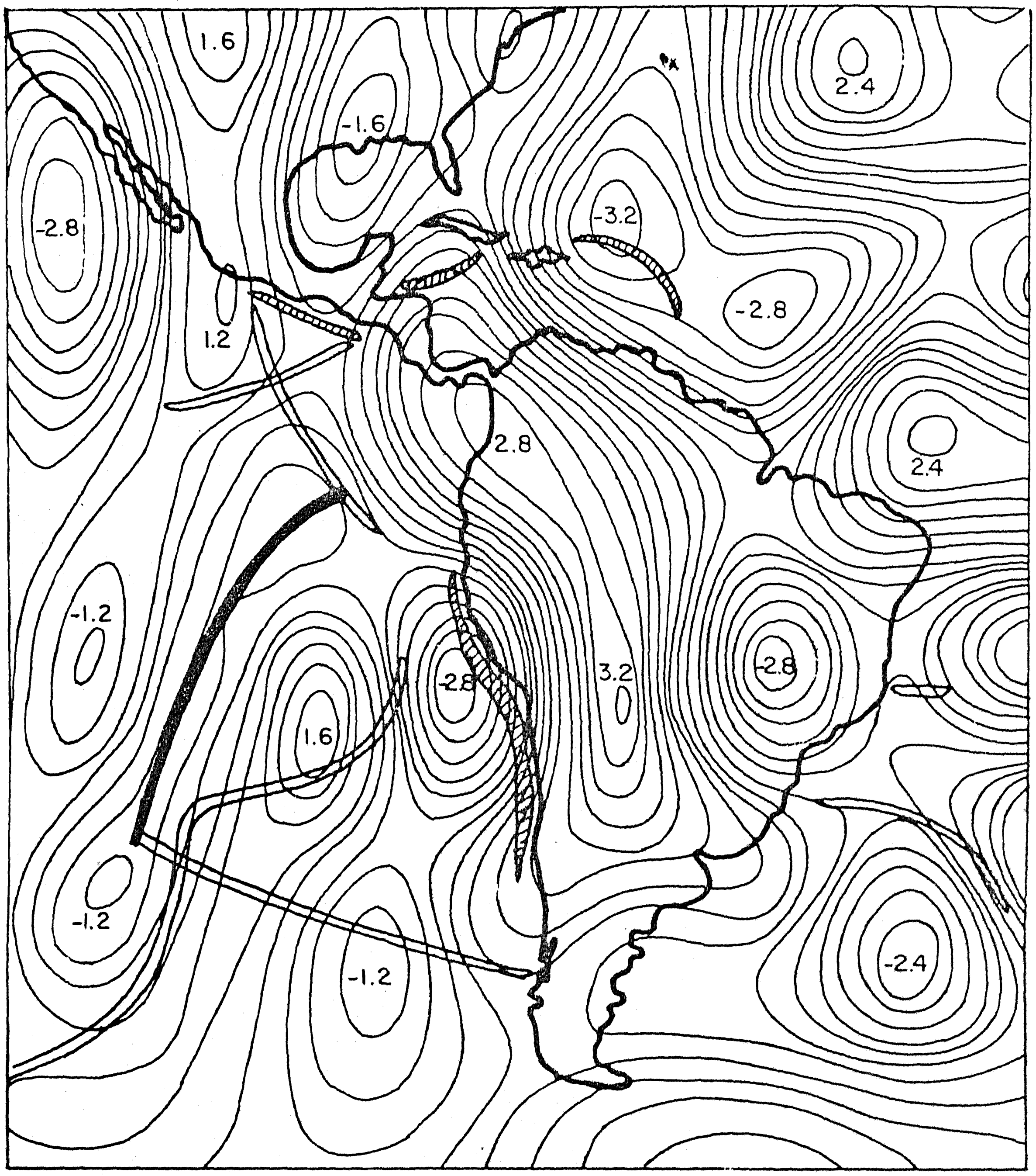

FIG. 1 Gravimetric anomalies from satellite observations, computed for the top of the upper mantle of the earth. Convection currents welling up under the East Pacific Rise (elongated negative anomaly at left) and converging under the Bolivian Altiplano (positive mid-continent maximum) may cause the floor of the Pacific Ocean to drift against the Chilean coast at a rate of 5 cm/year (after E. W. Schwiderski, J. Geoph. Res. 73, 2830, 1968).

FIG. 2 Earthquake Risk R in % for 30 years