STRUCTURAL RESPONSE TO NONSTATIONARY RANDOM EXCITATION

by

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SYNOPSIS

Earthquake motion is simulated by a nonstationary random process which is represented as a product of a deterministic function of time and a stationary random process with arbitrary power spectrum. The structure is replaced by a single degree of freedom system and its response to statistical excitation is treated as a problem of the threshold crossing. A response spectrum which contains the probabilistic quantity as a parameter is presented for evaluating the structural response under earthquake motion.

INTRODUCTION

There are two different approaches to the design of structures for seismic loadings. The first one consists in the investigation of the response of structures to earthquake records and the second is based on the probability method. Although accelerograms of the El Centro, May 18, 1940, the Taft, July 21, 1952 and so forth are frequently used as an input excitation to the numerical computation of a structural response, the use of the record of the ground movement of a certain earthquake is questionable for the analysis of response of structures on the ground with different dynamic properties. However, it seems reasonable that these strong motion accelerograms provide a character as a standard which can be used as common data among studies on structural response for the aseismic design of structures.

Since, on the other hand, the strong-motion earthquake records obtained in the past are known to have statistical properties and the response of structures to random excitations which are represented as the white noise or the train of random pulses is close to the response to actual earthquake records(1), the simulation of earthquakes by stochastic process has been carried out in the investigations of structural response(2-8). However, because the spectral composition of earthquake motion is influenced by the dynamic properties of ground, the simulation in which the frequency characteristics of ground is taken into consideration would be desirable. Then in this study the earthquake acceleration has been represented as a nonstationary random process which is described as a product of a nonstationary deterministic function and a stationary random process which has an arbitrary power spectrum. By this representation, the frequency characteristics of ground can be easily considered in the simulated earthquakes.

Usually the response spectrum is defined as the maximum value of response of the system. Therefore the other information which the response of

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structures provides does not contribute to the response spectrum and the information which is obtained from the structural response to a specific input is limited because of the statistical nature of earthquake records. The simulation of earthquake as a stochastic process enables us to introduce the probabilistic quantity as a parameter in the response spectrum.

SIMULATION OF EARTHQUAKE MOTION

Since the ground motion during earthquakes is a nonstationary phenomenon, the representation as a stochastic process is to be treated as a nonstationary stochastic process of which probabilistic quantity governing the spectral composition and duration time is dependent on time. However the record of an actual earthquake is, unfortunately, merely a member function from a point of view such that earthquake motion is a stochastic process. Then the variation of probabilistic quantity with respect to time can not be found out from an actual earthquake record. Because of these limitations, much is yet to be studied concerning the nonstationarity of earthquake records in connection with structural response.

In some references, the earthquake motion has been simulated by the nonstationary random process which is represented as a product of a nonstationary deterministic function of time and a stationary random process(5–8). This method is followed in this study also and hence the ground acceleration f(t) during earthquake is represented as a product of a deterministic function $\Psi(t)$ and a stationary random process $g(t)$ as follows;

$$f(t) = \Psi(t) \cdot g(t) \qquad (1)$$

where $\Psi(t)$ is a slowly varying function relative to the fluctuation of $g(t)$.

Herein the following representation of $g(t)$ is used

$$g(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \cos(\eta_n t + \varphi_n) \qquad (2)$$

in which η_n is a random variable with probabilistic density $p_g(\eta)$ and φ_n is a random phase angle uniformly distributed in $(0, 2\pi)$ and N is a large positive integer. By virtue of the central limit theorem it is evident that the random process expressed in Eq.(2) is Gaussian.

The autocorrelation function $R_g(\tau)$ of $g(t)$ is

$$R_g(\tau) = E\left[g(t)\, g(t+\tau)\right]$$
$$= \frac{1}{2N} \sum_{n=1}^{N} \cos \eta_n \tau \qquad (3)$$

For stationary random process the autocorrelation function constructs a pair of Fourier transform with the power spectral density $S_g(\omega)$. Therefore

$$S_g(\omega) = \frac{\pi}{2N} \sum_{n=1}^{N} \left[\delta(\omega - \eta_n) + \delta(\omega + \eta_n)\right]$$

where $\delta(\eta)$ is the Dirac's delta function. In this equation, η_n is a random variable with density $p_g(\eta)$ which is valid for positive value of η, then it can be written as follows;

$$S_g(\omega) = \pi p_g(\omega) \tag{5}$$

for large value of N. Since Eq.(5) indicates that the power spectrum of stationary random process expressed by Eq.(2) is similar to the probability density of variable η, the stationary random process with arbitrary power spectrum can be constructed by the harmonic function containing the probabilistic variable η with density $S(\eta)/\pi$ and the random phase angle.

On the other hand, the autocovariance of nonstationary random process f(t) for large value of N can be expressed as follows;

$$K_{ff}(\tau_1, \tau_2) = E\left[f(\tau_1)\cdot f(\tau_2)\right]$$
$$= \frac{1}{2}\Psi(\tau_1)\Psi(\tau_2)\int_0^\infty p_g(\eta)\cos\eta(\tau_1-\tau_2)d\eta \tag{6}$$

Therefore the variance of f(t) is equal to a half of the squared value of $\Psi(t)$.

The spectrum of f(t) will be distorted compared with that of g(t), then the generalized spectral density $\Phi_{ff}(\omega_1, \omega_2)$ can be introduced in place of the Fourier transform of the autocorrelation function provided that the nonstationary random process f(t) has the Fourier transform $F(\omega)$.

$$\Phi_{ff}(\omega_1, \omega_2) = E\left[F(\omega_1)\,F^*(\omega_2)\right] - E\left[F(\omega_1)\right]\cdot E\left[F^*(\omega_2)\right] \tag{7}$$

where $F^*(\omega)$ is the complex conjugate of $F(\omega)$. If the Fourier transform $\tilde{\Psi}(\omega)$ exists $F(\omega)$ may be readily obtained as follows;

$$F(\omega) = \frac{1}{2}\left[\exp(i\varphi)\tilde{\Psi}(\omega-\eta) + \exp(-i\varphi)\tilde{\Psi}(\omega+\eta)\right] \tag{8}$$

Thus

$$E\left[F(\omega)\right] = E\left[F^*(\omega)\right] = 0 \tag{9}$$

then the generalized spectral density $\Phi_{ff}(\omega_1, \omega_2)$ reduces to

$$\Phi_{ff}(\omega_1, \omega_2) = \frac{1}{4}\int_0^\infty \Big[\tilde{\Psi}(\omega_1-\eta)\tilde{\Psi}^*(\omega_2+\eta)$$
$$+\tilde{\Psi}(\omega_1+\eta)\tilde{\Psi}^*(\omega_2-\eta)\Big]p_g(\eta)\,d\eta \tag{10}$$

Consequently, by use of the representations of Eqs.(1) and (2), the autocovariance function and spectral density of nonstationary random process can be expressed by the deterministic function, the Fourier transform of it and probability density governing the frequency characteristics.

GENERATION OF ARTIFICIAL EARTHQUAKES

The stationary random process expressed in Eq.(2) has been generated on a digital computer. Numerical computation is carried out by the Monte Carlo method. A random number η with density $p_g(\eta)$ and φ with uniform distribution in $(0,2\pi)$ are computed from the pseudo-random numbers which are generated successively on a digital computer from a pair of preceding ones.

A set of fifteen stationary random process $g(t)$ were generated by this method. In these computation N was fixed to 200 and time step was 0.02 sec. From these sample functions the response spectra(7) were calculated individually. The results were similar to the spectra of strong-motion earthquakes and it seems that the artificial earthquakes posess the known properties of actual earthquake records in spite of the different frequency characteristics.

For the purpose of generation of nonstationary random process, the deterministic function $\psi(t)$ and the probability density $p_g(\eta)$ are assumed to be

$$\psi(t) = a(t/t_p)\ \exp(1-t/t_p)\ U(t) \tag{11}$$

$$p_g(\eta) = 2\eta^2 \exp(-2\eta/\eta_p)/(\pi\eta_p^3) \tag{12}$$

where t_p, η_p, $U(t)$ and a are, respectively, the peak time of $\psi(t)$, the peak frequency, unit step function and constant with the dimension of acceleration. In Fig.2 are shown two sample records. In these figures, the nondimensional time t_p^* and nondimensional frequency η_p^* are used, i.e., $t_p^* = t/t_p$ and $\eta_p^* = \eta_p t_p$. Consequently, the value of η_p^* represents the ratio of the peak time to the period corresponding to the peak frequency.

From these generation of artificial earthquake records, either stationary or nonstationary, it has been found that the rough estimation of the maximum value of the generated records is given by $3\sigma_g$ using the standard deviation σ_g of the stationary random process $g(t)$. In both examples shown in Fig.2, since σ_g is $a/\sqrt{2}$, the value of $3\sigma_g$ is about $2a$ which coincide with the maximum value of $f(\eta, \varphi; t)$.

R.M.S. DETECTION

Under the assumption that the ground acceleration $f(t)$ is expressed as a product of a deterministic function $\psi(t)$ and a stationary random process $g(t)$ as Eq.(1), then $\psi(t)$ is the envelope of earthquake records, which is a slowly varying continuous function of time. If we have many actual earthquake records which belong to the same sample space, the envelope function is given by the ensemble average of the squared value of these records. However, since an actual earthquake record is merely a member function from the view point of statistical method, an another method, alternative to the ensemble average, is necessary for the evaluation of the envelope. Thus the r.m.s. of a few earthquake records are calculated in place of envelope by the moving average method.

Let f_i be the time series of the equi-spaced accelerograms, then the r.m.s. $\bar{f}_i$ is calculated by the following formula

$$\bar{f}_i = \left(\frac{1}{2(i-1)\Delta t} \sum_{s=1}^{2i-1} f_s^2\right)^{\frac{1}{2}} \qquad \text{for } i < m$$

$$\bar{f}_i = \left(\frac{1}{T_m} \sum_{s=i-m}^{i+m} f_s^{\,2}\right)^{\frac{1}{2}} \qquad \text{for} \quad M-m \geq i \geq m \qquad (13)$$

$$\bar{f}_i = \left(\frac{1}{2(M-i)\,\Delta t} \sum_{s=2i-M}^{M} f_s^{\,2}\right)^{\frac{1}{2}} \qquad \text{for} \quad i > M-m$$

where Δt is the time interval adjacent value of equi-spaced records, M is number of data and T_m is the period of moving average, which is equal to $2m\Delta t$.

Figs.3 and 4 show the results for the accelerograms of the Taft, California, earthquake of July 21, 1952 and of the El Centro, California, earthquake of May 18, 1940, respectively. Although the curve for short period moving average is jagged, the bottom curves in both figures are fairly smooth and very slowly varying with the fluctuation of accelerograms. After observing these figures, it would seem that the suitable average period T_m is approximately equal to the decuple of the predominant period of accelerograms.

STRUCTURAL RESPONSE

Consider a viscously damped linear system with a single degree of freedom which is subjected to the ground acceleration f(t). If the system starts from rest it is well known that the response x(t) is written as a convolution integral of input acceleration and the unit impulse response function as follows

$$x(t) = \int_0^t h(t-\tau)\, f(\tau)\, d\tau \qquad (14)$$

where

$$h(t) = \frac{1}{\omega_d} \exp(-\gamma t) \sin \omega_d t$$

$$\omega_d = \omega_0(1-\gamma^2)^{\frac{1}{2}}, \quad \omega_0^{\,2} = k/m, \quad \gamma = c/2m\,\omega_0,$$

m, k and c are, respectively, mass, spring constant and damping coefficient of the system considered.

The response x(t) is to be stochastic process in case that the input acceleration f(t) is a stochastic process, and its probabilistic properties are described perfectly by expectations and covariances. For the nonstationary random process f(t) expressed in Eq.(1) which is Gaussian with zero mean, the response x(t) is also Gaussian and its expectation is zero. The autocovariance $K_{xx}(t_1,t_2)$ of x(t) is written as follows;

$$K_{xx}(t_1,t_2) = E\left[X(t_1)\, X(t_2)\right]$$

$$= \int_0^{t_1}\int_0^{t_2} K_{ff}(\tau_1, \tau_2)\, h(t_1-\tau_1)\, h(t_2-\tau_2)\, d\tau_1 d\tau_2 \qquad (15)$$

where $K_{ff}(\tau_1, \tau_2)$ is the autocovariance of input acceleration $f(t)$ represented in Eq.(6). Substituting Eq.(6) into Eq.(15), it yields to

$$K_{xx}(t_1,t_2) = \frac{1}{2}\int_0^{\infty}\int_0^{t_1}\int_0^{t_2} \psi(\tau_1)\ \psi(\tau_2)\ h(t_1-\tau_1)\ h(t_2-\tau_2)$$

$$\cdot p_g(\eta)\cos\eta(\tau_1-\tau_2)\ d\tau_1 d\tau_2 d\eta \qquad (16)$$

By use of the following notations

$$I_c(\eta;t) = \int_0^t h(t-\tau)\ \psi(\tau)\ \cos\eta\tau d\tau$$

$$I_s(\eta;t) = \int_0^t h(t-\tau)\ \psi(\tau)\ \sin\eta\tau d\tau \qquad (17)$$

Eq.(16) becomes finally,

$$K_{xx}(t_1,t_2) = \frac{1}{2}\int_0^{\infty} p_g(\eta)\left[I_c(\eta;t_1)I_c(\eta;t_2)+I_s(\eta;t_1)I_s(\eta;t_2)\right]d\eta \qquad (18)$$

Autocovariances of response velocity $\dot{x}(t)$ and acceleration $\ddot{x}(t)$ are calculated from above equation as follows;

$$K_{\dot{x}\dot{x}}(t_1,t_2) = \frac{\partial^2}{\partial t_1\ \partial t_2} K_{xx}(t_1,t_2)$$

$$K_{\ddot{x}\ddot{x}}(t_1,t_2) = \frac{\partial^2}{\partial t_1\ \partial t_2} K_{\dot{x}\dot{x}}(t_1,t_2) \qquad (19)$$

Substituting $t_1=t_2=t$ into above equation leads to the expressions for variances of response

$$\sigma_x^{\ 2}(t) = \frac{1}{2}\int_0^{\infty} p_g(\eta)\left[I_c^{\ 2}(\eta;t)+I_s^{\ 2}(\eta;t)\right]d\eta$$

$$\sigma_{\dot{x}}^{\ 2}(t) = \frac{1}{2}\int_0^{\infty} p_g(\eta)\left[\dot{I}_c^{\ 2}(\eta;t)+\dot{I}_s^{\ 2}(\eta;t)\right]d\eta \qquad (20)$$

$$\rho_{x\dot{x}}(t) = \frac{1}{2\,\sigma_x\,\sigma_{\dot{x}}}\int_0^{\infty} p_g(\eta)\left[I_c(\eta;t)\dot{I}_c(\eta;t)+I_s(\eta;t)\dot{I}_s(\eta;t)\right]d\eta$$

where $\dot{I}(\eta;t)$ is the derivative of $I(\eta;t)$ with respect to time and $\rho_{x\dot{x}}(t)$ is the correlation coefficient.

Although η included in Eq.(20) is a probabilistic variable, it can be treated as a parameter in $I_c(\eta;t)$ and $I_s(\eta;t)$. Accordingly, the analysis of structural response for nonstationary random process expressed in Eq.(1)

is transformed to that for a deterministic input excitation $\psi(t)\cos\gamma t$ and it reduces to the calculation of $I_c(\gamma;t)$ and $I_s(\gamma;t)$. By use of Eq.(12) for deterministic function $\psi(t)$, $I_c(\gamma;t)$ and $I_s(\gamma;t)$ are expresses as follows;

$$I_c(\gamma;t) = a\,t_p^{\,2}\left[J_c(\gamma^*,t^*)+J_c(-\gamma^*,t^*)\right]$$
$$I_s(\gamma;t) = a\,t_p^{\,2}\left[J_s(\gamma^*,t^*)-J_s(\gamma^*,t^*)\right] \tag{21}$$

where

$$J_c(\gamma^*;t^*)=\frac{\exp(-\gamma\omega_o^{*}t^*)}{2\,\omega_d^{*}}\left[-\frac{t^*\exp(1-t^*)}{(\alpha^2+\beta^2)^{\frac{1}{2}}}\sin(\gamma^*t^*-\gamma)+\frac{\exp(1-t^*)}{\alpha^2+\beta^2}\sin(\gamma^*t^*-\delta)+\frac{1}{\alpha^2+\beta^2}\sin(\omega_d^{*}t^*+\delta)\right]$$

$$J_s(\gamma^*;t^*)=\frac{\exp(-\gamma\omega_o^{*}t^*)}{2\,\omega_d^{*}}\left[\frac{t^*\exp(1-t^*)}{(\alpha^2+\beta^2)^{\frac{1}{2}}}\cos(\gamma^*t^*-\gamma)-\frac{\exp(1-t^*)}{\alpha^2+\beta^2}\cos(\gamma^*t^*-\delta)+\frac{1}{\alpha^2+\beta^2}\cos(\omega_d^{*}t^*+\delta)\right] \tag{22}$$

$$\alpha=\gamma\omega_o^{*}-1,\quad \beta=\omega_d^{*}+\gamma^*,\quad \gamma=\tan^{-1}(\beta/\alpha),\quad \delta=\tan^{-1}\left[2\alpha\beta/(\alpha^2-\beta^2)\right],$$
$$\omega_o^{*}=\omega_o t_p,\quad \omega_d^{*}=\omega_d t_p,\quad \gamma^*=\gamma t_p,\quad t^*=t/t_p.$$

In Fig.5 are shown the results of numerical computation as an example. The abscissa of these figures is nondimensional, normalized by peak time of input excitation. These figures, from top to bottom, show the nondimensional square root of ensemble average of input acceleration, response acceleration, response velocity, response displacement and expected rate of threshold crossing per unit time. It will be seen that the time lag between input excitation and responses is considerable and that the peak time of response displacement is about twice of that of input acceleration for the value of 0.8 of nondimensional natural period T^*. This tendency is the more remarkable, the longer is T^*.

RESPONSE SPECTRUM WITH PROBABILISTIC PARAMETER

If random process $x(t)$ and its derivative $\dot{x}(t)$ with respect to time are governed with two dimensional Gaussian distribution, the expected number of crossing over the level ξ within the time interval (t_1,t_2) has been given by S. O. Rice(9) as

$$N(\xi;t_1,t_2) = \int_{t_1}^{t_2}\int_0^{\infty}\dot{x}(t)\,p_{x\dot{x}}(\xi,\dot{x};t)\,d\dot{x}\,dt \tag{23}$$

where $p_{x\dot{x}}(\xi,\dot{x};t)$ is the joint probability density. Let $x(t)$ be a structural response to the nonstationary random process, then the response displacement $x(t)$ and response velocity $\dot{x}(t)$ are also Gaussian process, and as $p_{x\dot{x}}(x,\dot{x};t)$ appearing in Eq.(23) Gaussian joint distribution density is used,

$$p_{x\dot{x}}(x,\dot{x};t) = \frac{1}{2\pi\sigma_x\sigma_{\dot{x}}(1-\rho_{x\dot{x}}^2)^{\frac{1}{2}}} \exp\left[-\frac{\sigma_{\dot{x}}^2 x^2 - 2\sigma_x\sigma_{\dot{x}}\rho_{x\dot{x}} x\dot{x} + \sigma_x^2\dot{x}^2}{2\sigma_x^2\sigma_{\dot{x}}^2(1-\rho_{x\dot{x}}^2)}\right] \tag{24}$$

It is convenient to deal with the rate of crossing per unit time $n(\xi;t)$ defined as follows;

$$N(\xi;t_1,t_2) = \int_{t_1}^{t_2} n(\xi;t)dt$$

$$n(\xi;t) = \int_0^{\infty} \dot{x}(t)\, p_{x\dot{x}}(\xi,\dot{x};t)d\dot{x} \tag{25}$$

Substitution of Eq.(24) into Eq.(25) yields

$$n(\xi;t) = \frac{\sigma_{\dot{x}}}{2\pi\sigma_x} \exp\left[-\frac{1}{2}\left(\frac{\xi}{\sigma_x}\right)^2\right]\left[(1-\rho_{x\dot{x}}^2)^{\frac{1}{2}}\left\{-\frac{\rho_{x\dot{x}}^2}{2(1-\rho_{x\dot{x}}^2)^{\frac{1}{2}}}\left(\frac{\xi}{\sigma_x}\right)^2\right\}\right.$$

$$\left. +\left(\frac{\pi}{2}\right)^{\frac{1}{2}} \rho_{x\dot{x}}\left(\frac{\xi}{\sigma_x}\right)\left\{1+\mathrm{erf}\left(\frac{\rho_{x\dot{x}}}{\sqrt{2}\,(1-\rho_{x\dot{x}}^2)^{\frac{1}{2}}}\frac{\xi}{\sigma_x}\right)\right\}\right] \tag{26}$$

Therefore, calculating the responses, that is σ_x, $\sigma_{\dot{x}}$ and $\rho_{x\dot{x}}$, the expected rate of crossing over the displacement level ξ within unit time can be computed by the above equation.

If the expectation of response displacement and velocity are zero, a rate of positive-slope crossing of positive level is equal to that of negative-slope crossing of negative level. Therefore let $n_D(t)$ be a sum of these two quantities,

$$n_D(t) = 2\, n(\xi;t) \tag{27}$$

Since $n_D(t)$ represents the expectation per unit time, an expected number $N_D(t)$ for every set of excitation is given by integration in the time interval $(0,\infty)$,

$$N_D(T^*) = N(\xi;0,\infty) = \int_0^{\infty} n_D(t)dt \tag{28}$$

It is evident that $N_D(T^*)$ is a function of the displacement level and natural period of structures. Therefore, let S_D be a displacement level of structural response, then

$$N_D(T^*) = f_n(\, S_D\, ,\, T^*\,) \tag{29}$$

From above equation, it can be written alternatively,

$$S_D = f_n(\ T^*\ ,\ N_D(T^*)) \tag{30}$$

This statement enables us to interpret the $S_D \sim T^*$ curve as a kind of response spectrum representing the relation between response level and nondimensional natural period.

The way of analysis for response velocity is analogous to what has been stated previously on the response displacement.

The numerical computation on the structural response has been carried out and is illustrated in Figs.5~11. The seismic loading is assumed to be a product of deterministic time function expressed in Eq.(11) and stationary random process of which spectral composition is given by Eq.(12). The bottom gragh in Fig.5 shows the rate of crossing of displacement level $S_D=0.8\,\alpha t_p^2/\omega_d^{*2}$. Since the area enclosed by this curve, which coincides with $N_D(T^*)$, is 1.02, it means that the expectation of response over the displacement level S_D is about one. In the vicinity of $t^*=5$ in this figure, neverthless the value of $n_D(t^*)$ is zero, the value of σ_x, that is, the r.m.s. value of response displacement remains a half of the peak value. Therefore we know that the quantity $n_D(t^*)$ is very sensitive to the displacement level S_D. Fig.6 illustrates the effect of S_D exerting on $n_D(t^*)$. This figure shows that the expectation $N_D(T^*)$ of response over the level S_D decreases rapidly with increasing of S_D.

Figs.7 and 8 show the relation between the response displacement level S_D, velocity level S_V and the expectations $N_D(T^*)$, $N_V(T^*)$, respectively. Since, in these figures, $S_D \sim N_D(T^*)$ curve and $S_V \sim N_V(T^*)$ curve are almost linear when plotted on a semi-logarithmic paper, it is found that the expectation of response over the level decreases exponentially for increasing of the level S_D. The parameter T^*, representing the nondimensional undamped natural period, exert much more effet on the response displacement than on the response velocity and this on the high level rather than the low level. Therefore, as to the response displacement, if the level is held constant, the longer the natural period is, the higher becomes the expectation of response over the level.

Fig.9 is alternative to Figs.7 and 8, and illustrates the relation of response level S_D, S_V versus to period T^*. These figures, which correspond to the relationship in Eq.(30), are considered to be a kind of response spectrum which exhibits the characteristics of response spectra for actual earthquake records, because the response level S_D increases monotonously for increasing value of T^* as to response displacement and S_V remains almost constant as to response velocity. Since the $N_D(T^*)$ and $N_V(T^*)$ parameters in Fig.9 give an upper bound(10) of the probability that the response crosses over the level S_D, it is equivalent to the calculation of the expected maximum value of response for a set of T^* to find out the level S_D for which the expectations $N_D(T^*)$ and $N_V(T^*)$ are 1.0. Accordingly, the curves for $N_D(T^*)$, $N_V(T^*)=1.0$ correspond to the response spectrum which is calculated from actual earthquake records. Hence the curves for smaller value of parameter in Fig.9 represent the conservative response spectrum and on the contrary, the case of larger value of $N_D(T^*)$ and $N_V(T^*)$ is the reverse. Therefore it appears that the parameter in Fig.9 is a kind of safety factor on the structural response for seismic loadings. Thus the representation of earthquake motion by stochastic process enables us to introduce the probabilistic quantity as a parameter in response spectra.

In Figs.10 and 11 are shown the relation between the response and predominant period of input excitation. In both figures, the value of $N_D(T^*)$ and $N_V(T^*)$ are 1.0 and $f_p^*=f_p t_p$. Since the peak time t_p of ensemble average of input excitation is considered to be a constant, the value of parameter f_p^* is so changed as to indicate the effect of the predominant frequency of input excitation on structural response. In Fig.11, f_p and t_p are alternative to those of Fig.10. From these figures it follows that the response level S_D and S_V are inversely proportional to f_p. Accordingly, if t_p and intensity of input excitation are assumed to be constant, the higher the frequency of excitation is, the more the response decreases and if f_p and intensity of input excitation are constant value, the longer the duration time of excitation becomes, the more the response level increases.

It is evident that the frequency characteristics and nonstationarity of earthquake motion exert an influence on the response spectrum and that their effect on structural response can not be neglected. Neverthless, the response spectra which are calculated from actual earthquake records under different circumstances fails to take into account the frequency characteristics and dynamic properties which are inherent to observation sites and are revealed on earthquake records.

CONCLUDING REMARKS

Since earthquake motion is a complicated dynamic phenomenon which is essentially nonstationary and of a statistical nature, the representation of earthquake motion as a stochastic process is acceptable for the aseismic design of structures. From this point of view, the present paper concerns the simulation of earthquake motion by a nonstationary random process in which the frequency characteristics of ground can be taken into account. Moreover, an analysis of structural response to these excitation is performed by statistical method.

In this study, earthquake acceleration is represented as a nonstationary random process which is described as a product of a slowly varying continuous time function and a stationary random process which has an arbitrary power spectrum. For this excitation the structural response is very simplified and can be represented by the response to the deterministic function of time. By use of this representation of earthquake motion the standard deviation of responses are calculated analytically and a method of generation of an artificial earthquake based on the Monte Carlo method is presented.

The response of structure to nonstationary random excitation is treated as a problem of the threshold crossing and the expected number of excess of response over a certain level is calculated as a function of the natural period of the structures with single degree of freedom. With the aid of results of numerical computation the response spectra which contain a probabilistic quantity as a parameter are presented in the figures. From these response spectra the information related not only to the maximum value of responses but also to the value of safety or risk is obtained. Moreover, it is pointed out from the discussion on the effect of frequency characteristics and duration time of input excitation on response spectra that the usual response spectrum which is obtained as an average of some spectra calculated from actual earthquake records is inadequate.

The information for the dependence of intensity, energy and spectral composition of earthquake motion upon time are still insufficient for the simulation of earthquake motion by stochastic process. Moreover, there are many questions for the structural response of nonlinear and multi-degree of freedom system to nonstationary random process.

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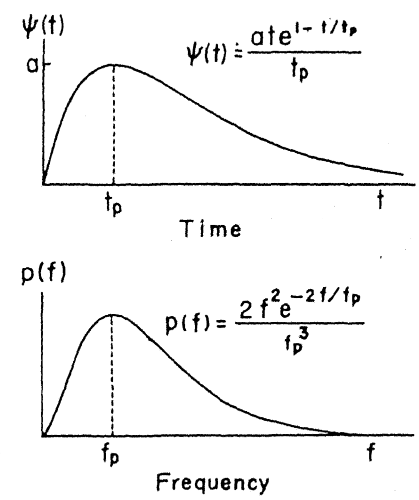

FIG. 1. DETERMINISTIC FUNCTION AND PROBABILITY DENSITY USED IN NUMERICAL COMPUTATION.

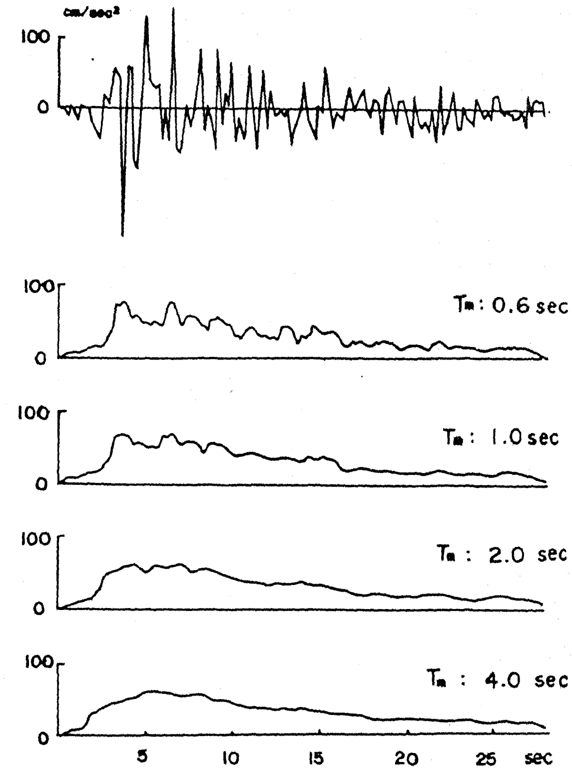

FIG.3 ENVELOPE OF ACCELEROGRAM FOR TAFT, JULY 21, 1952, (SEIII).

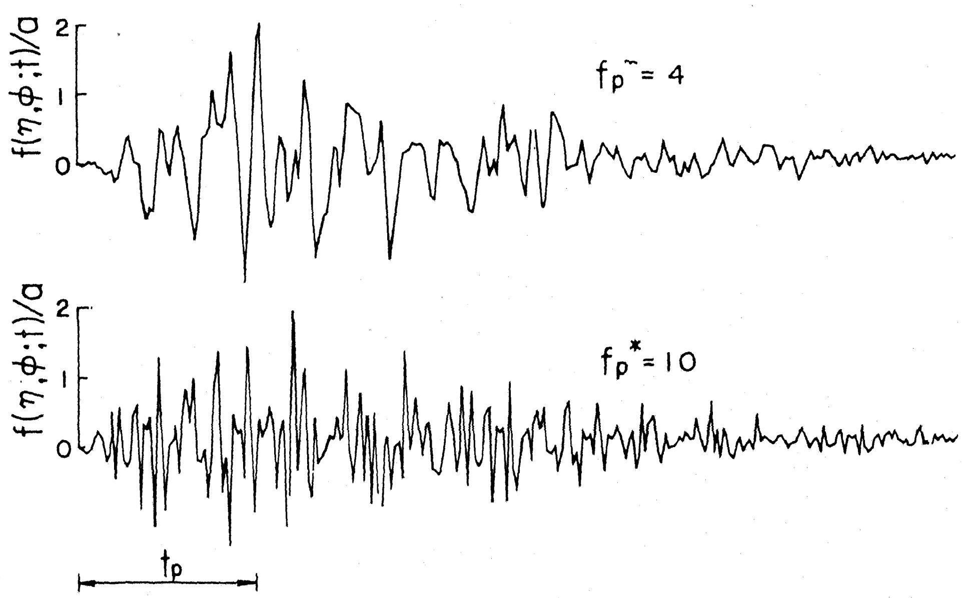

FIG. 2 EXAMPLES OF ARTIFICIAL EARTHQUAKES GENERATED ON DIGITAL COMPUTER.

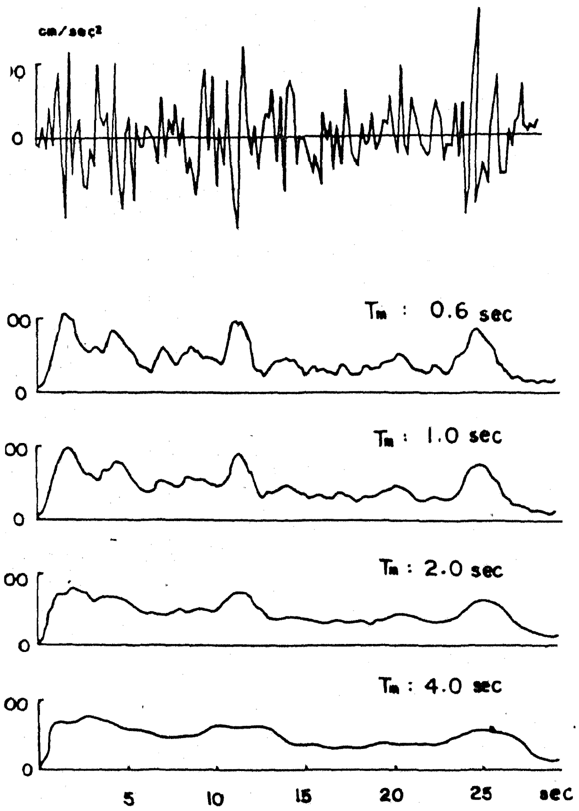

FIG. 4 ENVELOPE OF ACCELEROGRAM FOR EL CENTRO, MAY 18, 1940, (EW).

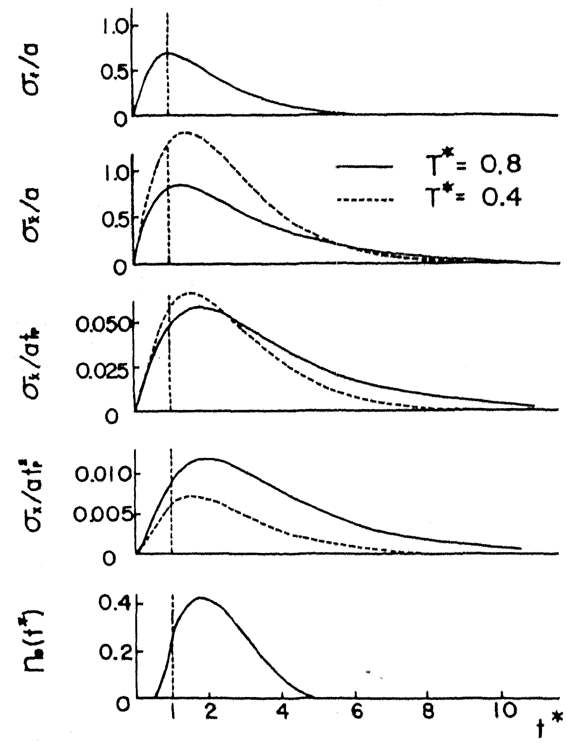

FIG. 5 R.M.S. OF INPUT EXCITATION AND RESPONSES AND RATE OF THRESHOLD CROSSING PER UNIT TIME.

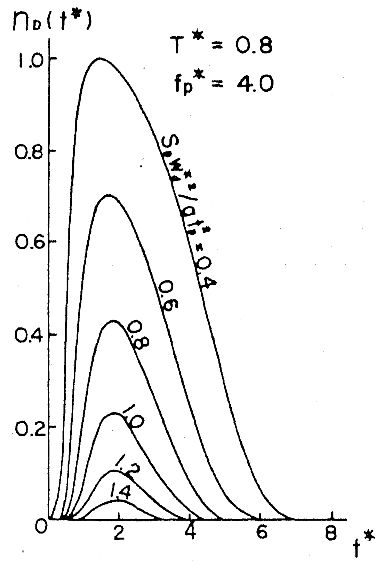

FIG.6 RATE OF THRESHOLD CROSSING PER UNIT TIME.

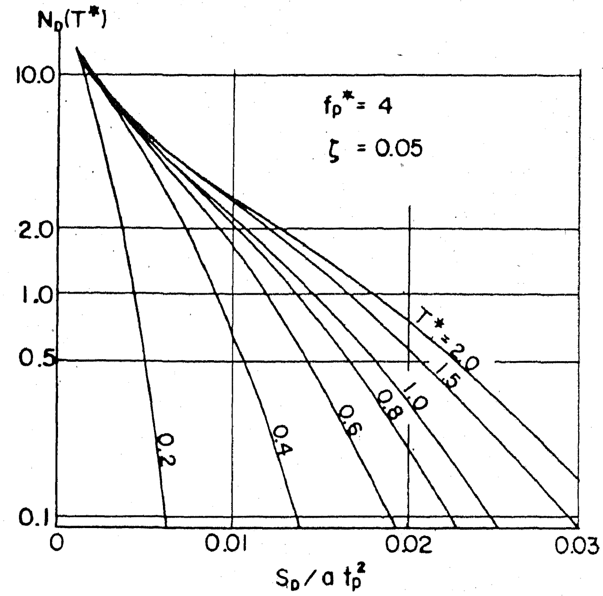

FIG.7 EXPECTED NUMBER OF THRESHOLD CROSSING VERSUS LEVEL OF RESPONSE DISPLACEMENT

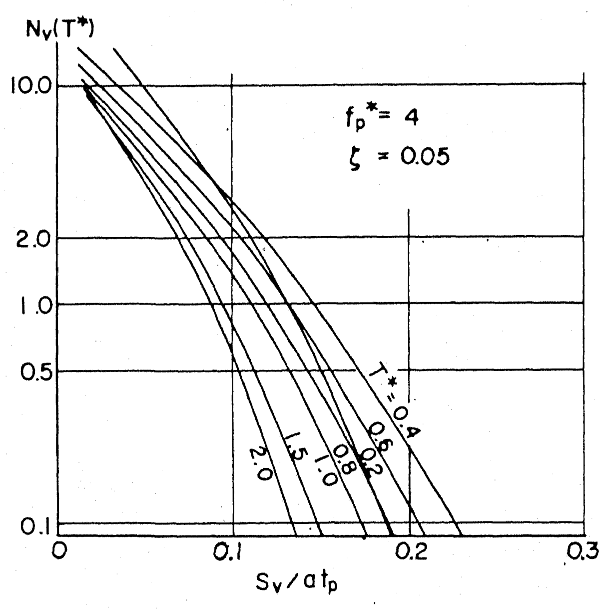

FIG. 8 EXPECTED NUMBER OF THRESHOLD CROSSING VERSUS LEVEL OF RESPONSE VELOCITY.

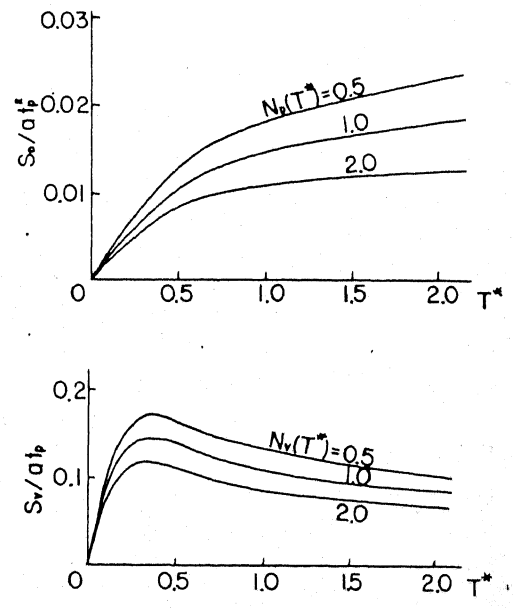

FIG. 9 RESPONSE SPECTRA WITH PROBABILISTIC PARAMETER

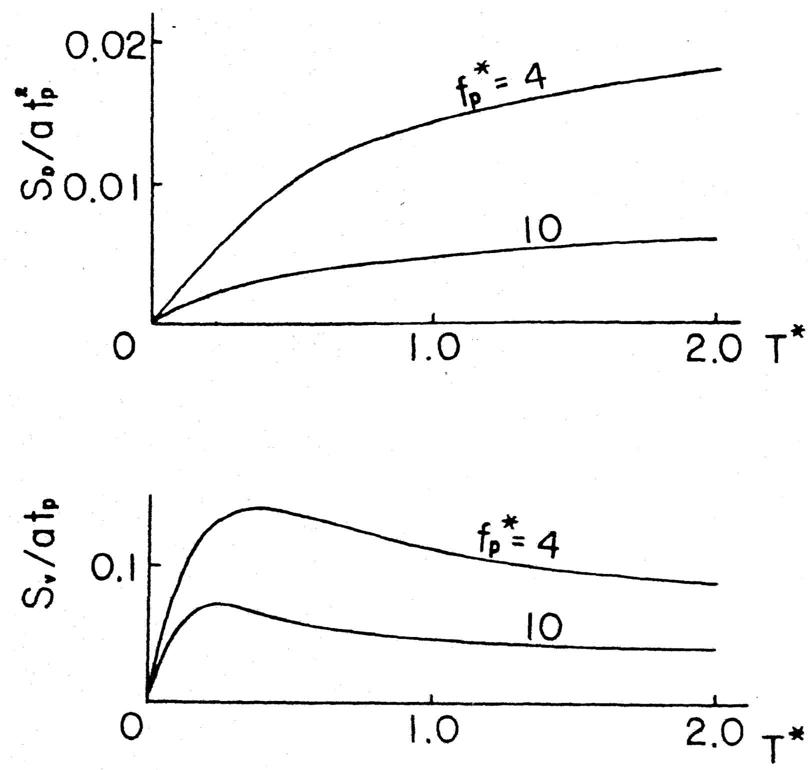

FIG.10 RESPONSE SPECTRA SHOWING EFFECT OF PREDOMINANT FREQUENCY

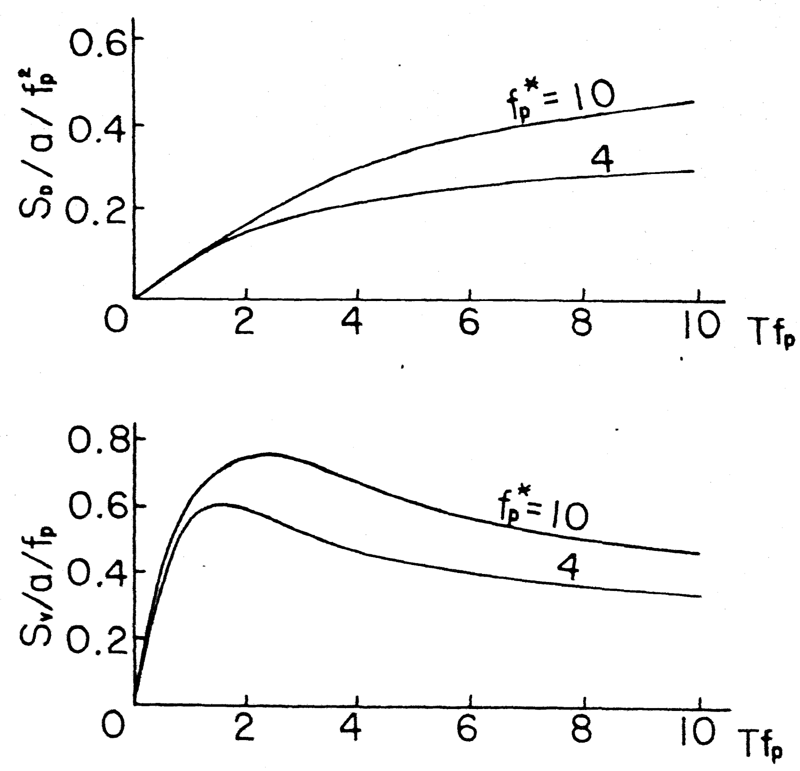

FIG.11 RESPONSE SPCTRA SHOWING EFFECT OF NONSTATIONARITY