ENGINEERING ESTIMATES OF GROUND SHAKING AND MAXIMUM EARTHQUAKE MAGNITUDE

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SYNOPSIS

Earthquake design of important structures requires the making of engineering decisions as to the frequency and intensity of ground shaking that should be considered in the design. For this purpose, idealized relations are given between fault-length and Magnitude, the spatial distribution of intensity of ground shaking and Magnitude, the felt area and Magnitude, and the frequency of occurrence and Magnitude. Values are presented for the probability of ground shaking for an average site in California, and estimates are given for the effective upper bound for Magnitude as a function of seismicity. These relations provide useful guides for making engineering decisions.

INTRODUCTION

The earthquake design of major structures is usually given special consideration as distinguished from the design of ordinary structures which is done according to the requirements of the building code. Dams, nuclear reactors, suspension bridges, off-shore oil drilling platforms, vehicular tunnels, and major industrial installations are examples. The special considerations that are given consist primarily of estimates of the maximum earthquake likely to be experienced and the frequency and severity of ground shaking to be expected, and the development of design criteria that are more realistic than those of the building code and that try to control damage and take into account the economics of cost of repair vs. initial investment. This paper is concerned with the first of these considerations, that is, what basis is there for making a sound engineering decision as to the frequency and intensity of ground shaking that should be used as a basis for design. The reasoning and data presented are for the United States but, presumably, the problem for other countries is basically the same.

The difficulty in arriving at suitable decisions results mainly from a lack of information about earthquakes. The generation of earthquakes is a completely deterministic process, presumably, viscous flow in the earth's interior generates strains in the rock that forms the earth's crust. The strain rates and stresses involved are sufficiently large so that at intervals the overstressed rock fails with consequent release of strain energy and propagation of seismic waves. The strains have been operative for a sufficiently long time so that it is thought that past failures have established systems of faults on which planes of weakness most future earthquakes will occur. In principle, if the existing state of stress and the strain rate in the earth's crust were known, and the strengths of the rocks and the locations of the faults were known, it would be possible to predict when and where earthquakes would occur and how large they would be. The information necessary for predicting earthquakes is not, at present, available,

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and estimates of ground shaking must be based essentially on 1) the earthquake history of the region, and 2) the geologic evidence of past and present straining of the earth's crust. In the United States, such information is most plentiful for California where earthquakes occur relatively frequently and surface evidence of faulting can be seen. For other parts of the country, there is relatively little information available. Since major engineering projects will be undertaken irrespective of the state of knowledge, the limited information available must be utilized to guide engineering decisions.

The present paper differs from approaches sometimes used in that it does not exploit mathematical statistics to make inferences about the earthquake process, nor is a detailed description of the earthquake process postulated, along the lines of operations research. The aim of the paper is to exploit the basic observational data that is available while making minimal assumptions about the process.

FAULT LENGTH VS. MAGNITUDE

From the engineering point of view, one of the most significant aspects of an earthquake is the area of the fault over which slip takes place. It appears that the stress drop across the slipped fault is approximately the same for all larger earthquakes and, hence, the area of slip is a measure of the total strain energy released. It can therefore be inferred that the length and depth of the fault indicate the extent of the surface area that is strongly shaken. It is for this reason that the Richter Magnitude of an earthquake is significant to the engineer; it is indicative of the size of the slipped area of the fault. Actually, the true length of slipped fault beneath the surface of the ground cannot be determined precisely, and neither can the vertical demension of the slipped area. From the surface fault displacement and from the clustering of aftershocks, the length of fault can be estimated. The curve shown in Figure 1 gives an idealized relation between the Magnitude and the length of fault. The portion of the curve for $M > 6.5$ is based on observations (1). The portion for $M < 6.5$ takes into account the fact that for most of these smaller shocks the vertical and horizontal dimensions of the slipped area must be approximately the same, whereas for larger shocks the vertical dimension is presumably independent of the length. This lower branch of the curve gives a length of 100 ft. for $M = 0$, but it is not known how realistic this is. It should be noted that the curve of Figure 1 is not a prediction of fault length, but is an idealized relation which is in approximate agreement with observations. Actually, since the determination of both Magnitude and fault length are subject to uncertainties, observations of particular earthquakes can be expected to have considerable scatter about the curve. Some reported fault lengths in miles for larger earthquakes are: Chile, 1960, $M = 8.6$, $L = 600\pm$; Alaska, 1964, $M = 8.4$, $L = 450\pm$; San Francisco, 1906, $M = 8.2$, $L = 250\pm$; El Centro, 1940, $M = 7.1$, $L = 40\pm$; Baja California, 1956, $M = 6.8$, $L = 15\pm$.

INTENSITY DISTRIBUTION

The ground shaking will be strongest in the general vicinity of the causative fault and the intensity will diminish with distance from the fault.

Figure 2 shows a set of idealized contour lines for the intensity of shaking. For a specific earthquake the contour lines would be irregular curves but in an average sense the contour lines of earthquakes of a given Magnitude would be smooth curves similar to those in Figure 1. Contour lines for intensity were constructed for west coast earthquakes of different Magnitudes so as to be consistent with the fault lengths shown in Figure 1 and to be consistent with ground accelerations recorded on firm ground at various distances from the causative fault. An upper bound of 45% to 50%g was taken for the maximum acceleration near the causative fault of a very large earthquake (2). The values of maximum acceleration and the shapes of the pulses have been correlated with these recorded in the western United States; maximum accelerations associated with very narrow pulses such as were recorded in Lima, Peru (1966) and Koyna, India (1967) are not covered by this analysis. The results are presented in Figure 3 which gives the area with maximum acceleration equal to or greater than a specified amount. Figure 3 is to be understood as an idealized self-consistent relation between Magnitude, acceleration and area, and it relates to actual ground shaking only in an average sense. Given the frequency of occurrence of earthquakes for a given region, Figure 3 can be used to calculate probabilities of ground shaking.

FREQUENCY OF OCCURRENCE

In order to utilize Figure 3, it is necessary to know the frequency of occurrence of earthquakes of various Magnitudes. The collecting of reliable statistics is so recent that in many regions the data are not adequate. For world earthquakes, the numbers are statistically sufficient even for large Magnitudes and their frequencies of occurrence are closely described by an exponential equation:

$$N = AN_o e^{-M/B} \tag{1}$$

$$n = -\frac{dN}{dM} = \frac{1}{B} AN_o e^{-M/B} \tag{1a}$$

where N is the number per year of shallow earthquakes having Magnitudes equal to or greater than M per unit area, and $n(dM)$ is the annual number of shocks having Magnitude between M and $M + dM$. The parameter N_o defines the average seismicity of the region. The parameter $B = -N/(dN/dM) = N/n$ describes the seismic severity of the earthquake process in that a larger value of B means a higher ratio of the number of large to small earthquakes. For world earthquakes $AN_o = (2.5)10^7$, $B = 0.48$ as determined on the basis of a 43-year period (3), 1904-1946. Figure 4 shows a plot of world earthquakes (3) and it is seen that $M = 8.7$ appears to be an upper bound. For a region including southern California and northern Mexico, over a 29-year period (4) the parameters have the values $A = 100,000$ square miles, seismicity $N_o = 1.7$ per square mile, $B = 0.48$. The data were well described by equation (1) down to Magnitude 3 and up to about Magnitude 6.5, for larger Magnitudes the data were too few to be statistically significant. It is assumed that the equation can be used for southern California for Magnitudes up to about 8.5. Equation (1) appears to describe the seismicity in most regions of the world with values of B in the range $0.4 < B < 0.6$.

The seismicity in a region is not constant and can be expected to fluctuate in time. In veiw of this, the values of the parameter N_o may change with size of data sample. To minimize this, the area of the region under consideration should not be taken too small. The data for California are not really sufficient to permit making a fine discrimination between the seismicities of small areas. Because of this, it is customary to consider the State of California as a single area of 150.000 square miles. Judgment must then be used to allow for different seismicities in different parts of the state. For the State of California (A = 150,000 sq miles) it is thought that a reasonable long-term seismicity is $N_o = 1.2$. When making probability calculations it is often assumed that the earthquakes occur randomly in time and space within the area. Any additional knowledge of regions of higher seismicity within the area can then be used to modify the results. However, it is clear that aftershocks tend to cluster in time and space so that a large earthquake will be followed by numerous smaller shocks in the same general vicinity. These shocks, which produce repetitive shaking of the same location should not be included among those that are assumed to occur at random within the state. To allow for this, the frequencies of occurrence shown in Table I have been taken to be the expectation of California earthquakes for making probability calculations. Shocks having $M = 4.75$ are about the lower limit of engineering significance.

With the frequencies shown in Table I and the areas shown in Figure 3, assuming random occurrence of earthquakes, there can be computed the probability of experiencing ground shaking. For example, for a site in California there is a certain probability of occurrence of, say, a Magnitude 7 shock and a certain probability that it will be located at such a distance that the site experiences ground shaking between 5% and 10%g. A simplified calculation of probabilities can be made on the basis of the following analogous problem. Given an area A, let a small area Δa be placed at random in it. After a number of Δa areas have been placed at random so that $\Sigma\Delta a = a$, the probability that a certain point in A has not been covered by any of the $\Sigma\Delta a$ can be written $p(0, a)$, that is, the probability of zero hits when a total area $\Sigma\Delta a = a$ has been placed. The probability of zero hits after $(a + \Delta a)$ has been placed is the product of the independent probabilities:

$$p(0, a + \Delta a) = p(0, a)\, p(0, \Delta a)$$

Noting that

$$p(0, \Delta a) = 1 - p(1, \Delta a) = 1 - \frac{\Delta a}{A}$$

the equation can be written in the form

$$\frac{p(0, a + \Delta a) - p(0, a)}{\Delta a} + \frac{1}{A}\, p(0, a) = 0$$

Letting $\Delta a \rightarrow 0$, the foregoing equation approaches in the limit

$$\frac{dp}{da} + \frac{1}{A} p = 0$$

which has the solution

$$p(0, a) = e^{-a/A}$$

which is the probability that a point in A is covered by the $\Sigma\Delta a$. The probability that the point is not missed, that is, that it is hit, is

$$p(h, a) = 1 - e^{-a/A} \qquad (2)$$

This equation can be used, for example, to calculate the probability that a certain site in California will experience maximum ground shaking between 5% and 10% g in a 50-year period, if the frequency of occurrence of earthquakes is as shown in Table I and the spatial distribution of ground shaking is as shown in Figure 3. The results of such calculations are shown in Figure 5. This shows, for example, a 50% probability of 18%g or greater at least once in a 50-year period. It is seen that the probability of weak shaking, 5% to 10%g, is relatively high, and the probability of very strong shaking, 30%g or greater, is quite small. There is, of course, a possibility that the site experiences more than once the specified acceleration and the probability of experiencing it n times is given by the well-known expression:

$$p(n, a) = \left(\frac{a}{A}\right)^{n} \frac{1}{n!} e^{-a/A} \qquad (3)$$

It should be noted that the probabilities given in Figure 5 are average values that are too high for some of the less seismic regions of California and too low for some of the highly seismic regions.

MAXIMUM EARTHQUAKE

For regions in the United States, other than California, the seismic history is so meager that it is often not sufficient to define a frequency vs. Magnitude curve, particularly for larger Magnitudes. For California or southern California, where large earthquakes are known to have occurred, it is customary to take M = 8.5 as the upper bound. The frequency distribution for southern California is given in Figure 6, with the dotted line showing the assumed drop-off at large Magnitudes (4). For a less seismic region, such as the State of Oregon or the eastern part of the United States, it is not known what is the correct shape of the frequency distribution curve at large Magnitudes. If the tectonic processes were the same in two regions, differing only in rate of straining, the shapes of frequency curves should be the same, but the tectonic processes are presumably not the same throughout the United States. In many regions of the United States the available geologic and seismic evidence indicates that large Magnitude earthquakes are not to be expected. However, the large New Madrid, Missouri earthquakes of 1811-1812 and the Charleston, South Carolina earthquake of 1886 occurred in regions of relatively low seismicity, which indicates that the possibility of large Magnitude shocks cannot be completely discounted just because a region has low seismicity.

From a practical veiwpoint, two different approaches may be used to deal with the question of the maximum earthquake. One, in effect, assumes different shapes for the frequency distribution curves for regions of different seismicity, whereas the other approach takes all regions to have frequency distribution curves of the same shape. The first method is based on a Seismic Probability Map such as that shown in Figure 7. The maximum intensity of shaking in the various zones and the approximate corresponding Magnitudes (2) are usually taken as shown in Table II. These values are more or less arbitrary and are used as a matter of expediency. The zones in the Uniform Building Code are based on this seismic probability map and the figures in Table IV indicate approximately the seismic significance of the building code requirements.

The second approach assumes that earthquakes of all Magnitudes are possible in any region of the world. This approach assumes the probability distribution to be the same in all regions, for M ranging from zero to infinity:

$$p(\geq M) = e^{-M/B} \quad (0 < M < \infty) \tag{4}$$

$$p(\geq M; \leq M + dM) = \frac{1}{B} e^{-M/B} dM \tag{4a}$$

With this ideal frequency distribution there is no upper bound for Magnitude. As a practical matter, however, a lower bound must be set for meaningful probability. For example, in California it seems most unlikely that an earthquake having $M > 8.5$ will occur, hence, when making probability calculations by means of equation (4), the probability for $M > 8.5$ can be considered to be negligible. This, then, specifies a "negligible probability" which may be applied when considering regions of lower seismicity than California. For example, the expectation of shocks having Magnitudes greater than M during Y years, per unit area, is

$$E = YN_o e^{-M/B}$$

and if this is less than some specified value E_1 corresponding to M_1, the hazard of earthquakes having Magnitudes greater than M_1 is considered to be negligible. Let $M_1 = 8.5$ for California which has seismicity N_{O1}, $B_1 = 0.48$ and $A_1 = 150,000$ square miles. A second region might have seismicity N_{O2}, severity B_2, and area A_2. The corresponding upper bound for M_2 is determined by the condition $E_2 = E_1$ which gives

$$M_2 = \frac{B_2}{B_1} M_1 - B_2 \log_e \left(\frac{N_{o1}}{N_{o2}} \right) \tag{5}$$

If both regions have $B = 0.48$, the upper bounds for Magnitude are as shown in Table III. This is also plotted in Figure 5, which shows curves of N vs. Magnitude for seismicities ranging from 0.0001 to 10 times that of southern California. These curves have been drawn to have upper bounds for Magnitudes corresponding to Table III. In Figure 6 the number N plotted as ordinate for any M is the number of shocks per square mile per year having Magnitudes between $M - \frac{1}{2}$ and $M + \frac{1}{2}$ that is

$$N = \int_{M - \frac{1}{2}}^{M + \frac{1}{2}} n dM$$

The values given in Table II are not predictions of maximum earthquakes but are a consistent set, in a probability sense, of effective upper bounds for Magnitudes. When making such probability calculations, it must be kept in mind that a certain minimum area is required for an earthquake, for example, a Magnitude 8.5 shock would require an area about that of California.

FELT AREA

Most of the earthquake data for the United States, exclusive of California, do not have instrumentally determined Magnitudes but instead the earthquakes are described by Modified Mercalli Intensity ratings (5). These are subjective measures based on the observed effects of the ground shaking and they are not reliable indicators of the Magnitude of the earthquake or of the maximum ground acceleration. Usually the total area over which the earthquake was felt is also given and this is a more reliable indicator of the Magnitude than is the Modified Mercalli Intensity at the epicenter As pointed out by Gutenberg and Richter (6), due to geologic conditions, the felt area in the eastern United States is greater for a given Magnitude than in the western United States. The following empirical formulas are idealized relations between the Magnitude, M and the felt area, A, in square miles (5). The reported values of A have, of course, considerable scatter about the curves shown in Figure 8.

a) Western United States

$$M = 2.3 \log_{10} (A + 3,000) - 5.1$$

b) Rocky Mountain region and central region

$$M = 2.3 \log_{10} (A + 14,000) - 6.6$$

c) Eastern region

$$M = 2.3 \log_{10} (A + 34,000) - 7.5$$

When the estimated Magnitudes for a given region have been computed by means of the foregoing relations, the distribution of Magnitudes should, presumably, be statistically consistent with equation (1), which fact may be used to check the data.

CONCLUSIONS

Relations are given between fault-length and Magnitude, the spatial distribution of intensity of ground shaking and Magnitude, the Felt area and Magnitude, and the frequency of occurrence of earthquakes and Magnitude. Calculations are made for the probability of ground shaking for an average site in California. Estimates are given for the effective upper bound for Magnitude as a function of seismicity. These relations provide useful guides for making engineering decisions as to the frequency and severity of ground shaking upon which the design of a project should be based.

ACKNOWLEDGEMENT

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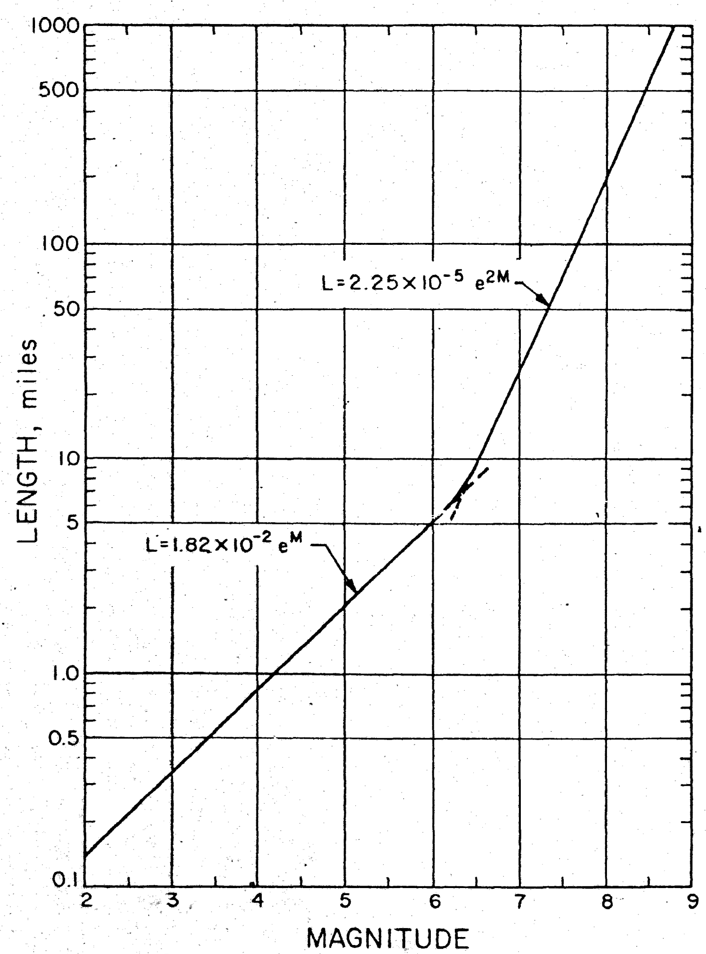

Figure 1. Idealized relation between length of slipped fault vs. Magnitude of earthquake.

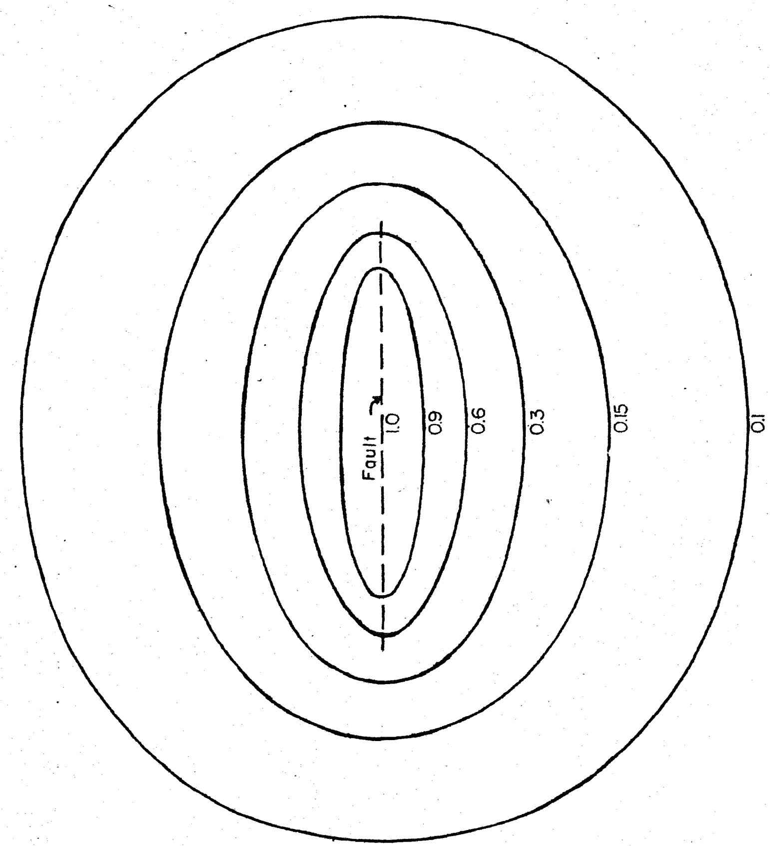

Figure 2. Idealized contour lines of intensity of ground shaking.

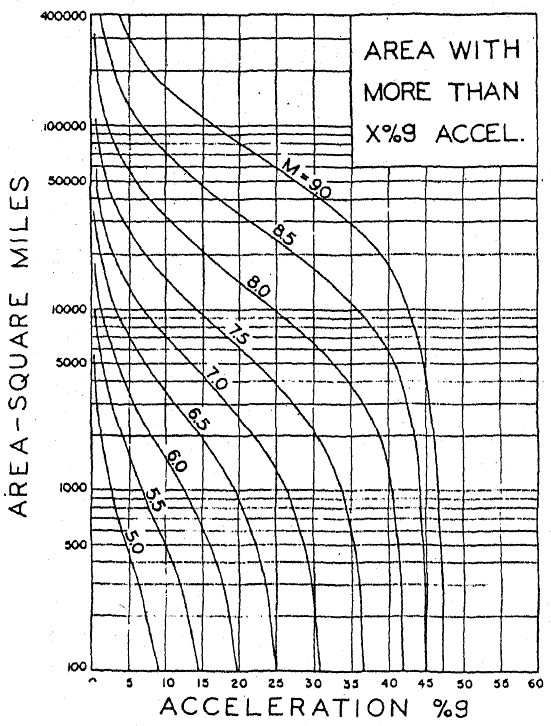

Figure 3. Area in square miles experiencing shaking of x%g or greater for shocks of different Magnitudes.

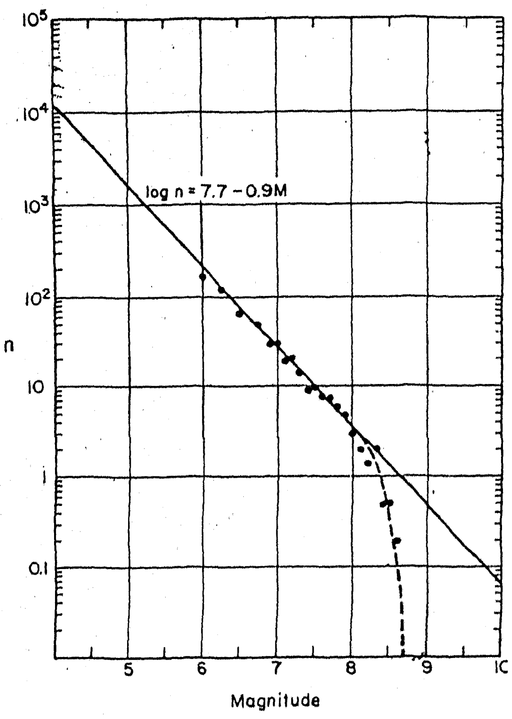

Figure 4. Average number of earthquakes per year in the world. The data points represent the averages of a 43-year period; ndM is the mean annual number of shocks having Magnitudes between M and M + dM.

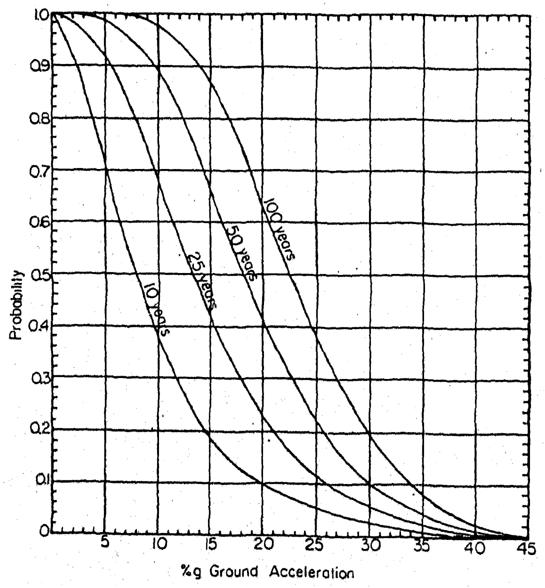

Figure 5. Probability of acceleration greater than x%g at a site in California (150,000 square miles), assuming earthquakes occur at random in the state.

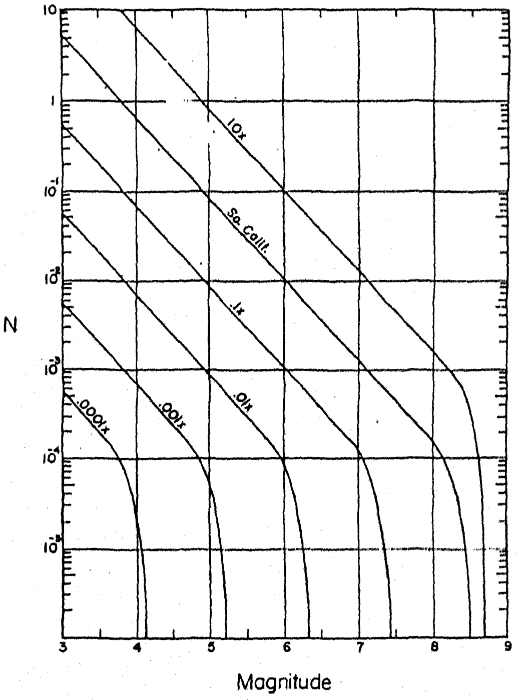

Figure 6. Plot of seismicity N, the number of earthquakes per square mile per year having Magnitudes between M - 1/2 and M + 1/2; that is,

$$N = \int_{M-\frac{1}{2}}^{M+\frac{1}{2}}$$

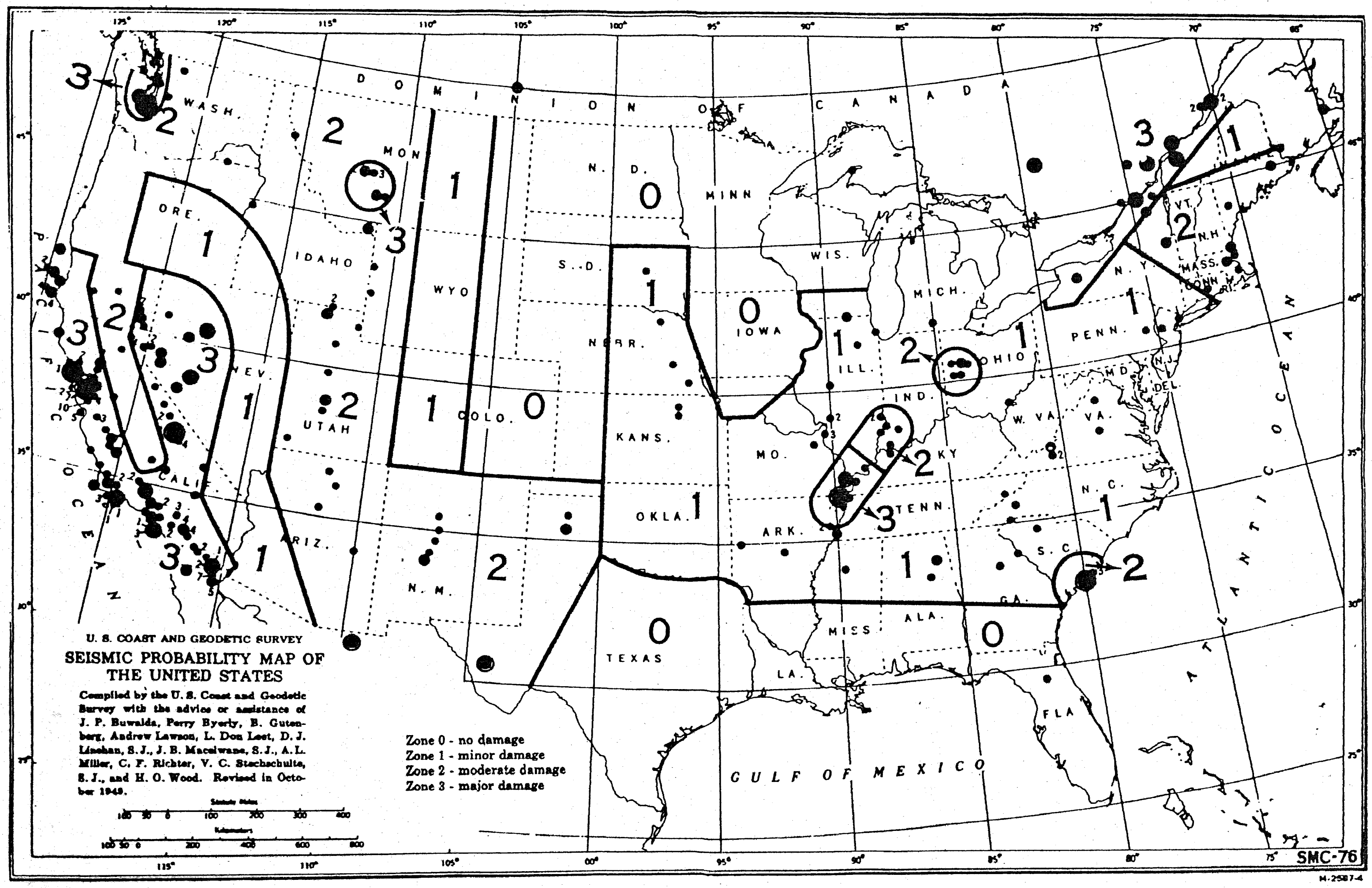

Figure 7. Seismic Probability Map showing the United States classified into four different zones of intensity of ground shaking.

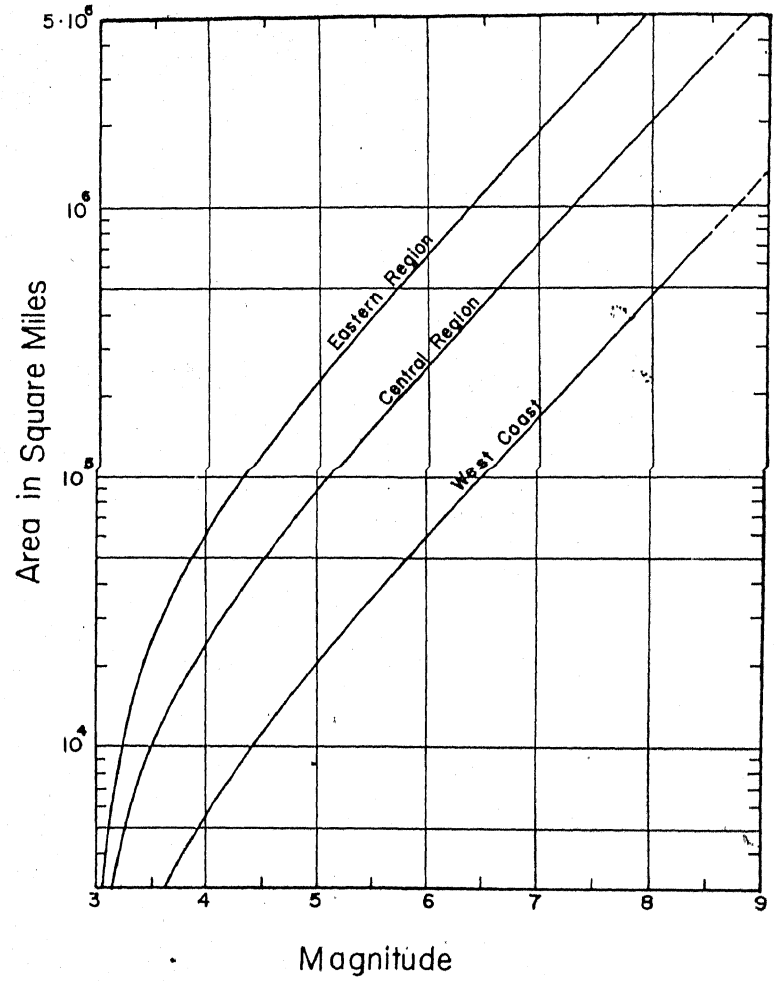

Figure 8. Idealized relation between Magnitude of an earthquake and the area over which it is felt. The relation is different in the different parts of the country.

TABLE I

Expectation of Earthquakes in California
(150,000 sq. miles)

Magnitude	No. per 100 Years
4.75-5.25	250
5.25-5.75	140
5.75-6.25	78
6.25-6.75	40
6.75-7.25	19
7.25-7.75	7.6
7.75-8.25	2.1
8.25-8.75	0.6

TABLE II

Estimated Maximum Zonal Accelerations

	Max. Accel.	M
Zone 3 (near a great fault)	50%g	8.5
Zone 3 (not near a great fault)	33%g	7.0
Zone 2	16%g	5.75
Zone 1	8%g	4.75
Zone 0	4%g	4.25

TABLE III

Upper Bound, M_2, for Magnitude as Determined by Seismicity Ratio

N_{o1}/N_{o2}	M_2
1	8.5
2	8.15
4	7.8
5	7.7
10	7.4
20	7.1
30	6.9
100	6.3